FACTORING A DIFFERENCE OF SQUARES

- It may be helpful to review these exercises first:
  Recognizing Products and Sums; Identifying Factors and Terms talks about the concept of factoring, and Writing Expressions in the Form A^2 helps you rename expressions as squares.

(more mathematical cats)

Recall that ***factoring*** is the process of taking a sum/difference (things added/subtracted) and renaming it as a product (things multiplied).

An expression of the form $A^2 - B^2$ is called a ***difference of squares***.
It's a ***difference***, because the last operation being performed is subtraction.
It's a difference ***of squares***, because both A^2 and B^2 are squares.

Using FOIL:

$$(A+B)(A-B) = A^2 - AB + AB - B^2 = A^2 - B^2$$

Thus, we have the following result:

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For all real numbers A and B:

$$A^2 - B^2 = (A+B)(A-B)$$

EXAMPLES:

Factor: $x^2 - 4$
Solution: $x^2 - 4 = x^2 - 2^2 = (x+2)(x-2)$

Factor: $9 - x^2$
Solution: $9 - x^2 = 3^2 - x^2 = (3+x)(3-x)$

Factor: $4x^2 - 9$
Solution: $4x^2 - 9 = (2x)^2 - 3^2 = (2x+3)(2x-3)$

Factor: $49x^2 - 64y^2$
Solution: $49x^2 - 64y^2 = (7x)^2 - (8y)^2 = (7x+8y)(7x-8y)$

Factor: $x^6 - 25$
Solution: $x^6 - 25 = (x^3)^2 - 5^2 = (x^3+5)(x^3-5)$

Factor: $x^2 - 5$
Solution:
Since 5 is not a perfect square, this cannot be factored using integers.
Note that it ***can*** be factored if we're allowed to use non-integers:

$x^2 - 5 = x^2 - (\sqrt{5})^2 = (x + \sqrt{5})(x - \sqrt{5})$

In this exercise, you are ***factoring over the integers***.
That is, you are to use ***only the integers*** for your factoring.
Recall that the integers are: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Factor: $x^2 + 4$
Solution:
This can't be factored using integers.
Usually, a ***sum*** of squares can't be factored.

(☆ The remaining discussion is beyond the scope of Algebra I;
it is included for the benefit of more advanced readers.)

The expression $x^2 + 4$ can't even be factored using real numbers.
It ***can*** be factored if we're allowed to use numbers that aren't real:

$x^2 + 4 = x^2 - (2i)^2 = (x + 2i)(x - 2i)$, where $i^2 = -1$

In general, a sum of squares can't be factored.
However, a sum of squares might ***also*** be a sum of cubes, which ***is*** factorable, like this:

$x^6 + 64$

$= (x^3)^2 + 8^2$	(so, it's a sum of squares)
$= (x^2)^3 + 4^3$	(it's also a sum of cubes, which ***can*** be factored)
$= (x^2 + 4)(x^4 - 4x^2 + 16)$	(use this: $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$)

So, you can't just make a blanket statement that sums of squares aren't factorable.