29. GREATEST COMMON FACTOR

Don't ever forget what factoring is all about!

In the previous section, you practiced factoring simple expressions. Keep in mind that *factoring* is the process of taking a sum and rewriting it as a product. Also keep in mind that the Zero Factor Law is one important reason why so much time is spent exploring techniques for renaming expressions as products: if a product is zero, then we know something about the factors—at least one of them must be zero.

greatest common factor

In this section, the concept of *greatest common factor* is explored, which is useful not only for factoring, but also in many other areas of mathematics. First we explore the greatest common factor of numbers, and then extend the ideas to variable expressions. Finally, you'll practice factoring expressions that are more complicated than those in the previous section.

a motivating example: cutting three boards of different lengths into same-length pieces

Here's an example that illustrates the usefulness of the concept of *greatest common factor*.

Suppose you're asked to make wooden blocks for a school fund-raising project. You go out to the garage and find three pieces of lumber, which have lengths 30, 36 and 45 inches. You want to cut these up into blocks that are all the same length (some even number of inches, like 4 inches, not 4.5), and you want the blocks to be as long as possible. You don't want any waste. What length should you make the blocks?

You could make 3 blocks that are each 10 inches long from the 30 inch board. However, the 36 and 45 inch boards can't be divided into 10 inch lengths without having some shorter pieces left over.

You could make blocks that are 6 inches long from both the 30 inch and 36 inch boards, but you'd still have waste from the 45 inch board.

What's the longest length block that you can make without having any waste?

solving the problem:

possible lengths of blocks from the 30 *inch board*

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30. These give the possible lengths of blocks that you could make from the 30 inch board. (Assume units of inches for all the lengths.)

1 block of length 30 (or 30 blocks of length 1);

2 blocks of length 15 (or 15 blocks of length 2);

3 blocks of length 10 (or 10 blocks of length 3);

5 blocks of length 6 (or 6 blocks of length 5).

possible lengths of blocks from the 36 *inch board*

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. These give the possible lengths of blocks from the 36 inch board:

1 block of length 36 (or 36 blocks of length 1);

2 blocks of length 18 (or 18 blocks of length 2);

3 blocks of length 12 (or 12 blocks of length 3);

4 blocks of length 9 (or 9 blocks of length 4);

6 blocks of length 6.

possible lengths of blocks from the 45 *inch board*

The factors of 45 are 1, 3, 5, 9, 15, and 45. These give the possible lengths of blocks from the 45 inch board:

1 block of length 45 (or 45 blocks of length 1);

3 blocks of length 15 (or 15 blocks of length 3);

5 blocks of length 9 (or 9 blocks of length 5).

finding the greatest common number in all three lists:

the greatest common factor

By looking at the factors of all three numbers together, it is clear that we want the greatest number that appears in all three lists:

factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

factors of 45: 1, 3, 5, 9, 15, 45

The number 1 appears in all three lists, as does 3. That is, 1 and 3 are the *common factors* of 30, 36, and 45. The number 3 is the greatest number that appears in all three lists, so it is the *greatest common factor* of 30, 36, and 45, and is the answer to the block problem. The longest block you can make from boards of lengths 30, 36, and 45 inches, without having any waste, is 3 inches. Here, 3 is the greatest common factor of 30, 36, and 45.

$\text{gcf}(30, 36, 45) = 3$

The notation

$$\text{gcf}(30, 36, 45) = 3$$

is used to denote that the greatest common factor of 30, 36, and 45 is 3.

EXERCISES

1. In the previous example, *how many* 3 inch blocks can be made from the boards of lengths 30, 36 and 45 inches?
2. After making your first batch of blocks, you go down to the cellar and find three more boards. These have lengths 90 inches, 60 inches, and 35 inches. What's the largest length of block that you can make from *these* three boards? (Again, you don't want to have any waste.) And, how many blocks can be made this time?
3. What notation can you use to report the greatest common factor of 35, 60, and 90?

methods for finding the greatest common factor:

the list method

the prime factorization method

Next, methods are discussed for finding the greatest common factor of two or more numbers.

The method illustrated in the previous example is called the "list method," and it works fine if the numbers aren't too big and there aren't too many of them. We'll talk about an efficient way to come up with the lists of factors for this method. However, the "list method" is terribly inefficient and tedious as the numbers get bigger or if there are lots of them. We'll see that the "prime factorization method" works much better in this latter situation.

listing the factors of a number in an efficient way

There's an easy way to list all the factors of a number, which is illustrated next by listing all the factors of 30.

Recall that the number $\sqrt{30}$ lies between 5 and 6, since $5^2 = 25$ is too small, and $6^2 = 36$ is too big. Using your calculator, check that $\sqrt{30} \approx 5.5$.

whenever two positive integers multiply to give 30, one is less than $\sqrt{30}$ and one is greater than $\sqrt{30}$

The key idea is this: whenever two positive integers multiply together to give 30, then one of the factors is less than $\sqrt{30}$, and the other is greater than $\sqrt{30}$.

For example, $2 \cdot 15 = 30$. Here, $2 < \sqrt{30}$ and $15 > \sqrt{30}$.

As a second example, $5 \cdot 6 = 30$. Here, $5 < \sqrt{30}$ and $6 > \sqrt{30}$.

The reason for this situation is simple:

if both numbers being multiplied are less than $\sqrt{30}$, then the product would be less than 30.

If both numbers being multiplied are greater than $\sqrt{30}$, then the product would be greater than 30.

Thus, one number must be less than $\sqrt{30}$, and one must be greater than $\sqrt{30}$.

Watch how this idea is used in the technique illustrated next.

LISTING ALL THE FACTORS OF A NUMBER:

Step 1: locate the square root of the number

- First, locate the square root of 30.

  You can use your calculator if desired, to find that $\sqrt{30} \approx 5.5$. Thus, $\sqrt{30}$ lies between 5 and 6.

  Or, you can say: $5^2 = 25$, which is too small; $6^2 = 36$, which is too big. Thus, $\sqrt{30}$ lies between 5 and 6.

Step 2: write down the factors, in pairs.

In each pair: one number is $\le$ the square root; one number is $\ge$ the square root

- The factors of 30 occur as pairs of numbers that multiply to 30.

  In each pair, one number is less than or equal to $\sqrt{30}$ and one number is greater than or equal to $\sqrt{30}$.

  Start with the factor 1, then check 2, then check 3, and so on. You can stop as soon as you reach the first number that is greater than the square root.

  So, start at 1 and write down the resulting pairs:

  $\mathbf{1} \cdot 30 = 30$; 1 and 30 are factors

  $\mathbf{2} \cdot 15 = 30$; 2 and 15 are factors

  $\mathbf{3} \cdot 10 = 30$; 3 and 10 are factors

  4 doesn't go into 30 evenly;

  $\mathbf{5} \cdot 6 = 30$; 5 and 6 are factors

  The next number going up is **6**, which is greater than $\sqrt{30}$, so you can stop.

  In the web exercises, you'll be listing the factors in the order that they are generated by this method: 1, 30, 2, 15, 3, 10, 5, 6

  (If necessary, you could easily rearrange them in increasing order.)

a second example

Here's a second example, where all the factors of 36 are listed.

The square root of 36 is 6.

You need only check if 1 through 6 go in evenly, and write down the resulting pairs of factors.

factors of 36: 1, 36, 2, 18, 3, 12, 4, 9, 6

Note that $6 \cdot 6 = 36$. You only need list the factor 6 once.

listing all factors is inefficient

Listing all the factors is a good way to illustrate the concept of greatest common factor, but in practice is very inefficient. A more efficient method is presented after the following exercise.

EXERCISES

4. a. List all the factors of 24.
   b. List all the factors of 18.
   c. List all the common factors of 24 and 18.
   d. What is the greatest common factor of 24 and 18?
5. Use the list method to find the greatest common factor of 50 and 36.
6. Use the list method to find $\text{gcf}(18, 30, 42)$.

getting the greatest common factor from the prime factorizations

In an earlier section, we saw that an efficient way to find the least common multiple is by using the prime factorizations of the numbers. The prime factorizations also give an efficient way to find the greatest common factor. The idea is illustrated in the following example.

PRIME FACTORIZATION METHOD for finding the greatest common factor

STEP 1: factor each number into primes

EXAMPLE: Find the greatest common factor of 24, 30, and 84 .

- First, factor each number into primes.

  Remember that the easiest way to do this is to come up with *any* two factors first. Then, keep breaking things down until only primes are in the list.

  In the final step, rewrite the factors from least to greatest, using exponent notation.

  $24 = 3 \cdot 8 = 3 \cdot 2 \cdot 4 = 3 \cdot 2 \cdot 2 \cdot 2 \overbrace{=}^{\text{rewrite}} 2^3 \cdot 3$

  $30 = 5 \cdot 6 = 5 \cdot 2 \cdot 3 \overbrace{=}^{\text{rewrite}} 2 \cdot 3 \cdot 5$

  $84 = 4 \cdot 21 = 2 \cdot 2 \cdot 3 \cdot 7 \overbrace{=}^{\text{rewrite}} 2^2 \cdot 3 \cdot 7$

Some people like to use "trees" to find the prime factorizations:

```
   24              30              84
  /  \            /  \           /    \
 3    8          5    6         4      21
     / \             / \       / \    /  \
    2   4           2   3     2   2  3    7
       / \
      2   2
```

the prime factorization of a number determines all the numbers that go into it evenly

The prime factorization of a number determines all the numbers that go into it evenly.

For example, look at $30 = 2 \cdot 3 \cdot 5$. Only numbers formed by multiplying a single factor of 2, 3, and 5 will go into 30 evenly:

$2 \cdot 3 = 6$ goes into 30 evenly;

$2 \cdot 5 = 10$ goes into 30 evenly;

$3 \cdot 5 = 15$ goes into 30 evenly;

$2 \cdot 3 \cdot 5 = 30$ goes into 30 evenly;

$2^2 = 4$ does not go into 30 evenly, because it uses more than one factor of 2 .

7 does not go into 30 evenly, because 30 does not have a factor of 7 .

Thus, any number that goes into 30 evenly can use at most one factor of 2 .

Any number that goes into 30 evenly can use at most one factor of 3 .

Any number that goes into 30 evenly can use at most one factor of 5 .

EXERCISES

7. What does the phrase "at most one factor of 2" mean? That is, what does "at most 1" mean?
8. $24 = 2^3 \cdot 3$; any number that goes into 24 evenly can use at most three factors of 2 . What does the phrase "at most three factors of 2" mean?
9. $84 = 2^2 \cdot 3 \cdot 7$; any number that goes into 84 evenly can use at most two factors of 2 . What does the phrase "at most two factors of 2" mean?

Summarizing:

A factor of 24 can't use more than 3 factors of 2 .
A factor of 30 can't use more than 1 factor of 2 .
A factor of 84 can't use more than 2 factors of 2 .

If we want a factor of *all three* numbers (24, 30, and 84), then the factor can't use more than 1 factor of 2 . These observations lead to the following strategy for finding the greatest common factor from the prime factorizations.

STEP 2: take the fewest number of times that each prime factor appears

The prime factorizations of 24, 30, and 84 are repeated below, with exponents of 0 put in for any missing factors. (Recall that any nonzero number to the zero power is 1).

In each column, locate the lowest exponent.
In the 2's column, the lowest exponent is 1 (circle 2^1).
In the 3's column, the lowest exponent is 1 (circle 3^1).
In the 5's column, the lowest exponent is 0 (circle 5^0).
In the 7's column, the lowest exponent is 0 (circle 7^0).

To find the greatest common factor, multiply together the circled entries from each column. Thus, you are multiplying together the fewest number of times that each prime factor appears.

$$\begin{aligned}
24 &= 2^3 \cdot 3^1 \cdot 5^0 \cdot 7^0 \\
30 &= 2^1 \cdot 3^1 \cdot 5^1 \cdot 7^0 \\
84 &= 2^2 \cdot 3^1 \cdot 5^0 \cdot 7^1 \\
\text{fewest} &= 2^1 \cdot 3^1 \cdot 5^0 \cdot 7^0 \\
\text{GCF} &= 6
\end{aligned}$$

MEMORY DEVICE	To find the greatest common **F**actor, take the **F**ewest number of times that each prime factor appears. Also recall this fact from an earlier section: To find the least common **M**ultiple, take the **M**ost number of times that each prime factor appears.

EXAMPLE

the prime factorization method for finding the greatest common factor

Here's a second example, presented more compactly. Once you've done this a few times, you won't have to line up the factors, and you won't have to write down exponents of 0 . However, you may still want to circle the factors that will contribute to the greatest common factor.

EXAMPLE: Find the greatest common factor of 540, 72, and 264 .

$$\begin{aligned}
540 &= 2^2 \cdot 3^3 \cdot 5 \\
72 &= 2^3 \cdot 3^2 \\
264 &= 2^3 \cdot 3 \cdot 11 \\
\text{fewest} &= 2^2 \cdot 3 \\
\text{GCF} &= 12
\end{aligned}$$

So, $\text{gcf}(540, 72, 264) = 12$.

EXERCISES

10. Use the prime factorization method to find the greatest common factor of:
    a. 210, 90, and 270
    b. 90, 585, and 675

finding the greatest common factor of variable expressions

The ideas just discussed are now very easily extended to variable expressions. You just identify the variable factors, and then take the *fewest* number of times that each type of factor appears.

EXAMPLE

EXAMPLE: Find the greatest common factor of x^2y^2, xy^3, and xy^4z.

The factor types involved in these three expressions are x, y, and z.

The factor x appears **two** times in the expression x^2y^2, **one** time in xy^3, and **one** time in xy^4z. The **fewest** number of times is **one**. The greatest common factor will use **one** factor of x.

The factor y appears **two** times in the expression x^2y^2, **three** times in xy^3, and **four** times in xy^4z. The **fewest** number of times is **two**. The greatest common factor will use **two** factors of y; that is, y^2.

The factor z appears **zero** times in the expression x^2y^2, **zero** times in xy^3, and **one** time in xy^4z. The **fewest** number of times is **zero**. The greatest common factor will use **zero** factors of z. That is, no z will appear in the greatest common factor.

Thus, $\text{gcf}(x^2y^2, xy^3, xy^4z) = xy^2$.

EXERCISES

11. Find the greatest common factor for each set of variable expressions:
    a. $xy^3z^5,\ \ y^2z^3,\ \ x^2y^3z^4$
    b. $abc^4,\ \ a^3bc^2,\ \ a^2b^3c$
    c. $x^4y^2z,\ \ x^5y,\ \ x^4z^4$

both numbers and variables

When both numbers and variables are involved, just use the greatest common factor of the numbers, and the greatest common factor of the variables:

EXAMPLE

EXAMPLE: Find the greatest common factor of $24x^2y^2$ and $30xy^2$.

Answer: $\text{gcf}(24x^2y^2, 30xy^2) = 6xy^2$

EXERCISES

12. Find the greatest common factor for each set of variable expressions:
    a. $6x^2,\ \ 2x,\ \ 10x^3$
    b. $15a^2b,\ \ 30ab^4$
    c. $8x^3y^5,\ \ 4xy^4,\ \ 16x^2y^7$

factoring out the greatest common factor

Here are some examples of factoring variable expressions. Find the greatest common factor, and then see what's left from each term.

EXAMPLE

factoring;

writing down an intermediate step

In this first example, an intermediate step is shown where each term is rewritten, showing the greatest common factor and what's left over. Eventually, you won't need to write down this intermediate step.

$$\begin{aligned} x^2y^2 + xy^3 + xy^4z &= \overbrace{xy^2}^{\text{gcf}} \cdot x + \overbrace{xy^2}^{\text{gcf}} \cdot y + \overbrace{xy^2}^{\text{gcf}} \cdot y^2z \qquad \text{(the gcf is } xy^2\text{)} \\ &= xy^2(x + y + y^2z) \end{aligned}$$

You should always check your result by multiplying it out! Usually, you just do the check mentally—don't write it down:

$$xy^2(x + y + y^2z) = x^2y^2 + xy^3 + xy^4z$$

It checks!

EXAMPLE

writing down the intermediate step;

both numbers and variables

Here's an example of writing down the 'intermediate step' when both numbers and variables are involved:

$$\begin{aligned} 6ab^2 - 18a^2b^3 + 6b^2 &= \overbrace{6b^2}^{\text{gcf}} \cdot a - \overbrace{6b^2}^{\text{gcf}} \cdot 3a^2b + \overbrace{6b^2}^{\text{gcf}} \cdot 1 \\ &= 6b^2(a - 3a^2b + 1) \end{aligned}$$

Notice the ' 1 ' that remains when the entire term is factored out.

EXERCISES

13. (Compare with Exercise 12). Factor out the greatest common factor. Write the 'intermediate step' that shows the greatest common factor, and what's left.
    a. $6x^2 - 2x + 10x^3$
    b. $15a^2b - 30ab^4$
    c. $-8x^3y^5 + 4xy^4 - 16x^2y^7$

EXAMPLE

factoring a simple expression in one step;

the thought process

Eventually, you should be able to factor fairly simple expressions in one step, like this:

$$30xy^3 - 24xy^2 = 6xy^2(5y - 4)$$

Here's the thought process involved:

- Identify the greatest common factor, which is $6xy^2$.
- Write down the greatest common factor. Open up a parenthesis.
- What's left from the first term, $30xy^3$?

  $\frac{30}{6} = 5$; write down the 5 .

  The x was factored out of $30xy^3$, so it's gone.

  There were 3 factors of y in $30xy^3$; 2 were factored out, leaving 1 factor of y ; write down the y .
- Write down the minus sign.
- What's left from $24xy^2$?

  $\frac{24}{6} = 4$; write down the 4 .

  The x was factored out, so it's gone.

  The y^2 was factored out, so it's gone. No variable part is left.
- Close the parenthesis.

EXERCISES

14. (Compare with Exercise 13). Factor out the greatest common factor, in one step. Use the thought process illustrated in the previous example.
    a. $6x^2 - 2x + 10x^3$
    b. $15a^2b - 30ab^4$
    c. $-8x^3y^5 + 4xy^4 - 16x^2y^7$

factoring in stages when things are complicated

In practice, people don't usually waste much time finding the *greatest* common factor, especially when the numbers are quite large. Instead, they just find *any* common factor, and factor it out. What's left inside the parentheses is then simpler, and the process is repeated:

$$\begin{aligned} 54x^3y^7 - 150xy^4 &= 2xy^4(27x^2y^3 - 75) \\ &= (2xy^4)(3)(9x^2y^3 - 25) \\ &= 6xy^4(9x^2y^3 - 25) \end{aligned}$$

Here's the thought process that was used:

- The numbers 54 and 150 are too big to waste time trying to find the greatest common factor; instead, just notice that 2 goes into each evenly. Factor out the 2, and the variable part.
- The numbers 27 and 75 that remain clearly have another common factor of 3. Factor it out.
- Multiply together both parts that were factored out.

In practice, the process is usually shortened like this:

$$\begin{aligned} 54x^3y^7 - 150xy^4 &= 2xy^4(27x^2y^3 - 75) \\ &= 6xy^4(9x^2y^3 - 25) \end{aligned}$$

EXERCISES

15. Factor in stages, as illustrated in the previous example. Be sure to write a complete mathematical sentence!
    a. $120x^2y - 105xy^2$
    b. $154ab^3 + 98b^5$

solving a simple equation

This section is concluded with an example that shows how factoring and the Zero Factor Law are used together to solve a simple equation. However, two other tools are needed in this example. These tools are explored in the next two sections: the Addition Property of Equality and the Multiplication Property of Equality. Each step in the process is numbered and commented on below:

EXAMPLE: Solve: $6x^2 = 3x$.

SOLUTION:

$$\begin{aligned} &6x^2 = 3x && (1)\\ &6x^2 - 3x = 0 && (2)\\ &3x(2x-1) = 0 && (3)\\ &3x = 0 \ \text{ or } \ 2x - 1 = 0 && (4)\\ &x = 0 \ \text{ or } \ 2x = 1 && (5)\\ &x = 0 \ \text{ or } \ x = \frac{1}{2} && (6) \end{aligned}$$

(1) Write the original equation.

(2) Get 0 on one side, by subtracting $3x$ from both sides. This uses the Addition Property of Equality, studied in the next section.

(3) Factor the left-hand side.

(4) Use the Zero Factor Law. Notice the mathematical word 'or'.

(5) Divide both sides of $3x = 0$ by 3 ; add 1 to both sides of $2x - 1 = 0$. This uses both the Addition and Multiplication Properties of Equality.

(6) Divide both sides of $2x = 1$ by 2 . This uses the Multiplication Property of Equality.

The solutions of $6x^2 = 3x$ are $x = 0$ and $x = \frac{1}{2}$. Check:

$$\begin{gathered} 6 \cdot 0^2 \stackrel{?}{=} 3 \cdot 0\\ 0 = 0 \end{gathered}$$

$$\begin{gathered} 6(\frac{1}{2})^2 \stackrel{?}{=} 3(\frac{1}{2})\\ 6(\frac{1}{4}) \stackrel{?}{=} \frac{3}{2}\\ \frac{3}{2} = \frac{3}{2} \end{gathered}$$

They check!

Soon, you'll be solving equations like these—and many more!

EXERCISES *web practice*	Go to my homepage **http://onemathematicalcat.org** and navigate to my Algebra I course, which has about 170 sequenced lessons. It can be used as a complete year-long high school course, or one semester in college. You're currently looking at the pdf version—you'll see that the HTML version has unlimited, randomly-generated, online and offline practice in every section. It's all totally free. Enjoy!

SOLUTION TO EXERCISES: GREATEST COMMON FACTOR

1. $\frac{30}{3} = 10$, $\frac{36}{3} = 12$, and $\frac{45}{3} = 15$, so you can make $10 + 12 + 15 = 37$ blocks, each of length 3 inches, from the three boards.

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2. factors of 35: 1, 5, 7, 35

factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

factors of 90: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

$\text{gcf}(35, 60, 90) = 5$; so the longest length block you can make (with no waste) is 5 inches.

$\frac{35}{5} = 7$, $\frac{60}{5} = 12$, and $\frac{90}{5} = 18$, so you can make $7 + 12 + 18 = 37$ blocks, each of length 5 inches, from the three boards.

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3. $\text{gcf}(35, 60, 90) = 5$

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4. a. factors of 24: 1, 24, 2, 12, 3, 8, 4, 6

b. factors of 18: 1, 18, 2, 9, 3, 6

c. common factors of 24 and 18: 1, 2, 3, 6

d. the greatest common factor of 24 and 18 is 6

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5. factors of 50: 1, 50, 2, 25, 5, 10

factors of 36: 1, 36, 2, 18, 3, 12, 4, 9, 6

the greatest common factor of 50 and 36 is 2

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6. factors of 18: 1, 18, 2, 9, 3, 6

factors of 30: 1, 30, 2, 15, 3, 10, 5, 6

factors of 42: 1, 42, 2, 21, 3, 14, 6, 7

$\text{gcf}(18, 30, 42) = 6$

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7. "at most 1" means "≤ 1"; in the current context, where we're counting the number of factors, only whole numbers are being allowed. Thus, "at most 1" means 1 or 0. There can be 1 factor of 2, or 0 (no) factors of 2.

8. "at most three factors of 2" means ≤ 3 factors of 2: 3 factors, or 2 factors, or 1 factor, or 0 factors

9. "at most two factors of 2" means ≤ 2 factors of 2: 2 factors, or 1 factor, or 0 factors

10a.

$$\begin{aligned}210 &= 21\cdot 10 = 3\cdot 7\cdot 2\cdot 5 = 2\cdot 3\cdot 5\cdot 7\\ 90 &= 9\cdot 10 = 3\cdot 3\cdot 2\cdot 5 = 2\cdot 3^2\cdot 5\\ 270 &= 27\cdot 10 = 3\cdot 9\cdot 2\cdot 5 = 3\cdot 3\cdot 3\cdot 2\cdot 5 = 2\cdot 3^3\cdot 5\end{aligned}$$

$\text{gcf}(210, 90, 270) = 2\cdot 3\cdot 5 = 30$

10b.

$$\begin{aligned}90 &= 9\cdot 10 = 3\cdot 3\cdot 2\cdot 5 = 2\cdot 3^2\cdot 5\\ 585 &= 5\cdot 117 = 5\cdot 3\cdot 39 = 5\cdot 3\cdot 3\cdot 13 = 3^2\cdot 5\cdot 13\\ 675 &= 5\cdot 135 = 5\cdot 5\cdot 27 = 5\cdot 5\cdot 3\cdot 3\cdot 3 = 3^3\cdot 5^2\end{aligned}$$

$\text{gcf}(90, 585, 675) = 3^2\cdot 5 = 45$

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11. a. $\text{gcf}(xy^3z^5, y^2z^3, x^2y^3z^4) = y^2z^3$

b. $\text{gcf}(abc^4, a^3bc^2, a^2b^3c) = abc$

c. $\text{gcf}(x^4y^2z, x^5y, x^4z^4) = x^4$

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12. a. $\text{gcf}(6x^2, 2x, 10x^3) = 2x$

b. $\text{gcf}(15a^2b, 30ab^4) = 15ab$

c. $\text{gcf}(8x^3y^5, 4xy^4, 16x^2y^7) = 4xy^4$

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13. a. $6x^2 - 2x + 10x^3 = 2x\cdot 3x - 2x\cdot 1 + 2x\cdot 5x^2 = 2x(3x - 1 + 5x^2)$

b. $15a^2b - 30ab^4 = 15ab\cdot a - 15ab\cdot 2b^3 = 15ab(a - 2b^3)$

c. $-8x^3y^5 + 4xy^4 - 16x^2y^7 = 4xy^4\cdot(-2x^2y) + 4xy^4\cdot 1 - 4xy^4\cdot 4xy^3 = 4xy^4(-2x^2y + 1 - 4xy^3)$

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14. a. $6x^2 - 2x + 10x^3 = 2x(3x - 1 + 5x^2)$

b. $15a^2b - 30ab^4 = 15ab(a - 2b^3)$

c. $-8x^3y^5 + 4xy^4 - 16x^2y^7 = 4xy^4(-2x^2y + 1 - 4xy^3)$

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15. a. $120x^2y - 105xy^2 = 5xy(24x - 21y) = (5xy)(3)(8x - 7y) = 15xy(8x - 7y)$

b. $154ab^3 + 98b^5 = 2b^3(77a + 49b^2) = (2b^3)(7)(11a + 7b^2) = 14b^3(11a + 7b^2)$