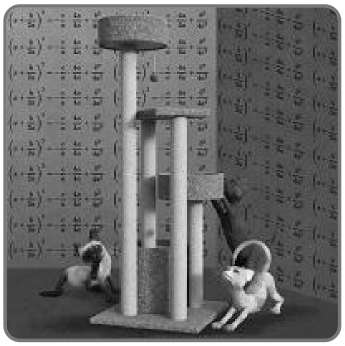
POINT-SLOPE FORM

- Want some other practice with lines?
  Introduction to the Slope of a Line
  Practice with Slope
  Graphing Lines
  Finding Equations of Lines


(more mathematical cats)

Suppose a line has slope m and passes through a known point (x_1, y_1).
That is, we ***know the slope of the line*** and we ***know a point on the line***.

We can get an equation that is ideally suited to these two pieces of information.
This equation is appropriately called the ***point-slope form of a line***.

Here's what to do:

- Recall that (x_1, y_1) is a known point on a line with slope m.
- Let (x, y) denote any other point on the line.
- Now, we have two points: the known point (x_1, y_1) and a 'generic' point (x, y).
- The slope of the line, computed using these two points, must equal m.
- Using the slope formula, we have:

$$m = \frac{y - y_1}{x - x_1}$$

or, equivalently,

$$y - y_1 = m(x - x_1)$$

This gives us an extremely useful equation of a line, as summarized below:

POINT-SLOPE FORM *line with slope* m*, passing through* (x_1, y_1)

The graph of the equation

$$y - y_1 = m(x - x_1)$$

is a line with slope m that passes through the point (x_1, y_1).
Since this equation is ideally suited to the situation where you know a ***point*** and a ***slope***, it is appropriately called ***point-slope form***.

IMPORTANT THINGS TO KNOW ABOUT POINT-SLOPE FORM:

- The ***variables*** in the equation $y - y_1 = m(x - x_1)$ are x and y.
  That is, this is an equation in two variables, x and y.
  Thus, its solution set is the set of all ordered pairs (x, y) that make it true.
- For a given equation:
  - the number m is a constant (a specific number) that represents the slope of the line;
  - the number x_1 (read as 'ex sub one') is a constant that represents the x-value of the known point;
  - the number y_1 (read as 'wye sub one') is a constant that represents the y-value of the known point
- As we vary the values of m, x_1, and y_1, we get lots of different equations.
  Here are some of them:

$y - 2 = 5(x - 3)$	($m = 5$, $x_1 = 3$, and $y_1 = 2$)
$y - \frac{1}{2} = \sqrt{2}(x - 3.4)$	($m = \sqrt{2}$, $x_1 = 3.4$, and $y_1 = \frac{1}{2}$)
$y = 5(x + 1)$	Rewrite the equation as: $y - 0 = 5(x - (-1))$ Thus, we see that: $m = 5$, $x_1 = -1$, and $y_1 = 0$

- So, even though the equation $y - y_1 = m(x - x_1)$ uses five different 'letters' (y, y_1, m, x, and x_1), they play very different roles:
  - x and y are the variables; they determine the nature of the solution set
  - m, x_1 and y_1 are called ***parameters***;
    they are constant in any particular equation, but vary from equation to equation.
- This is another beautiful example of the power/compactness of the mathematical language!
  The single equation $y - y_1 = m(x - x_1)$ actually describes an entire ***family*** of equations, which has infinitely-many members.
  We get the members of this family by choosing real numbers m, x_1 and y_1 to plug in.
- If you know the slope of a line and the y-intercept, then it's probably easiest to use slope-intercept form.
  But, if you know the slope of a line and a point that ***isn't*** the y-intercept, then it's easiest to use point-slope form.
- Remember—just as expressions have lots of different names, so do sentences.
  Every non-vertical line can be written in any of these forms:
  - point-slope form: $y - y_1 = m(x - x_1)$
  - slope-intercept form: $y = mx + b$
  - general form: $ax + by + c = 0$
- Here's an example. (Make sure you convince yourself that these are equivalent equations!)
  - point-slope form: $y - 2 = 5(x - 3)$
  - slope-intercept form: $y = 5x - 13$
  - general form: $5x - y - 13 = 0$

EXAMPLE:

Question:
Write the point-slope equation of the line with slope 5 that passes through the point $(3, -2)$.
Then, write the line in $y = mx + b$ form.

Solution:
Here, (x_1, y_1) is $(3, -2)$ and $m = 5$.
Substitution into $y - y_1 = m(x - x_1)$ gives:

$$y - (-2) = 5(x - 3)$$

y	minus	known y-value	equals	known slope	(	x	minus	known x-value	)
y	$-$	(-2)	$=$	5	(	x	$-$	3	)
y	$-$	y_1	$=$	m	(	x	$-$	x_1	)

Then, put it in slope-intercept form by solving for y:

$y - (-2) = 5(x - 3)$	(start with point-slope form)
$y + 2 = 5x - 15$	(simplify each side)
$y = 5x - 17$	(subtract 2 from both sides)