

THE DISTANCE FORMULA

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THE DISTANCE FORMULA

The distance between points (x_1, y_1) and (x_2, y_2) is given by *the Distance Formula*:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, x_1 (read as ‘ x sub 1’) denotes the x -value of the first point, and y_1 (read as ‘ y sub 1’) denotes the y -value of the first point. Similarly, x_2 and y_2 denote the x -value and y -value of the second point.

This formula involves differences (subtraction problems) that are squared. Notice that $(a - b)^2$ is equal to $(b - a)^2$.

Why?

Firstly, $a - b$ and $b - a$ are opposites, since $a - b = (-1)(b - a)$.

Secondly, when you square a number and its opposite, you get the same result.

Alternatively, you could FOIL both out and observe that they're the same:

$$(a - b)^2 = (a - b)(a - b) = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$$

$$(b - a)^2 = (b - a)(b - a) = b^2 - ab - ab + a^2 = a^2 - 2ab + b^2$$

Thus, in the Distance Formula, you can do the subtractions in any order you want, and it won't affect your result.

Also, because of the [Commutative Property of Addition](#), you can work with the x or y values first—your choice.

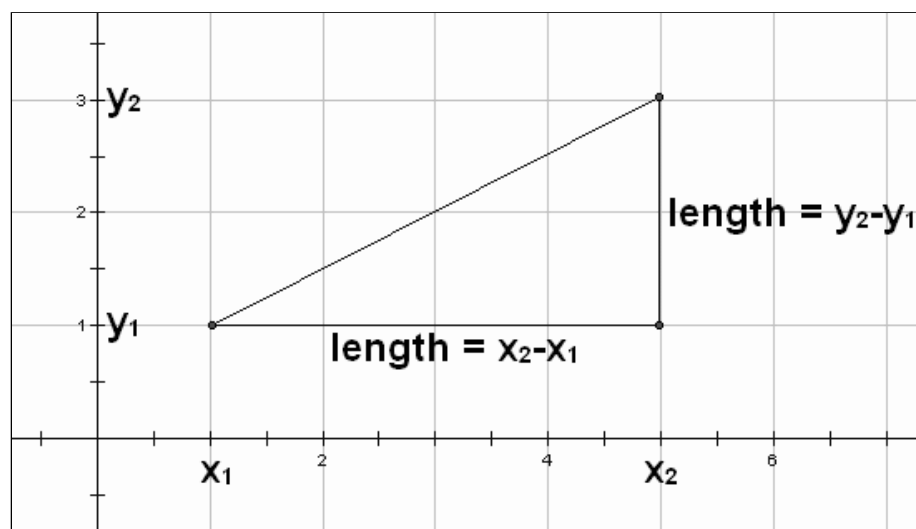
That is,

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

In words, to find the distance between two points, do the following:

- Subtract the x -values of the two points (in any order), and square this result.
- Subtract the y -values of the two points (in any order), and square this result.
- Add together the previous two quantities.
- Take the square root of the result.

The Distance Formula follows easily from the Pythagorean Theorem, as suggested by the picture below:



By the Pythagorean Theorem,

$$\text{hypotenuse}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Solve for the hypotenuse!

EXAMPLES:

Question: Find the distance between $(1, -3)$ and $(-2, 5)$.

Solution:

$$\begin{aligned} & \sqrt{(-2 - 1)^2 + (5 - (-3))^2} \\ &= \sqrt{(-3)^2 + 8^2} \\ &= \sqrt{9 + 64} \\ &= \sqrt{73} \end{aligned}$$

Question: Does the following formula represent the distance between points (a, b) and (c, d) ?
Answer YES or NO.

$$\sqrt{(a - c)^2 + (d - b)^2}$$

Solution: YES.

The order that you subtract the numbers does not affect the result.

Question: Does the following formula represent the distance between points (a, b) and (c, d) ?
Answer YES or NO.

$$\sqrt{(a - b)^2 + (c - d)^2}$$

Solution: NO.

You must subtract the x -values and y -values of the two points.

NOTE:

When the number inside a square root has a factor that is a perfect square, then it can be renamed.

For example, 4 goes into 8 evenly, and 4 is a perfect square.

Thus, we can rename as follows:

$$\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$$

In the solutions to this web exercise, both names ($\sqrt{8}$ and $2\sqrt{2}$) are given.