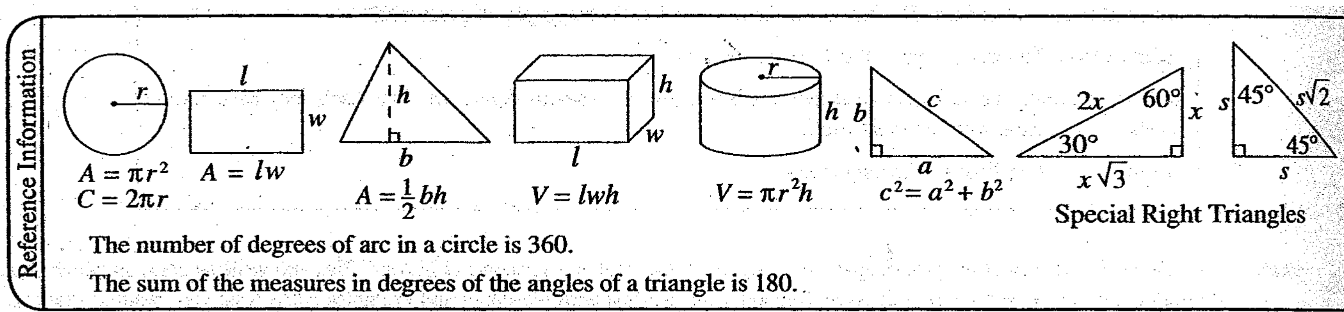
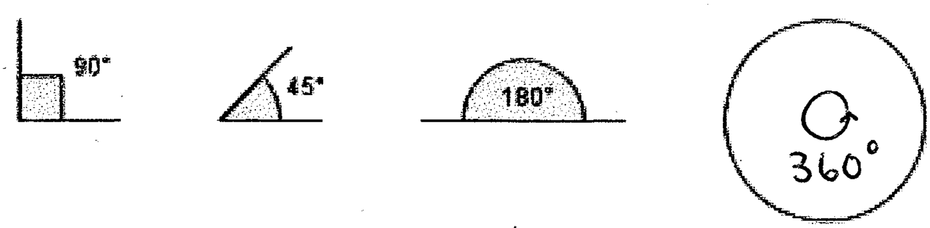
SATstuff#7

About one-third of the math problems on the SAT involve Geometry—mixed in with other ideas!

You are GIVEN many of the math formulas that you need at the beginning of each math portion:



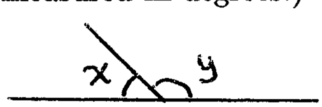
COMMON ANGLES
One complete revolution is 360 degrees (denoted 360°).
So, 1° is $\frac{1}{360}$ of a complete revolution.
A 90° angle is called a *right angle* and is always marked with a little box (see below).
Make sure you know what common angles look like:



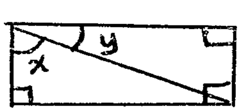
YOU TRY THESE:

(Assume all angles are measured in degrees.)

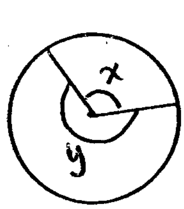
(1) What is $x + y$?



(2) What is $2(x + y)$?

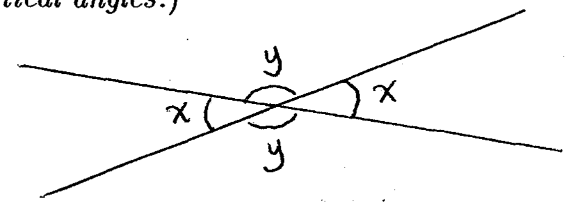


(3) What is $\frac{x + y}{3}$?



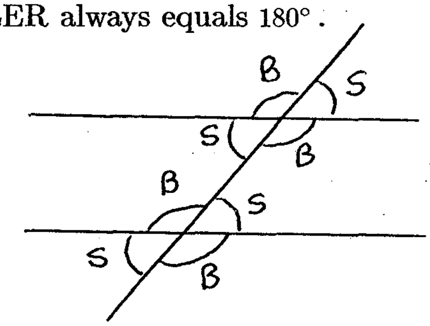
(1) 180° (2) 180° (3) 120°

INTERSECTING LINES
Whenever two lines intersect, the angles opposite each other always have the same measure. (These are called *vertical angles*.)



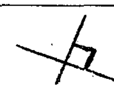
Also,
$x + y = 180°$

PARALLEL LINES
Lines that have the same "slant" (that is, the same slope) are called PARALLEL.
Whenever two parallel lines are cut by another (non-perpendicular) line, there are only TWO different angles that emerge!
All the bigger angles are the same, and all the smaller angles are the same.
BIGGER + SMALLER always equals 180°.

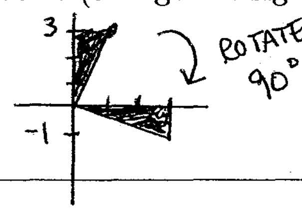


$B + S = 180°$

PERPENDICULAR LINES
Lines that meet at a 90° angle are called *perpendicular*.



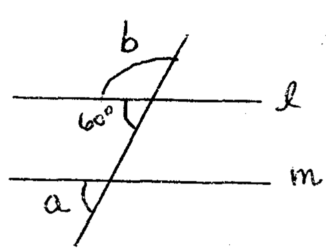
If two lines are perpendicular, then their slopes are opposite reciprocals (change the sign, and flip).
For example, lines with slopes of 3 and $-\frac{1}{3}$ are perpendicular.
Also, lines with slopes of $-\frac{2}{3}$ and $\frac{3}{2}$ are perpendicular.



YOU TRY THESE:
Assume all angles are measured in degrees.

(4) Find $3x + y$:

(5) Suppose line ℓ is parallel to line m.
Find $\frac{a}{2} + \frac{b}{3}$.



(6) Suppose a line has slope $-\frac{1}{7}$. What is the slope of a perpendicular line?

(7) Suppose a line has slope m. What is the slope of a perpendicular line?

(4) 210° (5) 70° (6) 7 (7) $-\frac{1}{m}$

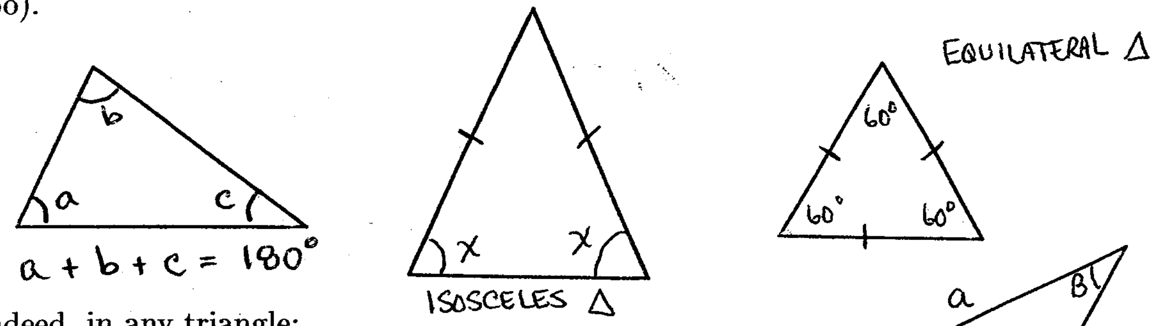
TRIANGLES

The sum of the interior angles in ANY triangle is 180°.

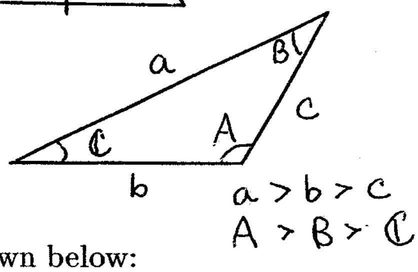
If angles are the same, then their opposite sides are the same (and vice versa).

An *equilateral* triangle has all sides the same length (so all angles are the same, too... so each angle is 60°.)

An *isosceles* triangle has two sides the same length (so the opposite angles are the same, too).

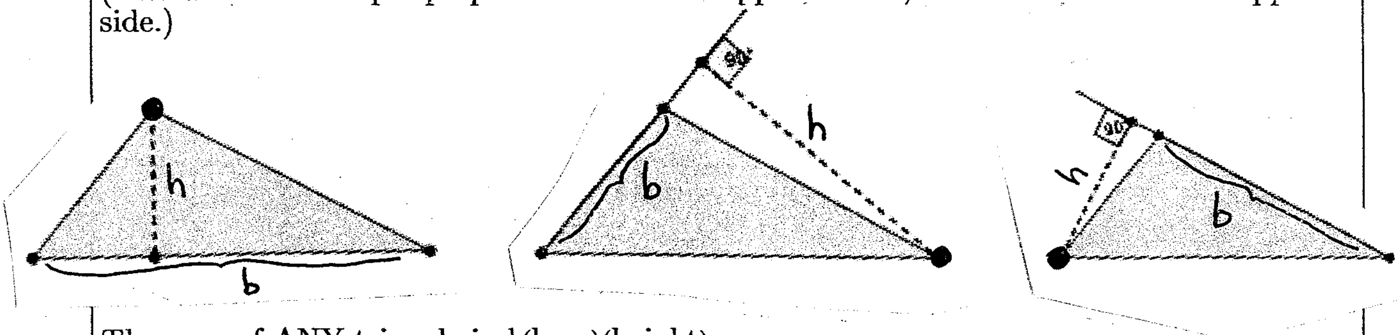


Indeed, in any triangle:
- the longest side is opposite the biggest angle
- the shortest side is opposite the smallest angle
- the medium side is opposite the medium angle



Every triangle has three different base/height pairs, as shown below:

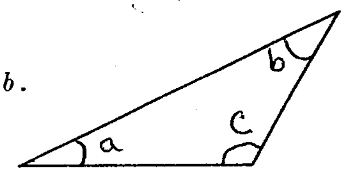
(Pick a vertex. Drop a perpendicular to the opposite side, or an extension of the opposite side.)



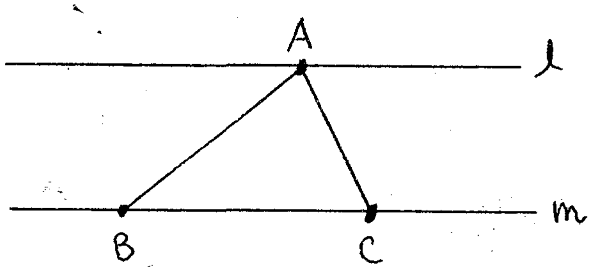
The area of ANY triangle is $\frac{1}{2}$(base)(height).

YOU TRY THESE:

(8) Give a formula for c, in terms of a and b.



(9) Line ℓ is parallel to line m, and the distance between them is 3.
The distance from B to C is 6.
Find the area of triangle ABC.



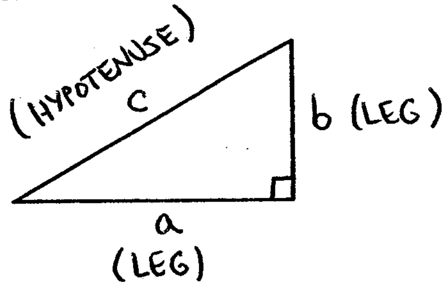
(8) $c = 180° - a - b$ (9) $\frac{1}{2}(6)(3) = 9$

RIGHT TRIANGLES

A *right triangle* has a 90° angle; the remaining two angles must add to 90°.
The longest side (the one opposite the 90° angle) is called the HYPOTENUSE.
The two shorter sides are called the LEGS.
The PYTHAGOREAN THEOREM gives a beautiful relationship between the lengths of the sides in any right triangle:
If a and b are the two shorter sides, and c is the hypotenuse, then:

$$a^2 + b^2 = c^2$$

Also, if the lengths of a triangle satisfy this equation (i.e., make it TRUE), then the triangle MUST BE a right triangle!

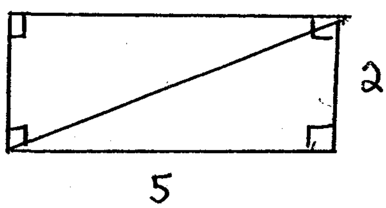


$a^2 + b^2 = c^2$

YOU TRY THESE:

(10) Suppose the lengths of the sides of a triangle are 1, 3, and $\sqrt{10}$.
Is is a right triangle?
Which side is the hypotenuse?

(11) Find the length of the diagonal of the rectangle:

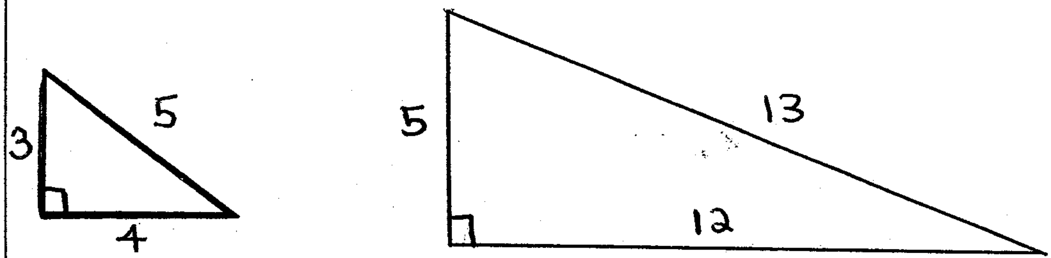


(10) $1^2 + 3^2 = 10$; $(\sqrt{10})^2 = 10$; Yes! The hypotenuse has length $\sqrt{10}$.

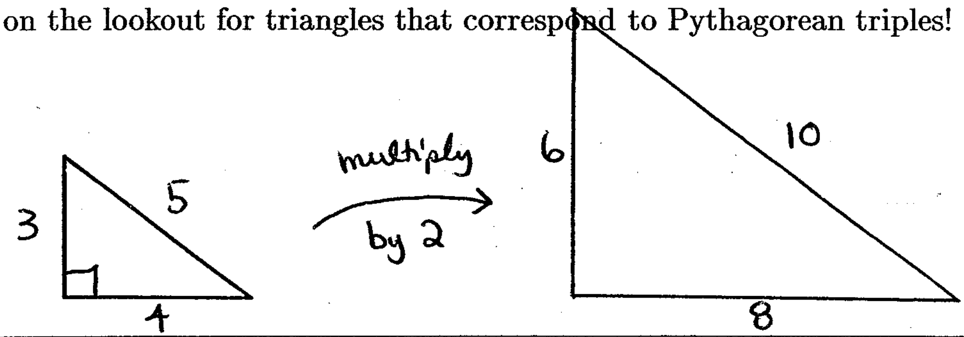
(11) $2^2 + 5^2 = x^2$
$x = \sqrt{29}$

SPECIAL TRIANGLES

Here are a couple right triangles with particularly simple sides:
notice that $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$.
(Numbers like 3-4-5 and 5-12-13 are called *Pythagorean triples*.)



When you *scale* a triangle (that is, multiply ALL its sides by the same number), then the angles don't change.
Triangles with the same angles are called SIMILAR triangles.
So, 6-8-10 and 9-12-15 and 10-24-26 are all Pythagorean triples. (Check them out!)
Be on the lookout for triangles that correspond to Pythagorean triples!



YOU TRY THESE:

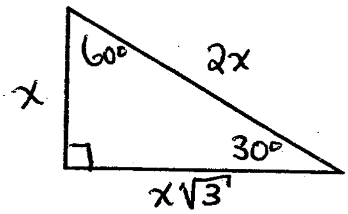
(12) Suppose you're given a right triangle.
The length of one leg is 4 and the hypotenuse has length 5.
What is the length of the remaining side?

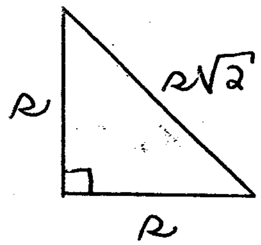
(13) Suppose you're given a right triangle.
The lengths of its legs are 5 and 12.
What is the length of the hypotenuse?

(14) Suppose you're given a triangle.
The lengths of its sides are 6, 8, and 10.
Is it a right triangle? If so, which side is opposite the right angle?

(12) 3 (13) 13 (14) Yes; 10 is opposite the right angle

MORE SPECIAL TRIANGLES
In the formula sheet, you're given two more special triangles:
the 30°-60°-90° triangle, and the 45°-45°-90° triangle.
They look like this:



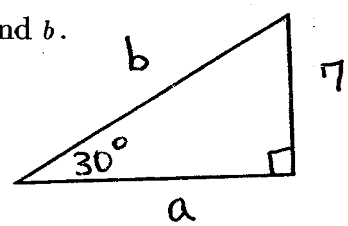


Here's how to use this information:

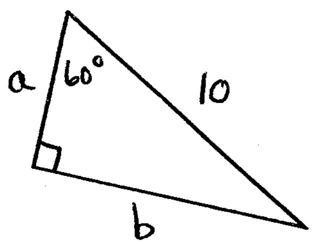
- In a 30°-60°-90° triangle, the shortest side is opposite the 30° angle.
  The hypotenuse is TWICE as long as the shortest side.
  The medium-length side is $\sqrt{3}$ times as long as the shortest side ($\sqrt{3} \approx 1.7$).
- In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg ($\sqrt{2} \approx 1.4$).

YOU TRY THESE:

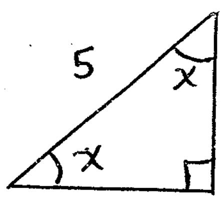
(15) Find a and b.



(16) Find a and b.



(17) Find x. Then find the length of each leg.



(15) $a = 7\sqrt{3}$
$b = 14$

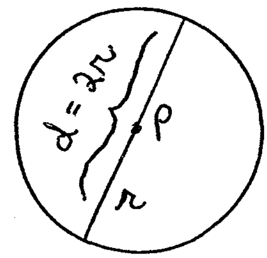
(16) $a = 5$
$b = 5\sqrt{3}$

(17) $x = 45°$
Each leg is $\frac{5}{\sqrt{2}}$

Note: $\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

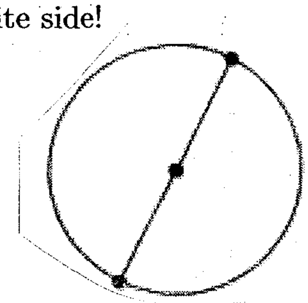
CIRCLES

Give me a POINT (call it P) and a POSITIVE NUMBER (call it r),
and I can make you a circle:
it's ALL the points that are distance r from P.
The point P is called the *center* of the circle,
and the number r is called the *radius* of the circle.



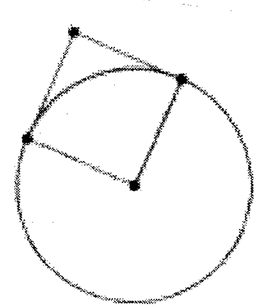
The *diameter* of a circle is twice the radius ($d = 2r$).

The CIRCUMFERENCE of a circle is the distance around: $C = 2\pi r$
Or, in terms of the diameter ($d = 2r$): $C = \pi d$ ($\pi \cong 3.14$; π is a little more than 3)
Here's a good way to think of this:
the distance AROUND any circle is a little more than three times the distance straight through the center to the opposite side!



DIAMETER = 2.5
THREE TIMES THE DIAMETER = 7.5
CIRCUMFERENCE = 7.9

The AREA of a circle is found by squaring the radius, and multiplying by π: $A = \pi r^2$
Since π is a little more than 3, here's a good way to think of this:
the area of the circle is a little more than 3 times the area of the square that is made from the radius.



AREA OF SQUARE = 2.4
THREE TIMES THE AREA OF THE SQUARE = 7.2
AREA OF CIRCLE = 7.54

(By the way, SAT gives you the circumference and area formulas for a circle.)

A *tangent* line to a circle is a line that is perpendicular to a radius; it touches the circle (as shown below) at only one point.