SOLVING MORE COMPLICATED EQUATIONS INVOLVING PERFECT SQUARES

- Need some simpler practice first?
  Solving Simple Equations involving Perfect Squares

(more mathematical cats)

Here, you will solve more complicated equations involving perfect squares.

As in the previous section, there are two basic approaches you can use. They're both discussed thoroughly on this page.

The two approaches are illustrated next, by solving the equation $(3x+2)^2 = 16$.

APPROACH #1 (FACTOR AND USE THE ZERO FACTOR LAW)

To use this approach, you must:

- Get 0 on one side of the equation.
- Factor to get a ***product*** on the other side of the equation.
- Use the Zero Factor Law:
  For all real numbers a and b, $ab = 0$ is equivalent to $(a = 0 \text{ or } b = 0)$.

To use this approach, you would write the following list of equivalent sentences:

$(3x+2)^2 = 16$	(original equation)
$(3x+2)^2 - 16 = 0$	(need 0 on one side; subtract 16 from both sides)
$(3x+2)^2 - 4^2 = 0$	(rewrite, so it's clear you have a difference of squares)
$(3x+2+4)(3x+2-4) = 0$	(factor the difference of squares)
$(3x+6)(3x-2) = 0$	(simplify)
$3x+6 = 0 \text{ or } 3x-2 = 0$	(use the zero factor law)
$3x = -6 \text{ or } 3x = 2$	(solve the simpler equations)
$x = -2 \text{ or } x = \dfrac{2}{3}$	(solve the simpler equations)

It's a good idea to check:

$$(3(-2)+2)^2 \stackrel{?}{=} 16$$
$$(-6+2)^2 \stackrel{?}{=} 16$$
$$(-4)^2 \stackrel{?}{=} 16$$
$$16 = 16 \quad \text{Check!}$$

$$(3(\tfrac{2}{3})+2)^2 \stackrel{?}{=} 16$$
$$(2+2)^2 \stackrel{?}{=} 16$$
$$(4)^2 \stackrel{?}{=} 16$$
$$16 = 16 \quad \text{Check!}$$

APPROACH #2 (USE THE FOLLOWING THEOREM)

THEOREM *solving equations involving perfect squares*

For all real numbers z and for $k \ge 0$:

$$z^2 = k \quad \text{is equivalent to} \quad z = \pm\sqrt{k}$$

The basic idea is that you're (correctly!) ‘undoing’ a square with the square root.
Notice that if $k < 0$, then the equation $z^2 = k$ has no real number solutions.
For example, consider the equation $z^2 = -4$.
There is no real number which, when squared, gives -4.

To use this approach, you must:

- Isolate a perfect square on one side of the equation.
- Check that you have a ***nonnegative*** number on the other side.
- Use the theorem.

To use this approach, you would write the following list of equivalent sentences:

$(3x+2)^2 = 16$	(original equation)
$3x+2 = \pm\sqrt{16}$	(check that $k \ge 0$; use the theorem)
$3x+2 = \pm 4$	(simplify: $\sqrt{16} = 4$)
$3x+2 = 4$ or $3x+2 = -4$	(expand the ‘plus or minus’ shorthand)
$3x = 2$ or $3x = -6$	(subtract 2 from both sides of both equations)
$x = \frac{2}{3}$ or $x = -2$	(divide both sides of both equations by 3)