SECTION 6.2 Some Basic Antidifferentiation Formulas

IN-SECTION EXERCISES:

EXERCISE 1.

1. $\int 3x^2\,dx = x^3 + C$

2.

$$\int 3x^2\,dx = x^3 + C \iff 3\int x^2\,dx = x^3 + C$$
$$\iff \int x^2\,dx = \frac{1}{3}(x^3 + C)$$
$$\iff \int x^2\,dx = \frac{1}{3}x^3 + K$$

3. $\int 5x^2\,dx = 5\int x^2\,dx = 5(\frac{x^3}{3}) + C = \frac{5}{3}x^3 + C$

EXERCISE 2.

1.

$$\int (ax^2 + bx + c)\,dx = a\frac{x^3}{3} + b\frac{x^2}{2} + cx + C = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$$

2.

$$\int \frac{2\sqrt{t} - 1}{t^2}\,dt = \int \frac{2\sqrt{t}}{t^2} - \frac{1}{t^2}\,dt = \int \frac{2t^{1/2}}{t^2} - t^{-2}\ dt$$
$$= \int 2t^{-3/2} - t^{-2}\ dt = 2(\frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}) - \frac{t^{-2+1}}{-2+1} + C$$
$$= 2\frac{t^{-1/2}}{-1/2} - \frac{t^{-1}}{-1} + C = -\frac{4}{\sqrt{t}} + \frac{1}{t} + C$$

3.

$$\int (1 + \sqrt[3]{x})^2\,dx = \int (1 + 2\sqrt[3]{x} + (\sqrt[3]{x})^2\ dx = \int 1 + 2x^{1/3} + (x^{1/3})^2\ dx$$
$$= \int (1 + 2x^{1/3} + x^{2/3})\ dx = x + 2(\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}) + \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + C$$
$$= x + \frac{2x^{4/3}}{4/3} + \frac{x^{5/3}}{5/3} + C = x + \frac{3}{2}\sqrt[3]{x^4} + \frac{3}{5}\sqrt[3]{x^5} + C$$

4.

$$\int \sqrt{\frac{3\pi}{y^4}} - e^y\ dy = \int \sqrt{3\pi}(y^4)^{-1/2} - e^y\ dy = \int \sqrt{3\pi}y^{-2} - e^y\ dy$$
$$= \sqrt{3\pi}\frac{y^{-1}}{-1} - e^y + C = -\frac{\sqrt{3\pi}}{y} - e^y + C$$

5.

$$\int \left(\frac{\sqrt{x} - 1}{x}\right)^2\ dx = \int \frac{x - 2\sqrt{x} + 1}{x^2}\ dx = \int \frac{1}{x} - 2x^{-3/2} + x^{-2}\ dx$$
$$= \ln|x| - 2\frac{x^{-1/2}}{-1/2} + \frac{x^{-1}}{-1} + C = \ln|x| + \frac{4}{\sqrt{x}} - \frac{1}{x} + C$$

EXERCISE 3.

1. First, find *all* antiderivatives of $2x - 3$:

$$\int (2x-3)\,dx = 2\frac{x^2}{2} - 3x + C = x^2 - 3x + C$$

Thus, every function $f(x) = x^2 - 3x + C$ has derivative $2x - 3$. Now, find C so that $f(0) = 4$:

$$f(0) = 4 \quad \iff \quad 0^2 - 3(0) + C = 4 \quad \iff \quad C = 4$$

Thus, $f(x) = x^2 - 3x + 4$ satisfies the desired properties.

2.

$$\int \sqrt{x}\;dx = \int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} + C = \frac{2}{3}\sqrt{x^3} + C$$

Let $f(x) = \frac{2}{3}\sqrt{x^3} + C$. Then:

$$f(1) = -2 \quad \iff \quad \frac{2}{3}\sqrt{1^3} + C = -2 \quad \iff \quad C = -2 - \frac{2}{3} \quad \iff \quad C = -\frac{8}{3}$$

Take $f(x) = \frac{2}{3}\sqrt{x^3} - \frac{8}{3}$.

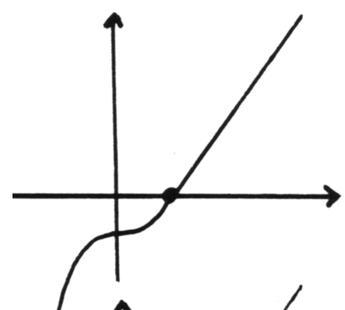
EXERCISE 4.

1. For $x > 1$, f must have the form: $f(x) = 2x + C$
For $x < 1$, f must have the form: $f(x) = x^3 + K$
Since $f(1) = 0$ and f is continuous at 1, we must have BOTH

$$2(1) + C = 0 \quad \text{and} \quad 1^3 + K = 0$$

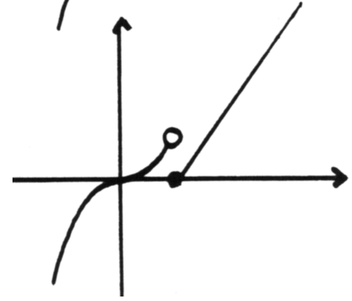
so that $C = -2$ and $K = -1$. Thus, define:

$$f(x) = \begin{cases} 2x - 2 & \text{for } x \ge 1 \\ x^3 - 1 & \text{for } x < 1 \end{cases}$$



2. Just move one 'piece' up or down! Say, define:

$$f(x) = \begin{cases} 2x - 2 & \text{for } x \ge 1 \\ x^3 & \text{for } x < 1 \end{cases}$$



EXERCISE 5.

1.

$$\int \frac{1-\sqrt{x}}{x}\;dx = \int \frac{1}{x} - \frac{x^{1/2}}{x}\;dx = \int \frac{1}{x} - x^{-1/2}\;dx$$

$$= \ln|x| - \frac{x^{1/2}}{1/2} + C = \ln|x| - 2\sqrt{x} + C$$

2.

$$\int \left(\frac{t+1}{t}\right)^2\;dt = \int \frac{t^2+2t+1}{t^2}\;dt = \int (1 + \frac{2}{t} + t^{-2})\,dt$$

$$= t + 2\ln|t| + \frac{t^{-1}}{-1} + C = t + 2\ln|t| - \frac{1}{t} + C$$

3.
$$\int \frac{1}{7x} + e^{-x} + 1 \ dx = \frac{1}{7}\ln|x| - e^{-x} + x + C$$

END-OF-SECTION EXERCISES:

A *radical* is an expression of the form: $\sqrt[n]{\text{something}}$

A *binomial* is a polynomial with two terms; a *binomial squared* is an expression of the form $(ax^n + bx^m)^2$ for real numbers a and b, and nonnegative integers n and m .

A *rational function* is a ratio of polynomials.