NAME: (1 pt)	NUMBER: (1 pt)

MAT 136, Dr. Carol JVF Burns, Exam #4, cards 65
This exam is closed book, closed notes, closed neighbor, and open mind.
Only a basic, four-function calculator is allowed (but is ***not*** required).
Show work leading to answers to receive full credit. Good luck!

1. (3 pts) In the space provided below, graph the function f defined by:

$$f(x) = \begin{cases} 2 & \text{if } 0 \le x < 1 \\ 3 - x & \text{if } 1 \le x < 3 \\ g(x) & \text{if } 3 \le x < 5 \end{cases},$$

where $g(x)$ is the upper half of a circle with center $(4,0)$ and radius 1.

Then, evaluate the following definite integrals:

(3 pts) $\int_0^1 f(x)\,dx$

(4 pts) $\int_3^5 f(t)\,dt$

(5 pts) $\int_3^1 f(x)\,dx$

2. Consider the definite integral $\int_c^d g(x)\,dx$.

(1 pt) What is the lower limit of integration?

(1 pt) What is the integrand?

(2 pts) Rewrite the integral using the dummy variable t.

3. (7 pts) Let $g(x) = \int_{2-x}^{3x} (t^2 - 1)^5 \, dt$.
   Find $g'(x)$.

4. (4 pts) Suppose that $f'(t) = \sin^3 t$, and $f(1) = 5$.
   That is, you know both the derivative of a function f, and a single point on the graph of f.
   Write a formula for $f(x)$. You do ***not*** need to evaluate any integral that might appear in your formula.

5. (6 pts) Estimate $\int_0^4 \sqrt{x} \, dx$ using right endpoints; use four approximating rectangles.
   Include a sketch that illustrates what you are finding. Give an exact answer, not a decimal approximation.

6. (5 pts) Suppose that $\int_0^1 f(t)\,dt = -2$, $\int_0^4 f(t)\,dt = 3$, and $\int_3^4 f(t)\,dt = 5$. Find $\int_1^3 f(t)\,dt$.

7. (5 pts) Use the technique of substitution to evaluate the following integral:

$$\int \frac{(\ln x)^5}{x}\,dx$$

8. (3 pts) In the space provided below, graph $y = \ln|x|$.

(2 pts) What is the slope of the tangent line to the graph of $y = \ln|x|$ when $x = -2$?

(3 pts) What is the most general antiderivative of $\frac{1}{x}$?

(3 pts) Evaluate: $\int_{-3}^{-1} \frac{1}{x}\,dx$

9. Compute the following integrals. Be sure to show work leading to your answers.

(6 pts) $\displaystyle\int \left(\frac{4}{1+x^2} - \sec x \tan x \right) dx$

(6 pts) $\displaystyle\int \frac{1-x^2}{x}\, dx$

(8 pts) $\displaystyle\int_0^1 x(1-x^2)^{13}\, dx$

(8 pts) $\displaystyle\int \ln x \, dx$

(6 pts) $\displaystyle\int \frac{3}{\sqrt[5]{x}}\, dx$

10. (5 pts) What is the Integration by Parts formula?

(8 pts) Use the Integration by Parts formula to find $\int xe^x\,dx$.

EXTRA CREDIT:

(6 pts) As on card 68b, use the ***definition*** of definite integral as a limit of Riemann sums to find $\int_0^2 f(x)\,dx$, where:

$$f(x) = \begin{cases} 5 & \text{if } x = 0 \\ 0 & \text{if } 0 < x < 2 \\ 3 & \text{if } x = 2 \end{cases}$$

(8 pts) As on card 69b, prove that if f' is integrable on $[a,b]$, then $\int_a^b f'(x)\,dx = f(b) - f(a)$.