MAT 136, Dr. Carol J.V. Fisher

This exam is closed book, closed notes, closed neighbor, and open mind. Only a basic, four-function calculator is allowed (but is *not* required). Show work leading to answers to receive full credit. Good luck!

1. (32 pts) The graph of a function f is shown below.

Read the following information from the graph; if something does not exist, write DNE. (1 pt each, unless otherwise specified)

f(1)	f(2)	f(1.9)	f(6)
f'(0)	$f'(rac{1}{2})$	f'(1.9)	f'(3)
$\lim_{x\to 1^+} f(x)$	$\lim_{x\to 1^-} f(x)$	$\lim_{x ightarrow 1}\!f(x)$	$\lim_{x\to 6^-} f(x)$
$\lim_{x\to 2^+} f(x)$	$\lim_{x\to 2^-} f(x)$	$\lim_{x ightarrow 2}\!f(x)$	$\lim_{x ightarrow 3}\!f(x)$
the domain of f (use interval notation)		the range of f (use interval notation)	
(2 pts) the average rate of change of f between $x = 0$ and $x = 2$			
the instantaneous rate of change of f at $x = 0.5$			
the slope of the tangent line to the graph of f at $x = 5$			
(2 pts) all value(s) of x where f is NOT continuous			
(2 pts) all value(s) of x where f is NOT differentiable			
(2 pts) the coordinates of a point (x, y) where f has a local maximum value (even if there is more than one, you only need to list one such point)			
(2 pts) the coordinates of a point (x, y) where f has a global minimum value (even if there is more than one, you only need to list one such point)			
(2 pts) the local linearization, $\ell_5(x)$, to the graph of f at $x = 5$			

2. (5 pts) Again refer to the graph at the top of the first page. Graph the function f' (the derivative of f) for -1 < x < 2 in the space provided below.

- 3. (7 pts) As we did in class, you will create a graph that matches the story below.
 You live on a long, straight, road (i.e., a number line). Your house is at position 0.
 You always leave your house and turn in the positive direction (i.e., towards 1,2,...).
 Let p(t) denote the position of the car at time t.
 Put p(t) along the vertical axis, and t along the horizontal axis.
 - Leave your house at t = 0. Gradually speed up to 70 mph at t = a.
 - Travel at 70mph until t = b; at this time, you see a police car ahead of you on the road, and quickly step on the brakes.
 - By t = c, you manage to be down to 50mph.
 - Too late! You hear the sirens. You pull over to the side of the road, coming to a complete stop at t = d, and remain there for a while while the policewoman writes out a speeding ticket (sigh).

4. (12 pts) Compute the following limits. If a limit does not exist, state DNE.

•
$$\lim_{x \to 1} (x^4 - 3x^2 + 5)$$

• $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$
• $\lim_{x \to \infty} \frac{2x - 3x^4 + 5}{7x^4 - 6x + 1}$
• $\lim_{x \to 0^+} \frac{|x|}{x}$

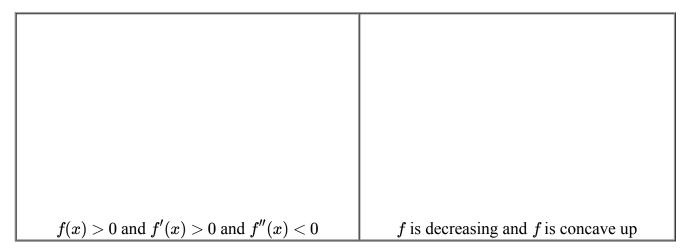
5. (2 pts) Start with the graph of $y = x^3$. Shift it to the right 4 units, and up 2 units. What is the equation of the resulting graph?

(2 pts) Explain, in words, what is done to the graph of y = f(x) to obtain the graph of y = -3f(x+5).

6. (8 pts) Use the definition of the derivative to find f'(x) if $f(x) = x^2 + 2x - 3$. (2 pts) Then, use your derivative formula to find the slope of the tangent line to f at x = 3. 7. (6 pts) Determine the values of a and b if f is continuous:

$$f(x) = egin{cases} x^2 & ext{if } x < -4 \ ax + b & ext{if } -4 \leq x < 5 \ \sqrt{x + 31}, & ext{if } x \geq 5 \end{cases}$$

8. In each spaces below, sketch the graph of a function f satisfying the following requirements:



9. (5 pts) Give a precise statement of the Squeeze Theorem. Include a sketch that illustrates what the theorem is saying.