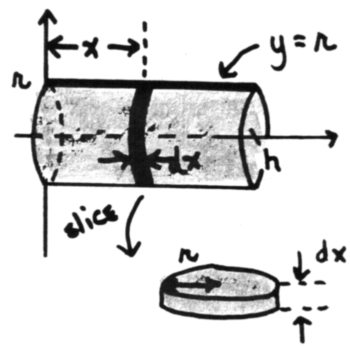
SECTION 7.6 Finding the Volume of a Solid of Revolution—Disks

IN-SECTION EXERCISES:

EXERCISE 2.

2. A cylinder of height h and radius r is easily generated by taking the graph of $y = r$, and rotating it about the x-axis on the interval $[0, h]$. A typical 'slice' is a disk with volume:

$$\pi r^2 \, dx$$



'Summing' these disks as x travels from 0 to h yields

$$\int_0^h \pi r^2 \, dx = \pi r^2 x \,\big|_0^h = \pi r^2 h \ ,$$

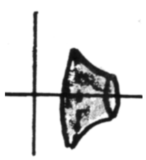
which is the desired volume.

END-OF-SECTION EXERCISES:

1. 

$$\int_0^1 \pi(2x)^2 \, dx = \int_0^1 4\pi x^2 \, dx = 4\pi \frac{x^3}{3} \,\big|_0^1 = \frac{4\pi}{3}(1-0) = \frac{4\pi}{3}$$

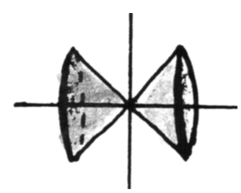
2. $$\int_1^2 \pi(x^3)^2 \, dx = \int_1^2 \pi x^6 \, dx = \pi \frac{x^7}{7} \,\big|_1^2 = \frac{\pi}{7}(2^7 - 1) = \frac{127\pi}{7}$$

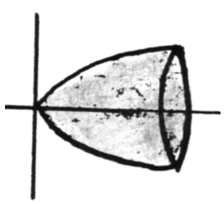
3. 

$$\int_1^2 \pi(\frac{1}{x})^2 \, dx = \int_1^2 \pi x^{-2} \, dx = \pi \cdot \frac{x^{-1}}{-1} \,\big|_1^2 = -\pi \cdot \frac{1}{x} \,\big|_1^2 = -\pi(\frac{1}{2} - 1) = \frac{1}{2}\pi$$

4. Take advantage of symmetry; find half the desired volume, then double. For $x \ge 0$, $|x| = x$. The desired volume is:

$$2 \int_0^1 \pi x^2 \, dx = 2\pi \frac{x^3}{3} \,\big|_0^1 = \frac{2\pi}{3}(1-0) = \frac{2\pi}{3}$$

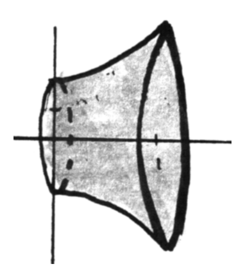


5. 

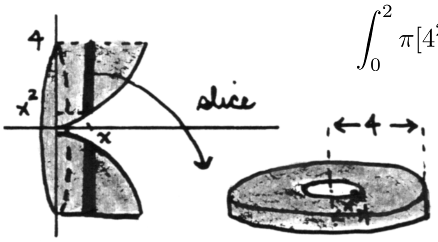
$$\int_0^4 \pi(\sqrt{x})^2 \, dx = \pi \frac{x^2}{2} \,\big|_0^4 = \frac{\pi}{2}(16 - 0) = 8\pi$$

6.

$$\begin{aligned}
\int_0^1 \pi(e^x + 1)^2 \, dx &= \pi \int_0^1 (e^{2x} + 2e^x + 1) \, dx \\
&= \pi(\frac{1}{2}e^{2x} + 2e^x + x)\big|_0^1 \\
&= \pi\big[(\frac{1}{2}e^2 + 2e + 1) - (\frac{1}{2}e^0 + 2e^0 + 0)\big] \\
&= \pi(\frac{1}{2}e^2 + 2e + 1 - \frac{1}{2} - 2) = \pi(\frac{1}{2}e^2 + 2e - \frac{3}{2})
\end{aligned}$$



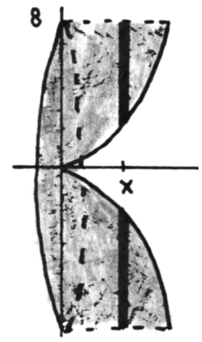
7. intersection point: $4 = x^2 \iff x = \pm 2$. A typical 'slice' has outer radius 4 and inner radius x^2. The desired volume is:

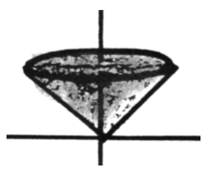


$$\begin{aligned}\int_0^2 \pi[4^2 - (x^2)^2]\,dx &= \pi \int_0^2 (16 - x^4)\,dx \\ &= \pi(16x - \frac{x^5}{5})\Big|_0^2 \\ &= \pi(32 - \frac{32}{5}) = \frac{128\pi}{5}\end{aligned}$$

8. intersection point: $x^3 = 8 \iff x = 2$. A typical 'slice' has outer radius 8 and inner radius x^3. The desired volume is:

$$\begin{aligned}\int_0^2 \pi(8^2 - (x^3)^2)\,dx &= \pi \int_0^2 (64 - x^6)\,dx \\ &= \pi(64x - \frac{x^7}{7})\Big|_0^2 \\ &= \frac{768\pi}{7}\end{aligned}$$



9. 

$$\int_0^2 \pi y^2\,dy = \pi \frac{y^3}{3}\Big|_0^2 = \frac{\pi}{3}(8 - 0) = \frac{8\pi}{3}$$

10. Note that $y = 2x \iff x = \frac{y}{2}$. Thus, a typical 'slice at a distance y has radius $\frac{y}{2}$. The desired volume is:

$$\int_1^2 \pi(\frac{y}{2})^2\,dy = \frac{\pi}{4}\int_1^2 y^2\,dy = \frac{\pi}{4}\cdot\frac{y^3}{3}\Big|_1^2 = \frac{\pi}{12}(8-1) = \frac{7\pi}{12}$$

11. Note that $y = \frac{1}{x} \iff x = \frac{1}{y}$. A typical 'slice' at a distance y has outer radius $\frac{1}{y}$ and inner radius $\frac{1}{2}$. The desired volume is:

$$\begin{aligned}\int_1^2 \pi([(\frac{1}{y})^2 - (\frac{1}{2})^2]\,dy &= \pi\int_1^2 (y^{-2} - \frac{1}{4})\,dy \\ &= \pi(\frac{y^{-1}}{-1} - \frac{1}{4}y)\Big|_1^2 \\ &= \pi(-\frac{1}{y} - \frac{1}{4}y)\Big|_1^2 \\ &= \pi[(-\frac{1}{2} - \frac{2}{4}) - (-1 - \frac{1}{4})] \\ &= \pi[(-1) - (-\frac{5}{4})] = \pi(-\frac{4}{4} + \frac{5}{4}) = \frac{\pi}{4}\end{aligned}$$