

NAME: (1 pt)	NUMBER: (1 pt)
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MAT 136, Dr. Carol J.V. Fisher

This exam is closed book, closed notes, closed neighbor, and open mind.

Only a basic, four-function calculator is allowed (but is **not** required).

Show work leading to answers to receive full credit. Good luck!

1. (32 pts) The graph of a function  $f$  is shown below.

Read the following information from the graph; if something does not exist, write DNE.  
(1 pt each, unless otherwise specified)

$f(1)$	$f(2)$	$f(1.9)$	$f(6)$
$f'(0)$	$f'(\frac{1}{2})$	$f'(1.9)$	$f'(3)$
$\lim_{x \rightarrow 1^+} f(x)$	$\lim_{x \rightarrow 1^-} f(x)$	$\lim_{x \rightarrow 1} f(x)$	$\lim_{x \rightarrow 6^-} f(x)$
$\lim_{x \rightarrow 2^+} f(x)$	$\lim_{x \rightarrow 2^-} f(x)$	$\lim_{x \rightarrow 2} f(x)$	$\lim_{x \rightarrow 3} f(x)$
the domain of $f$ (use interval notation)		the range of $f$ (use interval notation)	
(2 pts) the average rate of change of $f$ between $x = 0$ and $x = 2$			
the instantaneous rate of change of $f$ at $x = 0.5$			
the slope of the tangent line to the graph of $f$ at $x = 5$			
(2 pts) all value(s) of $x$ where $f$ is NOT continuous			
(2 pts) all value(s) of $x$ where $f$ is NOT differentiable			
(2 pts) the coordinates of a point $(x, y)$ where $f$ has a local maximum value (even if there is more than one, you only need to list one such point)			
(2 pts) the coordinates of a point $(x, y)$ where $f$ has a global minimum value (even if there is more than one, you only need to list one such point)			
(2 pts) the local linearization, $\ell_5(x)$ , to the graph of $f$ at $x = 5$			

2. (5 pts) Again refer to the graph at the top of the first page.  
Graph the function  $f'$  (the derivative of  $f$ ) for  $-1 < x < 2$  in the space provided below.

3. (7 pts) As we did in class, you will create a graph that matches the story below.  
You live on a long, straight, road (i.e., a number line). Your house is at position 0.  
You always leave your house and turn in the positive direction (i.e., towards 1, 2, ...).  
Let  $p(t)$  denote the position of the car at time  $t$ .  
Put  $p(t)$  along the vertical axis, and  $t$  along the horizontal axis.
- Leave your house at  $t = 0$ . Gradually speed up to 70 mph at  $t = a$ .
  - Travel at 70mph until  $t = b$ ; at this time, you see a police car ahead of you on the road, and quickly step on the brakes.
  - By  $t = c$ , you manage to be down to 50mph.
  - Too late! You hear the sirens. You pull over to the side of the road, coming to a complete stop at  $t = d$ , and remain there for a while while the policewoman writes out a speeding ticket (sigh).

4. (12 pts) Compute the following limits. If a limit does not exist, state DNE.

- $\lim_{x \rightarrow 1} (x^4 - 3x^2 + 5)$

- $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

- $\lim_{x \rightarrow \infty} \frac{2x - 3x^4 + 5}{7x^4 - 6x + 1}$

- $\lim_{x \rightarrow 0^+} \frac{|x|}{x}$

5. (2 pts) Start with the graph of  $y = x^3$ . Shift it to the right 4 units, and up 2 units. What is the equation of the resulting graph?

(2 pts) Explain, in words, what is done to the graph of  $y = f(x)$  to obtain the graph of  $y = -3f(x + 5)$ .

6. (8 pts) Use the definition of the derivative to find  $f'(x)$  if  $f(x) = x^2 + 2x - 3$ .

(2 pts) Then, use your derivative formula to find the slope of the tangent line to  $f$  at  $x = 3$ .

7. (6 pts) Determine the values of  $a$  and  $b$  if  $f$  is continuous:

$$f(x) = \begin{cases} x^2 & \text{if } x < -4 \\ ax + b & \text{if } -4 \leq x < 5 \\ \sqrt{x + 31}, & \text{if } x \geq 5 \end{cases}$$

8. In each spaces below, sketch the graph of a function  $f$  satisfying the following requirements:

$f(x) > 0$ and $f'(x) > 0$ and $f''(x) < 0$	$f$ is decreasing and $f$ is concave up
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9. (5 pts) Give a precise statement of the Squeeze Theorem.  
Include a sketch that illustrates what the theorem is saying.