NAME: (1 pt)	NUMBER: (1 pt)

MAT 136, Dr. Carol JVF Burns, Exam #4, Chapter 5, 100 pts
This exam is closed book, closed notes, closed neighbor, and open mind.
Only a basic, four-function calculator is allowed (but is ***not*** required).
Show work leading to answers to receive full credit. Good luck!

1. The graph of a function f is given below. The graph consists of straight-line segments and a semi-circle. Evaluate the following integrals:

   (2 pts) $\int_0^1 f(x)\,dx$

   (2 pts) $\int_1^4 f(t)\,dt$

   (2 pts) $\int_4^0 f(x)\,dx$

   (2 pts) Let $g(x) = \int_0^x f(t)\,dt$. Find $g(1)$.

2. Consider the definite integral $\int_a^b f(t)\,dt$.

   (1 pt) What is the lower limit of integration?

   (1 pt) What is the integrand?

   (2 pts) Rewrite the integral using the dummy variable w.

3. (7 pts) Let $g(x) = \int_{2x}^{x^2} \sqrt{1-t^3}\, dt$.
   Find $g'(x)$.

4. (5 pts) Evaluate the following definite integral by interpreting it in terms of area:
   $\int_{-3}^{3} \sqrt{9-x^2}\, dx$

5. (6 pts) Estimate $\int_0^2 x^2\, dx$ using left endpoints; use four approximating rectangles. Show work leading to your answer.

6. (5 pts) Suppose that $\int_0^1 f(t)\,dt = 5$, $\int_0^4 f(t)\,dt = -3$, and $\int_3^4 f(t)\,dt = 2$. Find $\int_3^1 f(t)\,dt$.

7. (4 pts) Suppose that the derivative of a function f is known, and one point on the graph of f is known. That is, suppose you are given both $f'(x)$ and a point $(a, f(a))$. Write a formula for $f(x)$.

8. (3 pts) In the space provided below, graph $y = \ln|x|$.

   (2 pts) What is the slope of the tangent line to the graph of $y = \ln|x|$ when $x = -3$?

   (3 pts) What is the most general antiderivative of $\frac{1}{x}$?

   (3 pts) Does $\int_{-1}^{3} \frac{1}{x}\,dx$ exist? Why or why not?

9. Compute the following integrals. Be sure to show work leading to your answers.

(5 pts) $\displaystyle\int \frac{1+x^3}{x}\,dx$

(5 pts) $\displaystyle\int \frac{2}{\sqrt[3]{x}}\,dx$

(4 pts) $\displaystyle\int \left(\frac{4}{\sqrt{1-x^2}} + 3\sec x \tan x\right) dx$

(8 pts) $\displaystyle\int_0^1 x(1+x^2)^{23}\,dx$

(8 pts) $\displaystyle\int x\mathrm{e}^{3x}\,dx$

10. (3 pts) What is the Integration by Parts formula?

(4 pts) Use the Integration by Parts formula to find $\int \ln(x)\,dx$.

11. (11 pts) (As promised!) Use words from the "word bank" given below to fill in the blanks, giving a discussion of the relationship between area and antiderivatives:

Let _____ be _____ on _____ (and assume _____ for this motivation).

Let _____ denote the _____ under the _____ of f on _____ .

It's _____ that _____ is an _____ for f,

since for _____ _____ ,

_____ $\simeq$ _____ _____ _____ _____ _____

WORD BANK

Δx A $\simeq$ $[a,x]$ f antiderivative $\dfrac{f(x)\Delta x}{\Delta x}$ area $[a,b]$ continuous $A'(x)$ $A(x)$ $=$ plausible small $f(x)$ $f \geq 0$ $\dfrac{A(x+\Delta x)-A(x)}{\Delta x}$ graph

EXTRA CREDIT (6 pts)

As on card 69b, prove that if f' is integrable on $[a, b]$, then $\int_a^b f'(x)dx = f(b) - f(a)$.