NAME: (1 pt)	NUMBER: (1 pt)

MAT 136, Dr. Carol J.V. Fisher
This exam is closed book, closed notes, closed neighbor, and open mind.
Only a basic, four-function calculator is allowed (but is ***not*** required).
Show work leading to answers to receive full credit. Good luck!

1. (24 pts) Fill in the following table of differentiation formulas (1 pt each).
   You may assume that all functions and constants have appropriate properties (e.g., are differentiable) and values (e.g., are nonzero) for the given situation.

$\frac{d}{dx} x^n =$	$\frac{d}{dx}\left(f(x)\right)^n =$
$\frac{d}{dx} kf(x) =$	$\frac{d}{dx}\left(f(x)+g(x)\right) =$
$\frac{d}{dx} f(g(x)) =$	$\frac{d}{dx}(f \circ g)(x) =$
$\frac{d}{dx}\left(kx^n\right) =$	$\frac{d}{dx} f(g(h(x))) =$
$\frac{d}{dx}\left(\mathrm{e}^x\right) =$	$\frac{d}{dx}\mathrm{e}^{f(x)} =$
$\frac{d}{dx} a^x =$	$\frac{d}{dx}\left(\log_a f(x)\right) =$
$\dfrac{d}{dx}\dfrac{f(x)}{g(x)} =$	$\dfrac{d}{dx}\log_a x =$
$\dfrac{d}{dx}\ln f(x) =$	$\dfrac{d}{dx}\sin f(x) =$
$\dfrac{d}{dx}\cos x =$	$\dfrac{d}{dx}\tan f(x) =$
$\dfrac{d}{dx}\sec x =$	$\dfrac{d}{dx} f^{-1}(x) =$
$\dfrac{d}{dx}\arcsin(x) =$	$\dfrac{d}{dx}\arccos f(x) =$
$\dfrac{d}{dx}\arctan(x) =$	$\dfrac{d}{dx} f(x)g(x) =$

2. (As promised!)
(10 pts) Suppose that f and g are differentiable functions, and let $P(x) = f(x)g(x)$.
Prove the product rule for differentiation.
I'll get you started:

$$P'(x) = \lim_{h \to 0}$$

3. (4 pts) In class, when we did a partial proof of the power rule, we used the 'telescopic identities', such as

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Verify this identity, by using long division of polynomials to show that $a-b$ goes into $a^3 - b^3$ evenly, as set up below:

4. Consider this mathematical statement:

$$\lim_{h\to 0} f(x+h) = f(x) \qquad (\dagger)$$

(4 pts) Under what condition(s) is (†) true?

(3 pts) Sketch the graph of a function f and a value of x for which (†) is false.

5. Suppose that f and g are differentiable functions, and $g(x)$ is always nonzero. Let $Q(x) = \frac{f(x)}{g(x)}$.

(2 pts) Rewrite $Q(x)$ as a product.

(4 pts) Use the product rule to find $Q'(x)$ (thus deriving the quotient rule for differentiation). Your final expression for $Q'(x)$ should NOT have any negative exponents.

6. Find each of the following limits.
Show work leading to your answers.

(5 pts) $\lim_{x \to 0^+} (1 + 7x)^{\frac{3}{x}}$

(5 pts) $\lim_{x \to 0} \dfrac{\sin(5x)}{7x}$

7. Suppose that y is a function of t.
Give two different notations for the third derivative of y with respect to t, evaluated at $t = 2$:
(2 pts) using prime notation

(2 pts) using Leibnitz notation

8. Let $f(x) = x^5$. Find $(f^{-1})'(x)$ in TWO WAYS:
(5 pts) By finding a formula for $f^{-1}(x)$, and then differentiating f^{-1} directly:

(5 pts) By using the formula for $(f^{-1})'(x)$:

9. Suppose that f is a differentiable, one-to-one function.
(2 pts) If (a, b) is a point on the graph of f, then what point is on the graph of f^{-1}?

(2 pts) Fill in the blanks:
If the slope of the tangent line to the graph of f at the point (a, b) is m,
then the slope of the tangent line to the graph of f^{-1} at $x =$ __________ is ________________.

10. For these problems, write derivatives in a form that matches the original function as closely as possible.

- (3 pts) $\frac{d}{dx} x^3 \cos(5x)$

- (3 pts) $\frac{d}{dx} \log_5(\sqrt{2x-1})$

- (3 pts) $\frac{d}{dx} 5^{x^2-1} \tan(3x)$

12. (3 pts) If $f(t) = \frac{1}{\sqrt[3]{4-t}}$, find $f'(5)$.

13. (6 pts) Let $f(x) = x^{3x}$. Find $f'(x)$. Be sure to show work leading to your answer.