NAME: (1 pt)	NUMBER: (1 pt)

MAT 136, Dr. Carol JVF Burns, Exam #3, Chapter 4, 100 pts
This exam is closed book, closed notes, closed neighbor, and open mind.
Only a basic, four-function calculator is allowed (but is ***not*** required).
Show work leading to answers to receive full credit. Good luck!

1. (7 pts) (As promised! One of the WebWorks optimization problems!)
   A Norman window has the shape of a semicircle atop a rectangle so that the diameter of the semicircle is equal to the width of the rectangle. What is the area of the largest possible Norman window with a perimeter of 35 feet?

2. (8 pts)
   Determine if each of the following is an indeterminate form.
   If it IS an indeterminate form, then give two values between which the outputs are "competing" (as discussed in class):
   - $0 \cdot \infty$
     circle one: IS an indeterminate form is NOT an indeterminate form
     If it IS an indeterminate form:
     the outputs are "competing" between __________ and __________.
   - 0^{∞}
     circle one: IS an indeterminate form is NOT an indeterminate form
     If it IS an indeterminate form:
     the outputs are "competing" between __________ and __________.
   - $\dfrac{\infty}{\infty}$
     circle one: IS an indeterminate form is NOT an indeterminate form
     If it IS an indeterminate form:
     the outputs are "competing" between __________ and __________.

3. Use l'Hopital's rule (if appropriate) to evaluate each of the following limits:
   - (4 pts) $\lim\limits_{x\to\infty} \dfrac{x^2+1}{e^x}$

   - (4 pts) $\lim\limits_{x\to 0^+} x\ln(2x)$

4. (3 pts) There are only two types of places where a function can change its sign (from positive to negative, or from negative to position). What are these two types of places?

   (6 pts) On the number line below, indicate the sign ($+$ or $-$) of the following function everywhere:

$$f(x) = \frac{3x^2(x-5)(x+1)^3}{(x-2)^2(7-x)}$$

   (Hint: Use the efficient technique discussed in class.)

5. (6 pts) There are only three types of places where a function can have a local maximum or a local minimum. What are these three types of places? In each case, include a sketch.

   (1)

   (2)

   (3)

6. Suppose g is a polynomial. The number lines below indicate the signs of its derivatives, g' and g'', for all real numbers x.

   Answer the following questions:

   - (2 pts) On what interval(s) is g increasing?
   - (2 pts) On what interval(s) is g concave down?
   - (2 pts) On what interval(s) is g both decreasing and concave up?
   - (2 pts) Give the x-value(s) of any local maxima for g.
   - (2 pts) Give the x-value(s) of any inflection point(s) for g.
   - (2 pts) On what interval(s) is g' (not g, but g') increasing?

7. TRUE or FALSE: (2 pts each)

T F If a function has a horizontal tangent line at c, then f has a max or min at c.

T F If $f'(c)$ does not exist, then f has a local extreme value at c.

T F Suppose f is differentiable on $\mathbb{R}$. If $f'(c) > 0$, then f does not have a max or min at c.

8. (4 pts) Give a precise statement of the Mean Value Theorem.

(4 pts) Find a value of c guaranteed by the Mean Value Theorem for the function $f(x) = 5 - 3x^2$ on the interval $[-3, 7]$.

9. (6 pts) Find the slope of the tangent line to the circle $x^2 + y^2 = 16$ at the point $(1, \sqrt{15})$.

10. (5 pts) Find all the critical point(s) for the function $f(x) = |x - 4|$, with $\mathrm{dom}(f) = [-2, 7]$.

11. (4 pts) Give a precise statement of the Extreme Value Theorem.

(2 pts) Give a sketch of a function with domain $(1, 3]$ that is continuous, has a global max, but no global min.

(2 pts) Give a sketch of a function with domain $[1, 3]$ that has a global min, but not a global max.

12. (7 pts) A street light is at the top of a 14 foot pole. A 6 foot tall woman walks away from the pole with a speed of 8 ft/sec along a straight path. How fast is the tip of her shadow moving when she is 35 feet away from the base of the pole?

12. (8 pts) As discussed in class, do a complete function analysis for $f(x) = \frac{x^3}{3} + x^2 - 3x + 7$.
Be sure to include a graph which clearly shows (as applicable): local max/min, global max/min, inflection point(s), y-intercept, and end behavior.
You do NOT need to find the x-intercept(s) for this function.
The summary table is started for you below; add more columns as needed.

x					
$f(x)$					
$f'(x)$					
$f''(x)$					

Please put your graph here: