There must be a better way—and there is.
It's called the 'Factor by Grouping' method,
and it's usually ***much*** more efficient than trial-and-error.

EXAMPLE: THE 'FACTOR BY GROUPING' METHOD

Here's an example of the 'factor by grouping' method.
Details follow, for inquiring minds.
However, some of you may want to just take this example, and run with it.

Factor: $6x^2 - 13x - 5$
Use the 'factor by grouping' method.
Solution:

- The number in front of x^2 isn't 1. Instead, it's a 6.
  So, we must modify the old method, which said:
  "find two numbers that multiply to the constant term, and add to the middle term"
  Somehow, we're going to have to use that 6.
- Here's how.
  Take the 6 and multiply it by the constant term, -5.
  You get $(6)(-5) = -30$.
- Now, find two numbers that multiply to -30 and that (still) add to -13.
- This should be a very familiar thought process—you've likely practiced it a lot.
  Use the PANS Memory Device, if you want.
- Come up with the two numbers that work—(thinking, thinking)—how about -15 and 2.
  Check them:
  Do they multiply to -30? $(-15)(2) = -30$ Check!
  Do they add to -13? $-15 + 2 = -13$ Check!
- Now, rename the original trinomial:

$6x^2 - 13x - 5$	(original trinomial)
$= 6x^2 \overbrace{-15x + 2x}^{=-13x} - 5$	(rename the middle term, using the two numbers you found)
$= (6x^2 - 15x) + (2x - 5)$	(group the first two terms, and the last two terms—hence the title of this technique!)
$= 3x(2x - 5) + 1(2x - 5)$	(factor the first part; put the 1 in for clarity in the second part)
$= 3x\,\overbrace{(2x - 5)}\,+1\,\overbrace{(2x - 5)}$	(Look carefully! There are two 'big' terms, with a common factor of $2x - 5$)
$= (2x - 5)(3x + 1)$	(factor out the common factor—done!)

It may ***seem*** like a lot of work to you.
But, over the years, I've divided my classes in half:
told one side to use trial-and-error, and the other side to use factor by grouping.
The factor by grouping side always wins.

Let's do the same example one more time.
This will show you that ***the order you write the two middle terms doesn't matter***.
Also, this version is much more compact, and is usually all you'll need to write down.

Factor: $6x^2 - 13x - 5$
Use the 'factor by grouping' method.
Solution:

- $(6)(-5) = -30$
  Find two numbers that multiply to -30 and add to -13.
- Try -15 and 2:
  $(-15)(2) = -30$ Check!
  $-15 + 2 = -13$ Check!
- Rename:

$6x^2 - 13x - 5$	(original trinomial)
$= 6x^2 \overbrace{+2x - 15x}^{\text{different order}} - 5$	(rename the middle term)
$= (6x^2 + 2x) + (-15x - 5)$	(group first two, last two)
$= 2x(3x + 1) + (-5)(3x + 1)$	(factor each group separately)
$= (3x + 1)(2x - 5)$	(factor again)

MOTIVATION FOR THE 'FACTOR BY GROUPING' METHOD

Here's the motivation for the technique.
Start by pulling a random problem out of the air, and FOILing it out:

$$(3x - 1)(5x + 7) = 15x^2 + 21x - 5x - 7$$

Don't combine the middle terms.
Instead, look at the four numbers generated in the resulting sum: 15, 21, -5, -7
The product of the first and last is: $(15)(-7) = -105$
The product of the middle two is: $(21)(-5) = -105$
Is this just a coincidence?

It's ***not*** a coincidence.
It's ***always*** true:

$$(ex + f)(gx + h) = (eg)x^2 + (eh)x + (fg)x + (fh)$$

The product of the first and last is: $(eg)(fh) = efgh$
The product of the middle two is: $(eh)(fg) = efgh$
Same result.
Other than being a curious observation, what good is this?

Ends up that it's a LOT of good.
If we're trying to factor a trinomial by grouping,
and we're looking for the right way to rename the middle term, then we know two things:

$$ax^2 + bx + c = ax^2 + \_\_\ x + \_\_\ x + c$$

- the two unknown numbers must add to b
- the two unknown numbers must multiply to ac

This is why we take the number in front of the x^2 term, and multiply it by the constant term!

The rest of the argument is more advanced, and is primarily included for the sake of the teacher.
Click here, if you're interested in taking a look.