SOLVING SIMPLE EQUATIONS INVOLVING PERFECT SQUARES

(more mathematical cats)

Here, you will solve simple equations involving perfect squares.

There are two basic approaches you can use.
They're both discussed thoroughly on this page.

Both approaches are illustrated next, using the equation $x^2 - 9 = 0$.

APPROACH #1 (FACTOR AND USE THE ZERO FACTOR LAW)

To use this approach, you must:

- Check that you have 0 on one side of the equation.
- Factor to get a ***product*** on the other side of the equation.
- Use the Zero Factor Law:
  For all real numbers a and b, $ab = 0$ is equivalent to $(a = 0 \text{ or } b = 0)$.

To use this approach, you would write the following list of equivalent sentences:

$x^2 - 9 = 0$	(original equation; check that 0 is on one side of the equation)
$(x+3)(x-3) = 0$	(factor to get a product on the other side)
$x + 3 = 0$ or $x - 3 = 0$	(use the zero factor law)
$x = -3$ or $x = 3$	(solve the simpler equations)

APPROACH #2 (USE THE FOLLOWING THEOREM)

THEOREM *solving equations involving perfect squares*

For all real numbers z and for $k \ge 0$:

$$z^2 = k \quad \text{is equivalent to} \quad z = \pm\sqrt{k}$$

Notice that if $k < 0$, then the equation $z^2 = k$ has no real number solutions.
For example, consider the equation $z^2 = -4$.
There is no real number which, when squared, gives -4.

To use this approach, you must:

- Isolate a perfect square on one side of the equation.
- Check that you have a ***nonnegative*** number on the other side.
- Use the theorem.

To use this approach, you would write the following list of equivalent sentences:

$x^2 - 9 = 0$	(original equation)
$x^2 = 9$	(isolate a perfect square by adding 9 to both sides)
$x = \pm\sqrt{9}$	(check that $k \geq 0$; use the theorem)
$x = \pm 3$	(rename: $\sqrt{9} = 3$)
$x = 3$ or $x = -3$	(expand the 'plus or minus' shorthand, if desired)

EXAMPLES:

Here are three slightly different approaches to solving the equation $16x^2 - 25 = 0$:

First Approach (use the Zero Factor Law)

$16x^2 - 25 = 0$	(original equation)
$(4x)^2 - 5^2 = 0$	(rewrite left-hand side as a difference of squares)
$(4x+5)(4x-5) = 0$	(factor the left-hand side)
$4x + 5 = 0$ or $4x - 5 = 0$	(use the Zero Factor Law)
$4x = -5$ or $4x = 5$	(solve the simpler equations)
$x = -\frac{5}{4}$ or $x = \frac{5}{4}$	(solve the simpler equations)

Second Approach (use the theorem; isolate x^2)

$16x^2 - 25 = 0$	(original equation)
$16x^2 = 25$	(add 25 to both sides)
$x^2 = \frac{25}{16}$	(divide both sides by 16: now, x^2 is isolated)
$x = \pm\sqrt{\frac{25}{16}}$	(use the theorem)
$x = \pm\frac{5}{4}$	(rename: $\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$)
$x = \frac{5}{4}$ or $x = -\frac{5}{4}$	(expand the 'plus or minus' shorthand, if desired)

Third Approach (use the theorem; isolate $(4x)^2$)

$16x^2 - 25 = 0$	(original equation)
$16x^2 = 25$	(add 25 to both sides)
$(4x)^2 = 25$	(rename left-hand side as a perfect square)
$4x = \pm\sqrt{25}$	(use the theorem)
$4x = \pm 5$	(rename: $\sqrt{25} = 5$)
$x = \frac{\pm 5}{4}$	(divide both sides by 4)
$x = \frac{5}{4}$ or $x = -\frac{5}{4}$	(expand the 'plus or minus' shorthand, if desired)