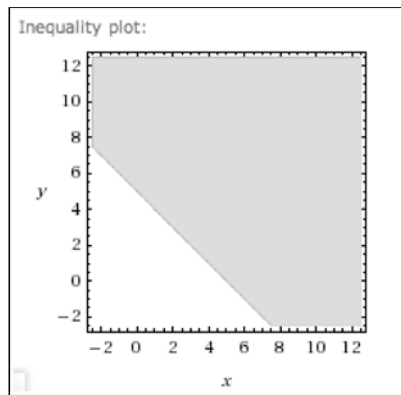


Here's what you'll see:



Of course, you can only see part of the graph—it goes on forever in all directions.

In this case, you're looking at all the points that are *on or above* the graph of $x + y = 5$.

Also, be careful—they're not showing the x -axis and the y -axis in this view:

the bottom line *isn't* the x -axis, and the left vertical line *isn't* the y -axis.

EXAMPLE (A MORE COMPLICATED EQUATION IN TWO VARIABLES)

The sentence $x^2 - 3xy + y^2 + 3x - 5y + 7 = 0$ is an equation in two variables. The variables can appear *any number of times*; there just can't be more than two different variables.

If you graph it at wolframalpha.com, type it in like this:

$$x^2 - 3xy + y^2 + 3x - 5y + 7 = 0$$

(You can just cut-and-paste, if you want.)

EXAMPLE (A TRICKY TYPE OF EQUATION IN TWO VARIABLES—AN 'INVISIBLE' VARIABLE)

One tricky type of 'sentence in two variables' is where you don't actually *see* two different variables, since one of them has a coefficient of 0.

What does this mean, exactly?

Consider the equation $x = 5$. It looks like there's only one variable, x .

Viewed as an equation in one variable, there's only one solution—the number 5.

In this case, the graph is very, very boring—a single dot, at location 5, on a number line.

However, the sentence $x = 5$ can also be viewed as an equation in two variables: $x + 0y = 5$.

We don't bother to write the $0y$, since it's just zero—but it changes the solution set completely.

Now, since it's an equation in two variables,

a solution is an ordered pair—a choice for x and a choice for y —that make the equation true.

What you notice pretty quickly is that x must be 5, but y can be anything:

$(5, 1)$ is a solution, since substituting 5 for x and 1 for y in ' $x + 0y = 5$ ' gives this true statement: $5 + 0(1) = 5$

Similarly, $(5, \frac{1}{2})$ is a solution, since $5 + 0(\frac{1}{2}) = 5$.

Indeed, $(5, \text{anything})$ is a solution, since $5 + 0(\text{anything}) = 5$.

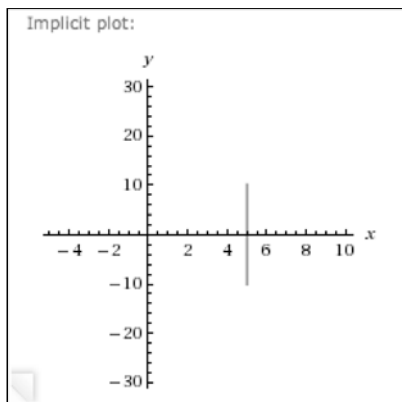
Thus, the solutions are ordered pairs of the form $(5, y)$, for all real numbers y .

What does this graph look like?

To get to any of these points, you start at the origin and move 5 units to the right.

Then, you can move up/down to your heart's content.

The graph is the vertical line that crosses the x -axis at 5.



Wolfram Alpha has a bit of trouble with this one.

Give it a break—it's a bit hard to see invisible things.

But, we can get a great approximation to the graph by being a bit clever.

Cut-and-paste the following into wolframalpha.com:

$$x + 0.00000001y = 5, -10 \leq y \leq 10$$

Notice that we've put a number really close to 0 in front of the y .

We're also specifying that we only want to see points

whose y -values are between -10 and 10 .

(Leave off the last part and see if you can figure out what's happening!)

By the way, wolframalpha.com can *plot* it easily with just one word's help:

plot $x = 5$

Try it!

So, what's a person to do when they see an equation like ' $x = 5$ '?

Treat it as an equation in one variable? In two variables? (In three variables!?)

Context, context, context.

If someone says 'graph $x = 5$ ' in high school,

then they're probably treating it as an equation in two variables.

If there's any doubt, just ask for clarification.

SOLUTIONS OF EQUATIONS/INEQUALITIES IN TWO VARIABLES

Let S denote an equation or inequality in two variables (x and y).

Then, the following are equivalent:

- (a, b) lies on the graph of S
- (a, b) satisfies S
- substitution of a for x , and b for y , makes the sentence S true

EXAMPLES:

Question: Does the point $(1, -2)$ lie on the graph of $2x + 3y = -4$?

Solution: Yes, since the equation ' $2(1) + 3(-2) = -4$ ' is true.

Question: Does the point $(0, -1)$ satisfy the equation $2x + 3y = -4$?

Solution: No, since the equation ' $2(0) + 3(-1) = -4$ ' is false.

Question: Does the point $(3, -1)$ lie on the graph of $x = 3$?

Solution: Yes, since substitution of 3 for x makes the equation true.

Observe that the equation $x = 3$ is being treated as an equation in two variables: $x + 0y = 3$

Here, we can ignore the y -value; all we care about is if x is equal to 3.

Question: Does the point $(3, 5)$ satisfy the inequality $y > 4$?

Solution: Yes, since substitution of 5 for y makes the inequality true.

Observe that the inequality $y > 4$ is being treated as an inequality in two variables: $0x + y > 4$

Here, we can ignore the x -value; all we care about is if y is greater than 4.

Question: Does the point $(3, 5)$ satisfy the inequality $x > 4$?

Solution: No, since substitution of 3 for x makes the inequality false.

Observe that the inequality $x > 4$ is being treated as an inequality in two variables: $x + 0y > 4$

Here, we can ignore the y -value; all we care about is if x is greater than 4.