MAT 136, Dr. Carol JVF Burns, EXAM \#2
This exam is closed book, closed notes, closed neighbor, and open mind.
Only a basic, four-function calculator is allowed (but is not required).
Show work leading to answers to receive full credit. Good luck!

1. (15 pts) (as promised!) Prove the PRODUCT RULE FOR DIFFERENTIATION. I'll get you started. Let $f$ and $g$ be differentiable at $x$, and let $P(x)=f(x) g(x)$. Then,

$$
P^{\prime}(x)=
$$

2. What does it mean when we say that ' $f$ is differentiable at $x$ '? Give two different answers: ( 3 pts ) a low-level answer:
(3 pts) the most precise answer (which involves a limit):
3. (24 pts) Fill in the following table of differentiation formulas (1 pt each). You may assume that all functions and constants have appropriate properties (e.g., are differentiable) and values (e.g., are nonzero) for the given situation. Assume that $a$ is a constant.

| $\frac{d}{d x} a^{f(x)}=$ | $\frac{d}{d x}\left(\log _{a} x\right)=$ |
| :--- | :--- |
| $\frac{d}{d x} f(g(x))=$ | $\frac{d}{d x}(f \circ g)(x)=$ |
| $\frac{d}{d x} \arcsin (f(x))=$ | $\frac{d}{d x} \arccos x=$ |
| $\frac{d}{d x}\left(k x^{n}\right)=$ | $\frac{d}{d x} f(g(h(x)))=$ |
| $\frac{d}{d x} x^{n}=$ | $\frac{d}{d x}(f(x))^{n}=$ |
| $\frac{d}{d x}\left(\mathrm{e}^{x}\right)=$ | $\frac{d}{d x} \mathrm{e}^{f(x)}=$ |
| $\frac{d}{d x} \frac{f(x)}{g(x)}=$ | $\frac{d}{d x} \log _{a} x=$ |
| $\frac{d}{d x} \ln x=$ | $\frac{d}{d x} \sin x=$ |
| $\frac{d}{d x} \cos f(x)=$ | $\frac{d}{d x} \tan x=$ |
| $\frac{d}{d x} \sec x=$ | $\frac{d}{d x} f^{-1}(x)=$ |
| $\frac{d}{d x} k f(x)=$ | $\frac{d}{d x}(f(x)-g(x))=$ |
| $\frac{d}{d x} \arctan (x)=$ | $\frac{d}{d x} f(x) g(x)=$ |

4. Consider this mathematical statement:

$$
\text { as } t \rightarrow d, f(t) \rightarrow f(d)
$$

(3 pts) Under what condition(s) is this statement true?
(3 pts) If possible, sketch the graph of a function $f$ and a value of $d$ in the domain of $f$ for which the statement is false.
5. Find each of the following limits.

Show work leading to your answers.
(5 pts) $\lim _{n \rightarrow \infty} 5\left(1+\frac{1}{n}\right)^{6 n}$
(5 pts) $\lim _{x \rightarrow 0^{+}} \frac{3 x}{\sin (5 x)}$
6. Let $f(x)=2 x^{3}$. Find $\left(f^{-1}\right)^{\prime}(x)$ in TWO WAYS:
( 5 pts ) By finding a formula for $f^{-1}(x)$, and then differentiating $f^{-1}$ directly:
(5 pts) By using the formula for $\left(f^{-1}\right)^{\prime}(x)$.
(3 pts) Rename things, as needed, to convince me that the answer you get in both cases is the same!
7. Suppose that $f$ is a differentiable, one-to-one function. Suppose $f(2)=3$ and $f^{\prime}(2)=5$.
(2 pts) Based on the information given, give a point $(x, y)$ on the graph of $f$.
(2 pts) Based on the information given, give a point $(x, y)$ on the graph of $f^{-1}$.
(2 pts) Write the equation, in $y=m x+b$ form, of the tangent line to the graph of $f$ at $x=2$.
(2 pts) From the information given, do we know ANY slopes of tangent line(s) to the graph of $f^{-1}$ ? If so, state the $x$-value and slope at any known point(s).
8. Differentiate.

- (5 pts) $\frac{d}{d x} \ln (\sin (2 x))$
- (5 pts) $\frac{d}{d t} \frac{t+7}{\mathrm{e}^{t}}$
- $(5 \mathrm{pts}) \frac{d}{d u} 8^{u} \tan u$
- (5 pts) $\frac{d}{d x} \frac{\cos t}{\sqrt{t-9}}$
- (5 pts) $\frac{d}{d t} \sqrt[7]{5 t}$

9. (8 pts) Let $f(x)=(2 x)^{x}$. Find $f^{\prime}(x)$. Be sure to show work leading to your answer.
10. (6 pts, EXTRA CREDIT)

Use the definition of derivative to prove that $\frac{d}{d x}(\ln x)=\frac{1}{x}$.

