SAMPLE PREREQUISITE PROBLEMS: PRECALCULUS

(no calculators allowed)

Multiplication Tables (through 12)

(You will have two minutes to do the following 24 multiplication problems.)

$$2 \times 6 =$$

$$3 \times 2 =$$

$$4 \times 9 =$$

$$5 \times 2 =$$

$$8 \times 8 =$$

$$9 \times 3 =$$

$$10 \times 7 =$$

$$2 \times 4 =$$

$$5 \times 1 =$$

$$6 \times 8 =$$

$$7 \times 9 =$$

$$8 \times 10 =$$

$$0 \times 10 =$$

$$1 \times 11 =$$

$$7 \times 3 =$$

$$11 \times 9 =$$

$$6 \times 4 =$$

$$7 \times 11 =$$

$$3 \times 7 =$$

$$4 \times 5 =$$

$$9 \times 5 =$$

$$10 \times 6 =$$

$$12 \times 10 =$$

$$9 \times 12 =$$

(Be sure that you can easily do problems like these: arithmetic with whole numbers, decimals, fractions; arithmetic with signed numbers)

$$\frac{0}{7.2} =$$

$$-\frac{(6)(-2)}{-3}=$$

$$-3 - (-2) =$$

$$1,000 \times 3.47 =$$

$$\frac{248.36}{100} =$$

$$\frac{1}{3} - \frac{1}{5} =$$

$$\frac{1}{3} \cdot \frac{1}{5} =$$

$$\frac{1}{3} \div \frac{1}{5} =$$

$$126 \times 24 =$$

SAMPLE PREREQUISITE PROBLEMS: PRECALCULUS

Problems 1–13 should be done WITHOUT A CALCULATOR.

1. For each expression given below, rename the expression as requested. If the requested name is not possible, so state. A few samples are done for you.

EXPRESSION	RENAME IN THIS FORM	ANSWER
(sample) 12 (sample) 12	a sum of even integers 2^x , where $x \in \{0, 1, 2, 3, \dots\}$	2 + 10 or $4 + 8$ etc. not possible
$\frac{1}{\sqrt{2}}$	a fraction with no radical in the denominator	
$23,070,000 \\ x^2 - y^2$	in scientific notation as a product (i.e., factor)	
$\frac{x^4x^{-1}}{(x^2)^3x}$	x^k	
$300 \mathrm{ft/sec}$	x mph (there are $5,280$ feet in one mile)	
7,036	$x \cdot 10^2 + y \cdot 10^{-1}$	
$8^{-2/3}$	as a simple fraction	
$x^2 + 2x + 3$	involving a perfect square, $(x+k)^2$	
$ 2x+3 $, for $x<-\frac{3}{2}$	without absolute values	
$2\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix} - \begin{bmatrix}3 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 4\end{bmatrix}$	$\left[\begin{matrix} a & b \\ c & d \end{matrix} \right]$	
$2+3i-i^2+(1-i)(3+4i)$, $(i=\sqrt{-1})$	a + bi	
$\frac{x^2 - x - 1}{x - 3}$	$Q(x) + \frac{R(x)}{D(x)}$	
$\log_7 5$	involving the natural log	
$\frac{4 \log 10^x}{3} \ln x^4 - \ln x^2 + \ln(x^2 + 1) (x - 2y)^4$	without logarithms	
	a single logarithm expanded form (Hint: use Pascal's triangle)	
$(-\infty, -2] \cap (-4, 5]$ $\{x \mid x \ge -2\}$	as a single interval using interval notation	

2. Solve each equation/inequality/system. Get EXACT answers, not decimal approximations. Report each solution set using correct set notation. A sample is done for you.

(sample)
$$x^2 - 2x > 3$$

Solution: Rewrite:

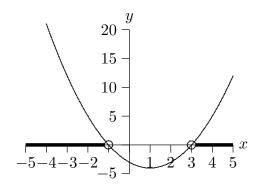
$$x^2 - 2x - 3 > 0$$

$$(x-3)(x+1) > 0$$

(graph y = (x-3)(x+1); see where graph lies above x-axis and read off solution set)

$$x < -1$$
 or $x > 3$

Solution set: $(-\infty, -1) \cup (3, \infty)$



(a)
$$3x(1-5x)(x^2-16)=0$$

(b)
$$\frac{1}{2}x - 7 = 3x + \frac{x}{5}$$

(c)
$$|2x-3| > 5$$

(d)
$$2 < |x| < 3$$

(e)
$$1 - 2x \le 3$$
 or $-3 \le x < -2$

(f)
$$x^2 = x + 2$$

(g)
$$2x - 3x^2 < -1$$

(h)
$$3^{2x-1} = 10$$

(i)
$$\log_3(x^2 - 1) = -2$$

(j)
$$\sqrt{3x^2 + 5x - 3} = x$$

(k)
$$y = x^2 + 1$$
 and $y = 2x + 4$

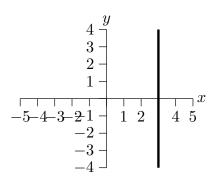
(1)
$$x+3 = \frac{-2x^2+7x-3}{x-3}$$

(m) Let
$$f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ 2, & \text{if } 0 \le x < 1 \end{cases}$$
. Solve the equation $f(x) = 1$. $x = 1$.

3. Graph each of the following equations/inequalities, where each sentence is viewed as a sentence in two variables. (That is, x=3 should be viewed as x+0y=3.) A sample is done for you.

(sample)
$$x = 3$$

Solution:



- (a) x > 3
- (b) 2y 3 = 0
- (c) x = 3 and y = 2
- (d) x = 3 or y = 2
- (e) y 2x + 1 = 0
- (f) $y = -2\sqrt{x+3} + 1$
- (g) |x| = 2
- (h) $y \le 2$
- (i) $\frac{y-2}{3} = 2x 1$
- $(j) \quad \frac{y-2}{3} \ge 2x 1$
- (k) $x^2 + 2x + y^2 6y 15 = 0$
- 4. Write a list of transformations that takes the graph of y = f(x) to the graph of y = 5 3|f(x+1)|. There may be more than one correct answer.

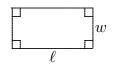
EQUATION TRANSFORMATION y = f(x) (starting place)

5. Starting with the equation $y = x^2 - 2x + 1$, apply the specified sequence of transformations.

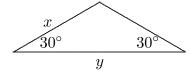
EQUATION TRANSFORMATION $y = x^2 - 2x + 1$ (starting place) up 1 left 3 reflect about the x-axis vertical scale by a factor of 2

6. Find the requested measurement(s) of each geometric figure.

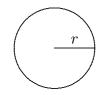
(a) PERIMETER and AREA:



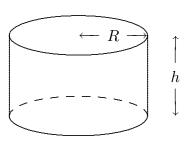
(b) PERIMETER and AREA:



(c) CIRCUMFERENCE and AREA:



(d) VOLUME:

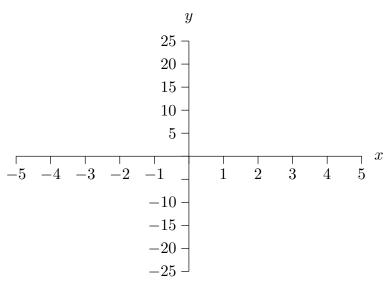


Which of the units below is a unit of length? Of area? Of volume? cubic feet cm^2 meter

7. (a) Let $f(x) = x^2 - 2x + 1$ and g(x) = 1 - 3x. Find both g(f(x)) and f(g(x)).

(b) Find functions f and g such that $f(g(x)) = \sqrt[3]{x^2 - 1}$.

8. Graph the rational function $g(x) = \frac{(x^2 - 1)(x + 2)}{(2x - 1)(x + 3)(x + 2)}$ in the space below.



If any of the following do not exist, so state:

x-intercept(s): _____

y-intercept(s): _____

Equation(s) of any horizontal asymptote(s):

Equation(s) of any vertical asymptote(s):

Equation(s) of any slant asymptote(s):

Puncture point(s):

Fill in the blank: as $x \to \infty$, $y \to$

Fill in the blank: as $x \to -3^+$, $y \to$ _____

9. Find the equation of a polynomial P satisfying the following properties: P(-3) = 0, 1 is a zero of P, the graph of P crosses the x-axis at x = 2, P has degree 5, and P(0) = 7.

- 10. Write an expression (using the variable x) to represent each sequence of operations.
 - (a) take a number, multiply by 2, then subtract 3
 - (b) take a number, subtract 3, then multiply by 2
 - (c) take a number, multiply it by 2, cube the result, add 1, then divide by the original number

Write the sequence of operations that is being described by each expression.

- (d) 3x 1
- (e) $2(x+1)^3 5$
- (f) $\frac{x-3}{7}-1$
- 11. Let $f(x) = x^2 2x + 1$. Evaluate each of the following expressions.
 - (a) f(0)
 - (b) f(1) 2
 - (c) f(f(-1))
- 12. Find the domain of the function $g(x) = \frac{1}{\sqrt{x-3}}$. Report your answer using interval notation.
- 13. Write the equation of the line, in y = mx + b form, that satisfies the given conditions.
 - (a) slope 3, passing through the point (2, -1)
 - (b) the horizontal line that crosses the y-axis at 2
 - (c) the line that is perpendicular to x 3y = 5 and passes through the point (0,3)
- 14. (Your calculator is needed for parts of this question.)
 - (a) What is the domain of the function $f(x) = \frac{1-3x}{x-2}$?
 - (b) Use your graphing calculator to graph the function f in the window -1 < x < 3 and -15 < y < 10.
 - (c) Find the x-intercept of the graph.
 - (d) Use your calculator to estimate a value for x for which f(x) = 5. (Zoom, as necessary, to get f(x) within 0.01 of 5.)
- 15. Estimate (where necessary) each of the following numbers on your calculator. For full credit, each answer must be correct to five decimal places.
 - (a) $\frac{1+\sqrt{2}}{\sqrt[3]{5}-7}$
 - (b) $3x^2 5x + 1$, where x = -1.8
 - (c) |1 2x|, where $x = \sqrt{3}$
 - (d) $(2.03 \times 10^{-9})(-4.1 \times 10^7)$

SOLUTIONS

Multiplication Tables:

$$0, -4, -1$$

$$3,470, 2.4836, \frac{2}{15}$$

$$\frac{1}{15}$$
, $\frac{5}{3}$, $3,024$

1. There are many possible correct answers for some of these problems, but these are the most obvious ones:

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$23,070,000 = 2.307 \times 10^7$$

$$x^2 - y^2 = (x - y)(x + y)$$

$$\frac{x^4x^{-1}}{(x^2)^3x} = \frac{x^3}{x^7} = x^{3-7} = x^{-4}$$

$$300 \frac{\mathrm{ft}}{\mathrm{sec}} = 300 \frac{\mathrm{ft}}{\mathrm{sec}} \cdot \frac{1 \mathrm{\ mile}}{5280 \mathrm{\ ft}} \cdot \frac{60 \mathrm{\ sec}}{1 \mathrm{\ min}} \cdot \frac{60 \mathrm{\ min}}{1 \mathrm{\ hr}} \approx 204.5 \frac{\mathrm{miles}}{\mathrm{hr}}$$

$$7,036 = 70 \cdot 10^2 + 360 \cdot 10^{-1}$$

$$8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{(8^{1/3})^2} = \frac{1}{2^2} = \frac{1}{4}$$

Use the technique of completing the square:

$$x^{2} + 2x + 3 = x^{2} + 2x + 1 - 1 + 3 = (x+1)^{2} + 2$$

When
$$x < -\frac{3}{2}$$
, $2x + 3 < 0$. Thus, $|2x + 3| = -(2x + 3) = -2x - 3$.

$$2\begin{bmatrix}1&-2\\-1&3\end{bmatrix}-\begin{bmatrix}3&-1\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&4\end{bmatrix}=\begin{bmatrix}2&-4\\-2&6\end{bmatrix}-\begin{bmatrix}3&2\\0&4\end{bmatrix}=\begin{bmatrix}-1&-6\\-2&2\end{bmatrix}$$

$$2 + 3i - i^2 + (1 - i)(3 + 4i) = 2 + 3i - (-1) + 3 + 4i - 3i - 4(-1) = 10 + 4i$$

Use long division of polynomials to get $\frac{x^2-x-1}{x-3}=x+2+\frac{5}{x-3}$. Do a "spot-check": when x=0, we have $\frac{x^2-x-1}{x-3}=\frac{0^2-0-1}{0-3}=\frac{1}{3}$; when x=0, we have $x+2+\frac{5}{x-3}=0+2+\frac{5}{0-3}=\frac{6}{3}-\frac{5}{3}=\frac{1}{3}$. They agree when x=0! (A "spot-check" like this catches lots of mistakes.)

Use the change of base formula for logarithms: $\log_b x = \frac{\log_a x}{\log_a b}$

Thus,
$$\log_7 5 = \frac{\ln 5}{\ln 7}$$
. Check that $7^{(\log_7 5)} = 5$.

$$\frac{4\log 10^x}{3} = \frac{4}{3}x\log 10 = \frac{4}{3}x(1) = \frac{4}{3}x$$

Use properties of logarithms:

$$\ln x^4 - \ln x^2 + \ln(x^2 + 1) = \ln \frac{x^4}{x^2} + \ln(x^2 + 1) = \ln x^2 + \ln(x^2 + 1) = \ln x^2(x^2 + 1)$$

Use the row of Pascal's triangle beginning with "1 4": Thus, $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$. Since $(x-2y)^4 = (x+(-2y))^4$, we apply this formula with a=x and b=-2y to get:

$$(x + (-2y))^4 = x^4 + 4x(-2y)^3 + 6x^2(-2y)^2 + 4x^3(-2y) + (-2y)^4$$

$$= x^4 - 32xy^3 + 24x^2y^2 - 8x^3y + 16y^4$$

$$(-\infty, -2] \cap (-4, 5] = (-4, -2]$$

$$\{x \mid x \ge -2\} = [-2, \infty)$$
2.
$$(a) \quad 3x(1 - 5x)(x^2 - 16) = 0$$

$$x = 0 \quad \text{or} \quad 1 - 5x = 0 \quad \text{or} \quad x^2 - 16 = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{1}{5} \quad \text{or} \quad x = \pm 4$$
Solution set: $\{0, \frac{1}{5}, 4, -4\}$

(b)
$$\frac{1}{2}x - 7 = 3x + \frac{x}{5}$$

$$5x - 70 = 30x + 2x \text{ (clear fractions; multiply by 10)}$$

$$-70 = 27x$$

$$x = \frac{-70}{27}$$
Solution set: $\{-\frac{70}{27}\}$

(c)
$$|2x-3| > 5$$

 $2x-3 > 5$ or $2x-3 < -5$
 $2x > 8$ or $2x < -2$
 $x > 4$ or $x < -1$
Solution set: $(-\infty, -1) \cup (4, \infty)$

(d)
$$2 < |x| < 3$$

solve by inspection; want all #s whose distance from 0 is between 2 and 3

$$-3 < x < -2$$
 or $2 < x < 3$

Solution set: $(-3, -2) \cup (2, 3)$

(e)
$$1-2x \le 3$$
 or $-3 \le x < -2$
 $x \ge -1$ or $-3 \le x < -2$
 Solution set: $[-3, -2) \cup (-1, \infty)$

(f)
$$x^2 = x + 2$$

 $x^2 - x - 2 = 0$
 $(x - 2)(x + 1) = 0$
 $x = 2$ or $x = -1$
Solution set: $\{-1, 2\}$

(g)
$$2x - 3x^2 \le -1$$

 $-3x^2 + 2x + 1 \le 0$
 $3x^2 - 2x - 1 \ge 0$
Note: $3x^2 - 2x - 1 = 0$
 $x = \frac{2 \pm \sqrt{4 - 4(3)(-1)}}{6} = \frac{2 \pm \sqrt{16}}{6} = \frac{2 \pm 4}{6} = 1, -\frac{1}{3}$

Thus, the graph of $y = 3x^2 - 2x - 1$ crosses the x-axis at $-\frac{1}{3}$ and 1, and holds water.

Solution set:
$$(-\infty, -\frac{1}{3}] \cup [1, \infty)$$

(h)
$$3^{2x-1} = 10$$

 $\ln 3^{2x-1} = \ln 10$
 $(2x-1)\ln 3 = \ln 10$
 $2x-1 = \frac{\ln 10}{\ln 3}$
 $2x = \frac{\ln 10}{\ln 3} + 1$
 $x = \frac{1}{2}(\frac{\ln 10}{\ln 3} + 1)$
Solution set: $\left\{\frac{1}{2}(\frac{\ln 10}{\ln 3} + 1)\right\}$

(i)
$$\log_3(x^2 - 1) = -2$$

 $3^{-2} = x^2 - 1$
 $\frac{1}{9} = x^2 - 1$
 $x^2 = \frac{10}{9}$
 $x = \pm \sqrt{\frac{10}{9}}$
Solution set: $\left\{\sqrt{\frac{10}{9}}, -\sqrt{\frac{10}{9}}\right\}$

(j)
$$\sqrt{3x^2 + 5x - 3} = x$$

square both sides; must check for extraneous solutions at the end

$$3x^2 + 5x - 3 = x^2$$

Solve using the quadratic formula to get:

$$x = \frac{1}{2}$$
 or $x = -3$

Discard x = -3; it is an extraneous solution.

Verify that $x = \frac{1}{2}$ is indeed a solution.

Solution set: $\{\frac{1}{2}\}$

(k)
$$y = x^2 + 1$$
 and $y = 2x + 4$

A quick sketch verifies that there are two solutions:

$$x^2 + 1 = 2x + 4$$

 $x = 3$ or $x = -1$

When x = 3, y = 10; when x = -1, y = 2.

Solution set: $\{(3, 10), (-1, 2)\}$

(1)
$$x+3 = \frac{-2x^2+7x-3}{x-3}$$

Clear fractions; potential for an extraneous solution when x=3:

$$(x+3)(x-3) = -2x^2 + 7x - 3$$

Solve the quadratic equation, yielding:

$$x = 3 \text{ or } x = -\frac{2}{3}$$

Discard x = 3; it is an extraneous solution.

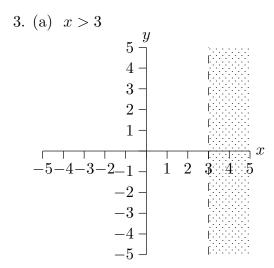
Solution set:
$$\left\{-\frac{2}{3}\right\}$$

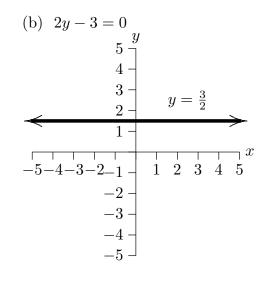
(m) Use a graphical approach to see that there are two solutions:

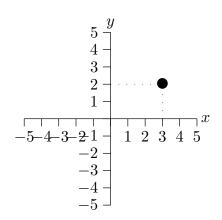
$$x + 2 = 1$$
 when $x = -1$

$$x - 1 = 1$$
 when $x = 2$

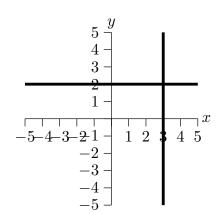
Solution set:
$$\{-1, 2\}$$



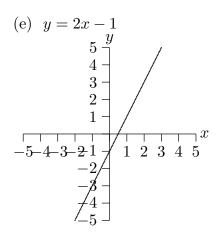


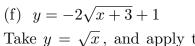


(c) x = 3 and y = 2

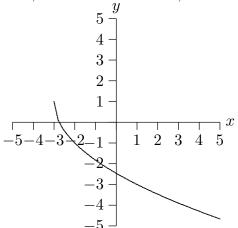


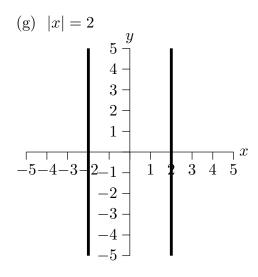
(d) x = 3 or y = 2

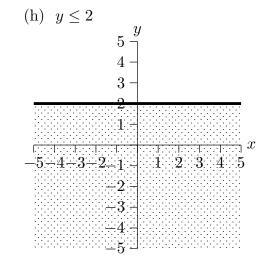




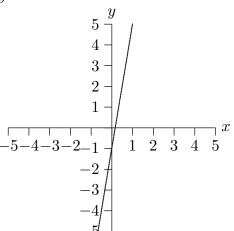
Take $y=\sqrt{x}$, and apply the following transformations: shift left 3; vertical stretch by a factor of 2; reflect about x-axis; move up 1. This gives:

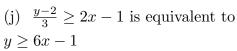


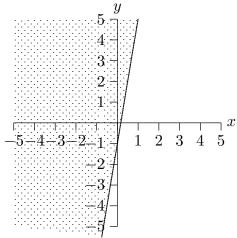


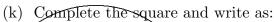


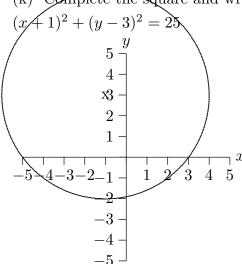
(i)
$$\frac{y-2}{3} = 2x - 1$$
 is equivalent to $y = 6x - 1$











Sample Prerequisite Problems: Precalculus—page 13

4.

$$\begin{array}{ll} \text{EQUATION} & \text{TRANSFORMATION} \\ y = f(x) & \text{(starting place)} \\ y = f(x+1) & \text{replace } x \text{ by } x+1 \text{; shift left 1} \\ y = |f(x+1)| & \text{take absolute value of } y\text{-values; any part below } x\text{-axis flips up} \\ y = 3|f(x+1)| & \text{multiply previous } y\text{-values by 3} \text{; vertical stretch} \\ y = -3|f(x+1)| & \text{multiply previous } y\text{-values by } -1 \text{; reflect about } x\text{-axis} \\ y = -3|f(x+1)| + 5 & \text{add 5 to previous } y\text{-values; move up 5} \end{array}$$

5.

EQUATION TRANSFORMATION
$$y = x^2 - 2x + 1$$
 (starting place)
$$y = x^2 - 2x + 2$$
 up 1
$$y = (x+3)^2 - 2(x+3) + 2$$
 left 3
$$y = -(x+3)^2 + 2(x+3) - 2$$
 reflect about the x-axis
$$y = -2(x+3)^2 + 4(x+3) - 4$$
 vertical scale by a factor of 2

6. PERIMETER = $2\ell + 2w$, AREA = ℓw PERIMETER = 2x + y, AREA = $\frac{1}{2}(y)(\frac{x}{2}) = \frac{1}{4}xy$ CIRCUMFERENCE = $2\pi r$, AREA = πr^2 VOLUME = (area of base)(height) = $\pi R^2 h$

Meter is a unit of length; cm² is a unit of area; cubic feet is a unit of volume.

7.

(a)
$$g(f(x)) = g(x^2 - 2x + 1)$$

 $= 1 - 3(x^2 - 2x + 1)$
 $= 1 - 3x^2 + 6x - 3$
 $= -3x^2 + 6x - 2$

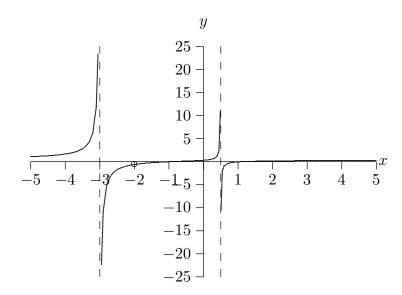
$$f(g(x)) = f(1 - 3x)$$

$$= (1 - 3x)^{2} - 2(1 - 3x) + 1$$

$$= 1 - 6x + 9x^{2} - 2 + 6x + 1$$

$$= 9x^{2}$$

(b) (There are other possible correct answers.) Let $g(x) = x^2 - 1$ and $f(x) = \sqrt[3]{x}$.



For
$$x \neq -2$$
, $g(x) = \frac{x^2 - 1}{(2x - 1)(x + 3)}$.

Note that the point $(-2, -\frac{3}{5})$ is a puncture point.

x-intercepts occur when $x = \pm 1$.

y-intercept: $(0,\frac{1}{3})$

8.

horizontal asymptote: $y = \frac{1}{2}$

vertical asymptotes: $x = \frac{1}{2}$ and x = -3

no slant asymptote

As $x \to \infty$, $y \to \frac{1}{2}$.

As $x \to -3^+$, $y \to -\infty$.

9. Since P(-3) = 0, P has a factor of x + 3.

Since 1 is a zero of P, x-1 is a factor.

Since the graph of P crosses the x-axis at x = 2, x - 2 is a factor.

Since P must have degree 5, I'll choose to make 1 a zero of multiplicity 3. (There are other possible choices here.) Thus, the polynomial takes on the following form:

$$P(x) = K(x+3)(x-1)^3(x-2)$$

Since P(0) = 7, we have:

$$K(3)(-1)^3(-2) = 7$$
$$6K = 7$$
$$K = \frac{7}{6}$$

Thus,
$$P(x) = (x + 3)(x - 1)^3(x - 2)$$
.

10. (a)
$$2x - 3$$

(b)
$$2(x-3)$$

(c)
$$\frac{(2x)^3+1}{x}$$

- (d) take a number, multiply by 3, then subtract 1
- (e) take a number, add 1, cube the result, multiply by 2, then subtract 5
- (f) take a number, subtract 3, divide by 7, then subtract 1

11. (a)
$$f(0) = 0^2 - 2(0) + 1 = 1$$

(b)
$$f(1) - 2 = (1^2 - 2 \cdot 1 + 1) - 2 = 0 - 2 = -2$$

(c)
$$f(f(-1)) = f((-1)^2 - 2(-1) + 1) = f(4) = 4^2 - 2(4) + 1 = 9$$

12. The function g is defined whenever x-3>0, that is, whenever x>3. The domain of g is the interval $(3,\infty)$.

13. (a)
$$y = 3x - 7$$

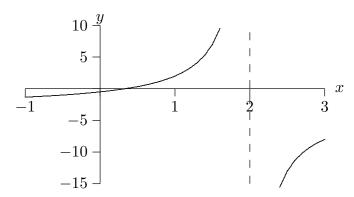
(b)
$$y = 2$$

(c) The line x - 3y = 5 has slope $\frac{1}{3}$; a perpendicular line will have slope -3.

The line with slope -3 passing through (0,3) has equation y = -3x + 3.

14. (a) The domain of f is the set of all real numbers except 2.

(b)



- (c) The graph crosses the x-axis at $\frac{1}{3}$. (Set 1-3x=0. Be sure you can get this exact answer, not just $x \approx 0.333333$.)
- (d) When x=1.375 (exactly), then f(x)=5. (You could check this, if desired, by solving the equation $5=\frac{1-3x}{x-2}$.)

15. (a)
$$\frac{1+\sqrt{2}}{\sqrt[3]{5}-7} \approx -0.45637$$

- (b) 19.72 (this is exact)
- (c) $|1 2\sqrt{3}| \approx 2.46410$
- (d) $(2.03 \times 10^{-9})(-4.1 \times 10^7) = -0.08323$ (this is exact)