ALGEBRA II OBJECTIVE: EXP3

algebraic definition of $|x|$:

$$|x| = \begin{cases} x\,, & \text{if } x \geq 0 \\ -x\,, & \text{if } x < 0 \end{cases}$$

READING ALOUD:

$-x$	negative x ; or, the opposite of x
$\lvert x\rvert$	the absolute value of x
$\lvert x\rvert = \begin{cases} x\,, & \text{if } x \geq 0 \\ -x\,, & \text{if } x < 0 \end{cases}$	the absolute value of x is x , if x is greater than or equal to zero; the absolute value of x is the opposite of x , if x is less than zero

DISCUSSION OF CONCEPT:

Sometimes we don't care if a number is positive (i.e., lies to the right of zero) or negative (i.e., lies to the left of zero), we just want to know its *size*—its distance from zero. In these situations, the concept of *absolute value* comes to the rescue!

The number $|x|$ gives the distance between x and 0 . Whenever you come across $|x|$, think to yourself: the distance between x and 0 .

Since the number 3 is three units from zero, $|3| = 3$. Since the number -3 is also 3 units from zero, $|-3| = 3$.

To solve the equation $|x| = 3$, think to yourself: What number(s) x have a distance from zero that is equal to 3 ? Answers: 3 and -3

To solve the inequality $|x| < 3$, think to yourself: What number(s) x have a distance from zero that is less than 3 ? Answer: all real numbers between -3 and 3 (not including the endpoints)

If a number is positive or zero, then the number itself tells us its distance from zero. That is, if $x \geq 0$, then $|x| = x$.

If a number is negative, then the opposite of the number gives its distance from zero. That is, if $x < 0$, then $|x| = -x$.

IMPORTANT!!! If x is negative, then $-x$ (the opposite of x) is POSITIVE.

Both cases can be summarized compactly like this:

ALGEBRAIC DEFINITION OF ABSOLUTE VALUE:

$$|x| = \begin{cases} x\,, & \text{if } x \geq 0 \\ -x\,, & \text{if } x < 0 \end{cases}$$

This is called a 'piecewise-defined function', because there are two PIECES to the function: one piece applies when x is nonnegative (the first row); the second piece applies when x is negative (the second row).

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ALGEBRA II OBJECTIVE: EXP3—SAMPLE PROBLEMS and SOLUTIONS

READ ALOUD:

1. $|x|$
2. $|t|$
3. $|x-1|$
4. $|x^2+1|$

WRITE EACH EXPRESSION WITHOUT ABSOLUTE VALUES, as simply as possible:

5. $|x|$, if $x > 0$
6. $|x|$, if $x < 0$
7. $|t|$, if $t > 5$
8. $|t|$, if $t < -2$
9. $|t-1|$, if $t \geq 1$
10. $|t-1|$, if $t < 1$
11. $|t-1|$, if $t > 4.2$
12. $|x^2+1|$

WRITE EACH ABSOLUTE VALUE EXPRESSION AS A PIECEWISE-DEFINED FUNCTION, as simply as possible:

13. $|y|$
14. $|x-2|$
15. $|3-2x|$

SOLUTIONS:

1. the absolute value of x
2. the absolute value of t
3. the absolute value of x minus one
4. the absolute value of x squared plus one
5. if $x > 0$, then $|x| = x$
6. if $x < 0$, then $|x| = -x$
7. if $t > 5$, then $|t| = t$
8. if $t < -2$, then $|t| = -t$
9. if $t \geq 1$, then $t - 1 \geq 0$, so $|t-1| = t-1$
10. if $t < 1$, then $t - 1 < 0$, so $|t-1| = -(t-1) = -t+1 = 1-t$
11. If $t > 4.2$, then $|t-1| = t-1$
12. since x^2+1 is ALWAYS positive, $|x^2+1| = x^2+1$ for all values of x
13. $|y| = \begin{cases} y\,, & \text{if } y \geq 0 \\ -y\,, & \text{if } y < 0 \end{cases}$
14. $|x-2| = \begin{cases} x-2\,, & \text{if } x \geq 2 \\ 2-x\,, & \text{if } x < 2 \end{cases}$
15. $|3-2x| = \begin{cases} 3-2x\,, & \text{if } x \leq \frac{3}{2} \\ 2x-3\,, & \text{if } x > \frac{3}{2} \end{cases}$

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