| NAME: (1 pt) | NUMBER: (1 pt) |
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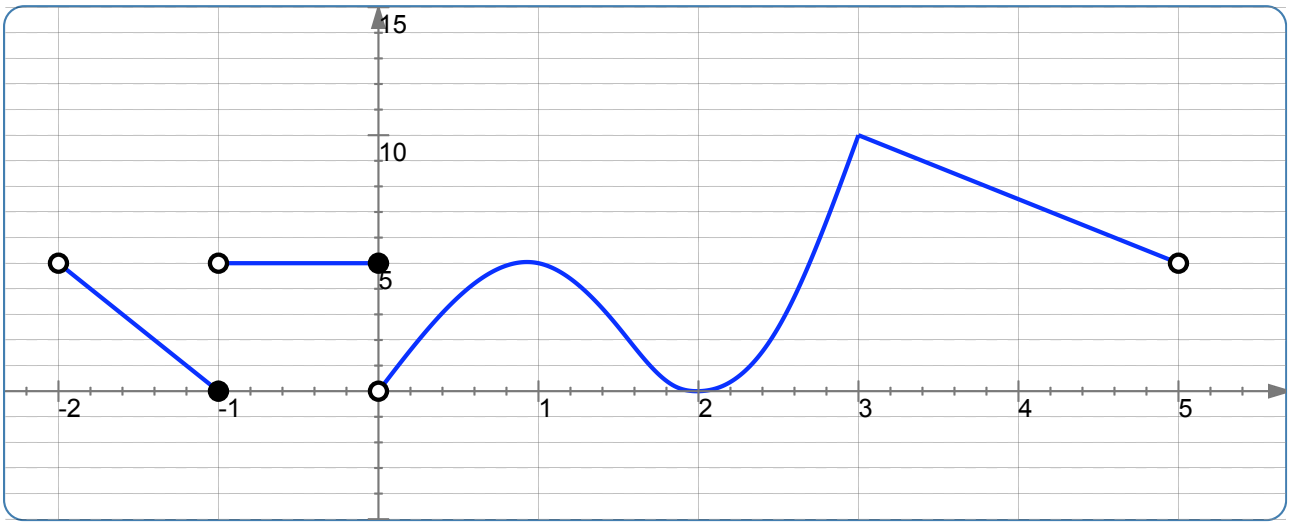
MAT 136, Dr. Carol JVF Burns, EXAM #1
This exam is closed book, closed notes, closed neighbor, and open mind.
Only a basic, four-function calculator is allowed (but is ***not*** required).
Show work leading to answers to receive full credit. Good luck!

1. (48 pts) The graph of a function $f$ is shown below.
Read the following information from the graph; if something does not exist, write DNE.
(2 pts each)




| | | | |
|---|---|---|---|
| $f(0)$ | $f(-1.5)$ | $f'(-1.5)$ | $f(2)$ |
| $f'(2)$ | $f(3)$ | $f'(3)$ | $f(5)$ |
| $\text{dom}(f)$ (use interval notation) | | $\text{ran}(f)$ (use interval notation) | |
| $\lim_{x\to -1^+} f(x)$ | $\lim_{x\to -1^-} f(x)$ | $\lim_{x\to -1} f(x)$ | $\lim_{x\to 2} f(x)$ |
| $\{x \mid f(x) = 0\}$ | | $\{t \mid f(t) < 0\}$ | |
| the coordinates of a point $(x,y)$ where $f$ has a global maximum value (if such a point exists) | | | |
| the coordinates of a point $(x,y)$ which is a local max, but not a global max | | | |
| give a value of $x$ in the domain of $f$ where $f$ is continuous, but not differentiable | | | |
| average rate of change of $f$ on $[1,2]$ | | | |
| instantaneous rate of change of $f$ at $x = 1$ | | | |
| all value(s) of $x$ in the domain of $f$ where $f$ is NOT continuous | | | |
| slope of tangent line to $f$ at $x = 4$ | | | |
| the local linearization, $\ell_2(x)$, to the graph of $f$ at $x = 2$ | | | |

2. (3 pts) Give a precise definition: ‘ $f$ is continuous at $a$ ’ if and only if _____ .

   (2 pts) When is evaluating a limit (as $x$ approaches $a$) as easy as direct substitution?
   Answer (fill in the blank): when $f$ is _____ at _____ .

   (2 pts) Under what condition(s) (if any) is the following statement true?

$$\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

3. As we did in class, create a graph that matches the story below.
   You live on a long, straight, road (i.e., a number line). Your house is at position 0.
   You always leave your house and turn in the positive direction (i.e., towards $1, 2, \ldots$).
   Let $p(t)$ denote the position of the car at time $t$.
   Put $p(t)$ along the vertical axis, and $t$ along the horizontal axis.
   “ I ALWAYS FORGET SOMETHING! ”
   - (2 pts) Leave your house at $t = 0$.
     Gradually speed up, reaching 40 miles per hour at $t = a$.
   - (2 pts) Drive at a constant speed of 40 mph, with your thoughts wandering, until $t = b$.
   - (2 pts) You suddenly realize that you've forgotten a homework assignment that needs to be passed in today!
     So, at $t = c$, start slowing down, and come to a complete stop by $t = d$.
   - (2 pts) Turn around, and start speeding up, heading back towards home.
     At $t = e$ you reach 40 mph, and remain at 40 mph until you arrive back home at $t = f$.
   - (2 pts) Run into your house and grab your homework.
     While there, the phone rings, which takes a few more minutes.
     Then, get back out to your car, and at $t = g$ take off again!

   PUT YOUR GRAPH HERE:

4. (16 pts) Compute the following limits. If a limit does not exist, state DNE.
   - $\lim_{x \to 1} \sqrt{x^3 - 2x^2 + 5}$
   - $\lim_{x \to 3} \dfrac{x - 3}{x^2 - 9}$
   - $\lim_{x \to -\infty} \dfrac{1 - 6x^2 + 3x}{7 - x^5}$
   - $\lim_{x \to 0^-} \dfrac{x}{|x|}$

5. (10 pts) Use the **definition** of the derivative (not a differentiation shortcut!) to find $f'(x)$ if $f(x) = x^2 - 3x + 2$.

7. (6 pts) Determine the values of $a$ and $b$ if $f$ is continuous:

$$f(x) = \begin{cases} x^2 & \text{if } x < -4 \\ ax + b & \text{if } -4 \leq x < 5 \\ \sqrt{x+31}, & \text{if } x \geq 5 \end{cases}$$

8. (9 pts) In each space below, sketch the graph of a function $f$ satisfying the given requirements:

| | | |
|---|---|---|
| $f$ increasing, $f'$ increasing | $f$ decreasing, $f'$ increasing | $f$ both decreasing and concave down |

9. (6 pts) Give a precise statement of the Squeeze Theorem.
Include a sketch that illustrates what the theorem is saying.

10. (6 pts) Suppose that $h(2) = 5$ and the average rate of change of $h$ on $[2, 7]$ is 3. Find $h(7)$.
Be sure to show some work leading to your answer.