5. SUBTRACTION OF SIGNED NUMBERS

every subtraction problem is an addition problem in disguise

In this section, we'll study problems like $-3-(-5)$; that is, problems of the form $x-y$. The good news is that every subtraction problem is an addition problem in disguise! In one easy step, every subtraction problem is changed to an addition problem, which you already know how to solve.

opposites

To understand subtraction, you must understand *opposites*. Recall that opposites are the same distance from zero, but on opposite sides of zero. Thus, the opposite of 3 is -3, the opposite of -5 is 5, and the opposite of 0 is 0.

EXERCISES

1. Find the opposites:
   a. the opposite of -3 is _____
   b. the opposite of 2 is _____
   c. the opposite of $-\frac{1}{2}$ is _____
   d. the opposite of 0 is _____

recognizing subtraction problems

It's important that you can recognize subtraction problems, and read them aloud correctly. There are several things that you should notice as you study the examples below:

- when a negative number is being subtracted, it goes inside parentheses
- the subtraction sign is read as 'minus'
- a negative number like -3 is read as 'negative three'
- even though the same symbol ' $-$ ' is used both for subtraction and for finding the opposite of a number, it is read in different ways

EXAMPLES:

$3-5$: the number being subtracted is 5 ; read aloud as 'three minus five'

$2-(-3)$: the number being subtracted is -3 ; read aloud as 'two minus negative three'

$-1-6$: the number being subtracted is 6 ; read aloud as 'negative one minus six'

$-2-(-7)$: the number being subtracted is -7 ; read aloud as 'negative two minus negative seven'

EXERCISES

2. Identify the number being subtracted, and state how the problem should be read aloud:
   a. $-1-3$
   b. $1-(-3)$
   c. $1-3$
   d. $-1-(-3)$

to subtract a number, add its opposite

To subtract a number, you add its opposite. To subtract 3, you add -3. To subtract -3, you add 3.

That is, $x-y=x+(-y)$ for all real numbers x and y. In words, 'x minus y equals x plus the opposite of y'. Here's a diagram of the sentence:

$$x \overbrace{-\ y}^{\text{to subtract } y,} = x \overbrace{+\ (-y)}^{\text{add the opposite of } y}$$

three steps in a subtraction problem

There are three steps in a subtraction problem. These steps are illustrated using the example $-3-(-5)$:

(1) identify the number being subtracted (Answer: -5)

(2) find the opposite of the number being subtracted (Answer: 5)

(3) rewrite the subtraction problem as addition of the opposite

(Answer: $-3-(-5)=-3+5=2$)

EXERCISES

SAMPLE:

Consider the subtraction problem $3-(-6)$:

What number is being subtracted? Answer: -6

What is the opposite of the number being subtracted? Answer: 6

To subtract the number -6, instead add the number 6 .

Thus, $3-(-6)=3+6=9$.

3. Consider the subtraction problem $2-(-3)$:

What number is being subtracted? _____

What is the opposite of the number being subtracted? _____

To subtract the number _____ instead add the number _____.

Thus, $2-(-3)=$ _____.

4. Consider the subtraction problem $-2-3$:

What number is being subtracted? _____

What is the opposite of the number being subtracted? _____

To subtract the number _____ instead add the number _____.

Thus, $-2-3=$ _____.

more examples

For a familiar problem like $5-1=4$, you certainly won't need to use the process outlined above, but you should understand that it still works.

Consider $5-1$:

(1) identify the number being subtracted (Answer: 1)

(2) find the opposite of the number being subtracted (Answer: -1)

(3) rewrite the subtraction problem as addition of the opposite

(Answer: $5-1=5+(-1)=4$)

more than two numbers

Here are a couple problems with more than two numbers. Notice that every subtraction is turned into an addition in the first step.

$$\begin{aligned} -3-5+(-2)-(-7)+4 &= -3+(-5)+(-2)+7+4 \\ &= -10+11 \\ &= 1 \end{aligned}$$

$$\begin{aligned} 2-(-3)-1+5+(-4) &= 2+3+(-1)+5+(-4) \\ &= 10+(-5) \\ &= 5 \end{aligned}$$

EXERCISES *web practice*	Go to my homepage **https://onemathematicalcat.org** and navigate to my Algebra I course, which has about 170 sequenced lessons. It can be used as a complete year-long high school course, or one semester in college. You're currently looking at the pdf version—you'll see that the HTML version has unlimited, randomly-generated, online and offline practice in every section. It's all totally free. Enjoy!

SOLUTIONS TO EXERCISES:
SUBTRACTION OF SIGNED NUMBERS

1. a. the opposite of -3 is 3

b. the opposite of 2 is -2

c. the opposite of $-\frac{1}{2}$ is $\frac{1}{2}$

d. the opposite of 0 is 0

2. a. $-1-3$: the number being subtracted is 3; read aloud as 'negative one minus three'

b. $1-(-3)$: the number being subtracted is -3; read aloud as 'one minus negative three'

c. $1-3$: the number being subtracted is 3; read aloud as 'one minus three'

d. $-1-(-3)$: the number being subtracted is -3; read aloud as 'negative one minus negative three'

3. Consider the subtraction problem $2-(-3)$:

What number is being subtracted? Answer: -3

What is the opposite of the number being subtracted? Answer: 3

To subtract the number -3 instead add the number 3.

Thus, $2-(-3)=2+3=5$.

4. Consider the subtraction problem $-2-3$:

What number is being subtracted? Answer: 3

What is the opposite of the number being subtracted? Answer: -3

To subtract the number 3 instead add the number -3.

Thus, $-2-3=-2+(-3)=-5$.