NAME: (1 pt)	NUMBER: (1 pt)

MAT 136, Dr. Carol JVF Burns, Exam #3, Chapter 4 and 5.1
This exam is closed book, closed notes, closed neighbor, and open mind.
Only a basic, four-function calculator is allowed (but is ***not*** required).
Show work leading to answers to receive full credit. Good luck!

1. (As promised! One of the WebWorks optimization problems!)
   Find the point on the line $-2x + 3y + 3 = 0$ that is closest to the point $(-3, 1)$.
   As you solve this problem, record various parts of your solution below:

   (3 pts) THE FUNCTION TO BE MAXIMIZED/MINIMIZED. (Be sure to clearly label, on a sketch, what your input variable represents.)

   (2 pts) The DOMAIN of the function above. Use correct set notation to report this domain.

   (3 pts) A JUSTIFICATION that the answer you get actually corresponds to the desired max/min.

   (8 pts) Your work, and the answer:

2. (6 pts) Use l'Hopital's rule (if appropriate) to evaluate the following limit:
$\lim_{x \to 0} \frac{\sin(5x)}{3x}$

3. Evaluate by interpreting in terms of area:
(3 pts) $\int_0^2 3\,dx$

(3 pts) $\int_{-1}^1 5x\,dx$

4. (4 pts) There are only two types of places where a function can change its sign (from positive to negative, or from negative to positive). What are these two types of places? Include both a sketch and words for each case.

(6 pts) On the number line below, indicate the sign (+ or −) of the following function everywhere:

$$f(x) = \frac{5x^2(x+1)^3}{2x-1}$$

$\longrightarrow$ SIGN (+/−) of $f(x)$

5. (6 pts) There are only three types of places where a function can have a local maximum or a local minimum. What are these three types of places? In each case, include a sketch.
   (1)

   (2)

   (3)

6. Suppose g is a twice differentiable function. The number lines below indicate the signs of its derivatives, g' and g'', for all real numbers x.

   −−−−−−−−−− ++++++++++ −−−−−−−−−−> SIGN of $g'(x)$

   -1 3

   −−−−−−−−−−−−−−−− +++++++++++++++> SIGN of $g''(x)$

   1

   Answer the following questions:

   - (2 pts) On what interval(s) is g increasing?
   - (2 pts) On what interval(s) is g concave up?
   - (2 pts) On what interval(s) is g both decreasing and concave down?
   - (2 pts) Give the x-value(s) of any local minima for g.
   - (2 pts) Give the x-value(s) of any inflection point(s) for g.
   - (2 pts) On what interval(s) is g' (not g, but g') decreasing?

7. TRUE or FALSE: (2 pts each)

T F If a function f has a local max or min at c, then $f'(c) = 0$ or $f'(c)$ does not exist or c is an endpoint of the domain of f.

T F If f has a global max at c, then f has a local max at c.

T F If $f'(c) = 0$, then f must have a local max or min at c.

8. (6 pts) Give a precise statement of the Mean Value Theorem. Include a sketch which illustrates what the theorem is saying.

(6 pts) Find a value of c guaranteed by the Mean Value Theorem for the function $f(x) = x^2 + 1$ on the interval $[0, 3]$.

9. (3 pts) Show that the point $(1, 2)$ lies on the graph of $x^2 - 2xy + y = -1$.

(8 pts) Find the slope of the tangent line to the graph at this point.

10. (6 pts) Give a precise statement of the Extreme Value Theorem.

(3 pts) Give a sketch of a function with domain $(1,3]$ that is continuous and has a global min, but no global max.

(3 pts) Give a sketch of a function with domain $[1,3]$ that has a global max, but no global min.

11. (10 pts) A twenty foot ladder rests against a vertical wall. If the bottom slides away from the wall at one foot per second, then how fast is the top sliding down the wall, when the bottom is sixteen feet from the wall? (Be sure to show work leading to your answer.)

12. (12 pts) As discussed in class, do a complete function analysis for $f(x) = \frac{x^3}{3} + x^2 - 3x + 7$.
Be sure to include a graph which clearly shows (as applicable): local max/min, global max/min, inflection point(s), y-intercept, and end behavior.
You do NOT need to find the x-intercept(s) for this function.
The summary table is started for you below; add more columns as needed.

x					
$f(x)$					
$f'(x)$					
$f''(x)$					

Please put your graph here: