17. ROUNDING

when an approximation suffices, the most common request is . . .

When you're solving a problem where an approximation suffices for the answer, the most commonly-requested approximation looks something like this:

'please round to 2 decimal places'

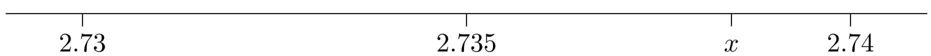
or equivalently,

'please round to the hundredths place'.

The purpose of this section is to discuss the concept and the technique of *rounding*.

first example

Let's start with an example. Suppose you're asked to round $x = 2.73845$ to two decimal places. The number 2.73845 lies between 2.73 and 2.74 on the number line. Indeed, it lies just about here:



choose the closest candidate

The two 'candidates' for the desired approximation are 2.73 and 2.74.

Which candidate is x closest to?

Answer: 2.74

So we say:

2.73845, rounded to 2 decimal places, is 2.74

or equivalently,

2.73845, rounded to the hundredths place, is 2.74.

rounding up

You could now be asked the follow-up question:

'Did you round up, or round down?'

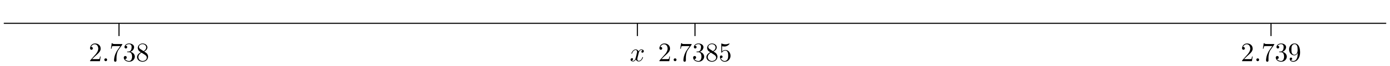
When the candidate to the right of the original number is chosen, we say that we are *rounding up*, because we're moving 'up' (to the right) on the number line to get our approximation.

round to n decimal places	To 'round to n decimal places' means to choose the *closest* neighbor that uses exactly n decimal places.

second example

As a second example, suppose you're asked to round $x = 2.73845$ to 3 decimal places.

The number 2.73845 lies between 2.738 and 2.739. Indeed, it lies just about here:



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This time, it lies closest to 2.738 , so we say:

2.73845 , rounded to 3 decimal places, is 2.738

or equivalently,

2.73845 , rounded to the thousandths place, is 2.738 .

rounding down

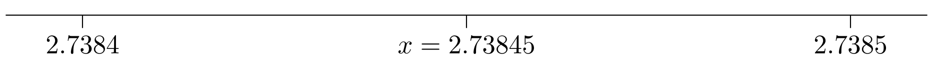
When the candidate to the left of the original number is chosen, we say that we are *rounding down*, because we're moving 'down' (to the left) on the number line to get our approximation.

EXERCISES

1. State the two candidates for each rounding problem. Which candidate is the number closest to?
   a. Round 4.937 to 2 decimal places.
   b. Round 943.009241 to 5 decimal places.
   c. Round 0.069238 to 1 decimal place.

What happens if the two choices are equidistant?

If asked to round $x = 2.73845$ to 4 decimal places, a slight problem arises:



if candidates are equidistant, round up

In this case, the two candidates are equidistant. Which one should be chosen? The normal convention is to round UP in this situation—that is, choose the candidate on the right. So,

2.73845 , rounded to 4 decimal places, is 2.7385

or equivalently,

2.73845 , rounded to the ten-thousandths place, is 2.7385

efficient technique for rounding to n decimal places

The ideas illustrated in the previous examples can be efficiently implemented as follows. The procedure is illustrated with the problem:

Round 2.73845 to 2 decimal places.

To round to n decimal places, do the following:

rounding digit

- Go to the digit in the n^{th} decimal place. This will be called the *rounding digit*.

  Example: Because we're rounding to 2 decimal places, the rounding digit is 3 :

$$2.7\underline{3}845$$

↑

rounding digit

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decider digit

- Look at the next digit to the right. This will be called the *decider digit*.

  Example: The decider digit is 8:

$$2.7\underline{3}\overset{\curvearrowright}{8}45$$

decider digit

decider digit < 5, *round down*

- If the decider digit is less than 5, round DOWN. That is, the rounding digit remains the same, and all the digits to the right are dropped.

decider digit ≥ 5, *round up*

- If the decider digit is 5 or more, round UP. That is, the rounding digit increases by 1, and all the digits to the right are dropped.

  Example: $2.73845 \cong 2.74$

What if the rounding digit is 9 *and you're rounding up?*

If the rounding digit is 9 and you're rounding up, then the rounding digit becomes 0, and the next digit to the left increases by 1. You may need to apply this rule more than once.

Whenever a 9 *must be increased by* 1, *then the* 9 *turns into a* 0, *and the next digit to the left increases by* 1.

Here are some examples:

293.4991927, rounded to 3 decimal places, is 293.499. (The decider digit is 1; round down.)

293.4699927, rounded to 3 decimal places, is 293.470. (The decider digit is 9; round up.) Notice that you must include the '0' in the thousandths place.

293.4999927, rounded to 3 decimal places, is 293.500. (The decider digit is 9; round up.) Notice that you must include the two trailing zeros.

999.9999992, rounded to 6 decimal places, is 999.999999. (The decider digit is 2; round down.)

999.9999998, rounded to 6 decimal places, is 1000.000000. (The decider digit is 8; round up.) Notice that you must include all the trailing zeros.

CAREFUL!

Be careful! When you are asked to round a number to n decimal places, then your answer MUST have n decimal places, even if there are zeros in those positions.

EXERCISES
web practice

Go to my homepage `https://onemathematicalcat.org` and navigate to my Algebra I course, which has about 170 sequenced lessons. It can be used as a complete year-long high school course, or one semester in college. You're currently looking at the pdf version—you'll see that the HTML version has unlimited, randomly-generated, online and offline practice in every section. It's all totally free. Enjoy!

156.523, rounded to the nearest whole number

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SOLUTION TO EXERCISES: ROUNDING

1. a. The candidates are 4.93 and 4.94; the number is closest to 4.94.

b. The candidates are 932.00924 and 932.00925; the number is closest to 932.00924.

c. The candidates are 0.0 and 0.1; the number is closest to 0.1.

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