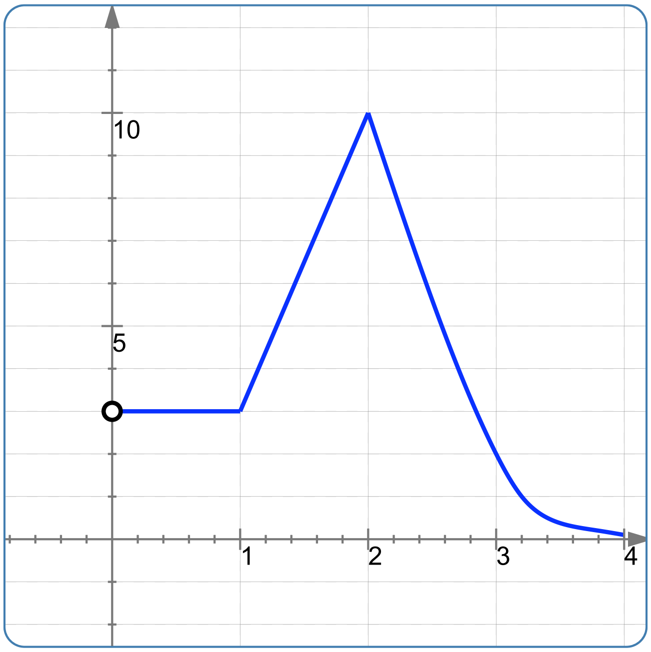
NAME: (1 pt)	NUMBER: (1 pt)

MAT 136, Dr. Carol JVF Burns, Final Exam
This final exam will be scaled so that it is 20% of your final grade.
This exam is closed book, closed notes, closed neighbor, and open mind.
Only a basic, four-function calculator is allowed (but is ***not*** required).
Show work leading to answers to receive full credit. Good luck!

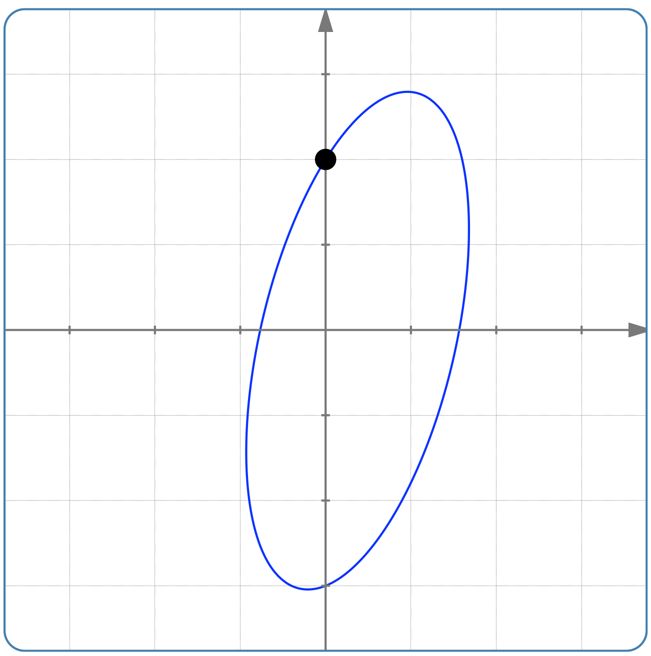
1. Let $f(x) = 3x^2 - 5$.

- (2 pts) Find $f'(x)$ using differentiation shortcuts.
- (6 pts) Use the limit definition of derivative to find $f'(x)$.

2. (6 pts) The graph of a function f is given below.
In the space provided, graph its derivative, f'.



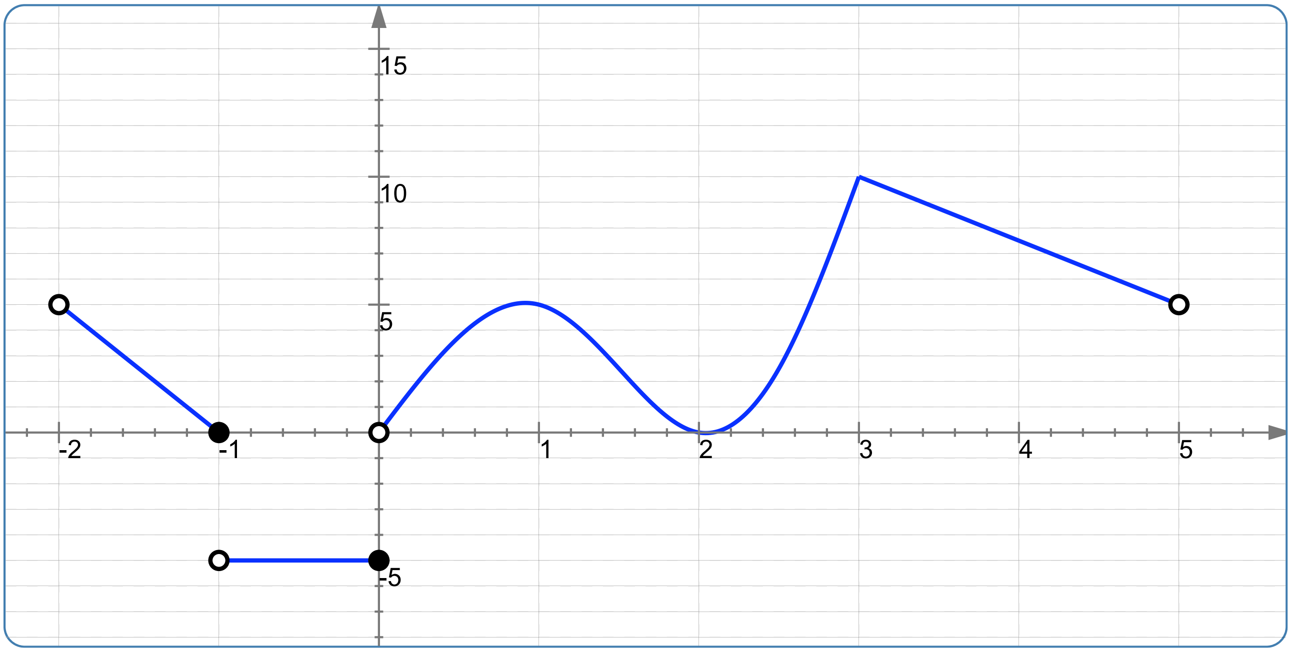
3. The graph of the equation $5x^2 - 2xy + y^2 = -y + 4x + 6$ is given below.



(2 pts) Verify that the point $(0, 2)$ lies on the graph.

(6 pts) Find the slope of the tangent line to the graph at $(0, 2)$.
(Hint: use implicit differentiation.)

4. (40 pts) The graph of a function f is shown below.
Read the following information from the graph; if something does not exist, write DNE.
(2 pts each)



$f(0)$	$f(-1.5)$	$f'(-1.5)$	$f''(4)$
$\lim\limits_{x\to -1^+} f(x)$	$\lim\limits_{x\to -1} f(x)$	$\mathrm{dom}(f)$	$\mathrm{ran}(f)$
$\{x \mid f(x) = 0\}$	$\int_3^5 f(x)\,dx$	$\int_{-1}^{-2} f(t)\,dt$	$\lim\limits_{t\to 1} f(t)$
the coordinates of a point (x, y) where f has a global maximum value (if such a point exists)			
the coordinates of a point (x, y) which is a local min, but not a global min			
give a value of x in the domain of f where f is continuous, but not differentiable			
average rate of change of f on $[0, 2]$			
instantaneous rate of change of f at $x = 1$			
all value(s) of x where f is NOT continuous			
slope of tangent line to f at $x = 4$			
$\{t \mid t > 0 \text{ and } f'(t) = 0\}$			

5. (8 pts) An open rectangular box with a square base is to be made from 48 square feet of material. Find the dimensions of the box that gives the largest possible volume.

DIMENSIONS OF BOX (use correct units): ______________________________________

VOLUME OF DESIRED BOX (use correct units): __________________________________

6. (6 pts) We have studied several "named theorems" this term.
Choose your favorite, and give a precise statement.
Include a sketch, if appropriate.

7. DIFFERENTIATION PROBLEMS:

- (5 pts) $\dfrac{d}{dx}\left(\mathrm{e}^x - 5^x + \dfrac{1}{x} + 7x^5 - \sqrt{3}\right)$

- (4 pts) $\dfrac{d}{dx}\sqrt[3]{5x^2 - 1}$

- (5 pts) $\dfrac{d}{dt}\left(\ln(\cos(3t - 1))\right)$

- (4 pts) $\dfrac{d}{dx}\left(\dfrac{2x - 1}{x + 5}\right)$

- (4 pts) Let $f(x) = 5\sin(x)\tan(x)$. Find $f'(0)$.

8. INTEGRATION PROBLEMS:

- (5 pts) $\displaystyle\int \left(\frac{2 - x^3}{x^2} + 5 \right) dx$

- (4 pts) $\displaystyle\int \frac{x}{\sqrt{5x^2 - 1}}\, dx$

- (5 pts) $\displaystyle\int x\mathrm{e}^x\, dx$

- (4 pts) $\displaystyle\int \frac{1}{1 + t^2}\, dt$

- (4 pts) $\displaystyle\int_{2/5}^{7} \frac{1}{5x - 1}\, dx$

9. LIMITS:

- (3 pts) $\lim_{x \to 0} \frac{3 + \sin(x)}{\cos(x)}$

- Find $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$ in two ways:

  (3 pts) using l'Hopital's rule

  (3 pts) not using l'Hopital's rule

- (3 pts) $\lim_{x \to \infty} \frac{1 - 3x^5 + 4x}{7x^5 + x^2 - 2}$

- (2 pts) $\lim_{n \to \infty} (1 + \frac{1}{n})^n$

- (3 pts) $\lim_{x \to 0^-} \frac{|x|}{x}$

10. Let $f(x) = 2x^3 - 9x^2 + 12x - 4 = (x-2)^2(2x-1)$.

The following questions guide you through a complete function analysis for f:

- (1 pt) $\mathrm{dom}(f) =$
- (2 pts) $f'(x) =$
- (3 pts) critical points for f (both x and y values):
- (2 pts) $f''(x) =$
- (2 pts) candidate(s) for inflection point(s) for f (both x and y values):
- (8 pts) Fill in the function analysis table. Some columns may be left blank. Add additional columns if needed.

x								
$f(x)$								
$f'(x)$								
$f''(x)$								

- (2 pts) x-intercept(s) of f:
- (1 pt) y-intercept of f:
- (2 pts) end behavior of f:
  as $x \to \infty$, $y \to$ ___________
  as $x \to -\infty$, $y \to$ ___________
- (4 pts) sketch a graph of f: