SECTION 5.5 More Graphing Techniques

IN-SECTION EXERCISES:

EXERCISE 1.

1. Find A and B for which:

$$AB = (3)(-1) = -3 \text{ and } A + B = 2$$

Choosing $A = 3$ and $B = -1$ works. Then:

$$\begin{aligned} 3x^2 + 2x - 1 &= 3x^2 + 3x - x - 1 \\ &= 3x(x+1) - (x+1) \\ &= (3x-1)(x+1) \end{aligned}$$

2. Find A and B for which:

$$AB = (10)(-3) = -30 \text{ and } A + B = -13$$

Choosing $A = -15$ and $B = 2$ works. Then:

$$\begin{aligned} 10x^2 - 13x - 3 &= 10x^2 - 15x + 2x - 3 \\ &= 5x(2x-3) + (2x-3) \\ &= (5x+1)(2x-3) \end{aligned}$$

3. Find A and B for which:

$$AB = (14)(-3) = -42 \text{ and } A + B = 19$$

Choosing $A = 21$ and $B = -2$ works. Then:

$$\begin{aligned} 14x^2 + 19x - 3 &= 14x^2 + 21x - 2x - 3 \\ &= 7x(2x+3) - (2x+3) \\ &= (7x-1)(2x+3) \end{aligned}$$

EXERCISE 2.

1. The solutions to $3x^2 + 2x - 1 = 0$ are given by:

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{2^2 - 4(3)(-1)}}{2 \cdot 3} \\ &= \frac{-2 \pm \sqrt{16}}{6} \\ &= \frac{-2 \pm 4}{6} = \frac{1}{3},\ -1 \end{aligned}$$

Then:

$$3x^2 + 2x - 1 = 3(x - \frac{1}{3})(x+1) = (3x-1)(x+1)$$

2. The solutions to $10^2 - 13x - 3 = 0$ are given by:

$$\begin{aligned} x &= \frac{13 \pm \sqrt{(-13)^2 - 4(10)(-3)}}{2 \cdot 10} \\ &= \frac{13 \pm \sqrt{289}}{20} \\ &= \frac{13 \pm 17}{20} = \frac{3}{2}, \ -\frac{1}{5} \end{aligned}$$

Then:

$$\begin{aligned} 10x^2 - 13x - 3 &= 10(x - \frac{3}{2})(x + \frac{1}{5}) \\ &= 2(x - \frac{3}{2}) \cdot 5(x + \frac{1}{5}) \\ &= (2x - 3)(5x + 1) \end{aligned}$$

3. The solutions to $14^2 + 19x - 3 = 0$ are given by:

$$\begin{aligned} x &= \frac{-19 \pm \sqrt{(19)^2 - 4(14)(-3)}}{2 \cdot 14} \\ &= \frac{-19 \pm \sqrt{529}}{28} \\ &= \frac{-19 \pm 23}{28} = \frac{1}{7}, \ -\frac{3}{2} \end{aligned}$$

Then:

$$14x^2 + 19x - 3 = 14(x - \frac{1}{7})(x + \frac{3}{2}) = 7(x - \frac{1}{7})2(x + \frac{3}{2}) = (7x - 1)(2x + 3)$$

EXERCISE 3.

A	B	A∨B	¬(A∨B)	¬A	¬B	¬A∧¬B
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

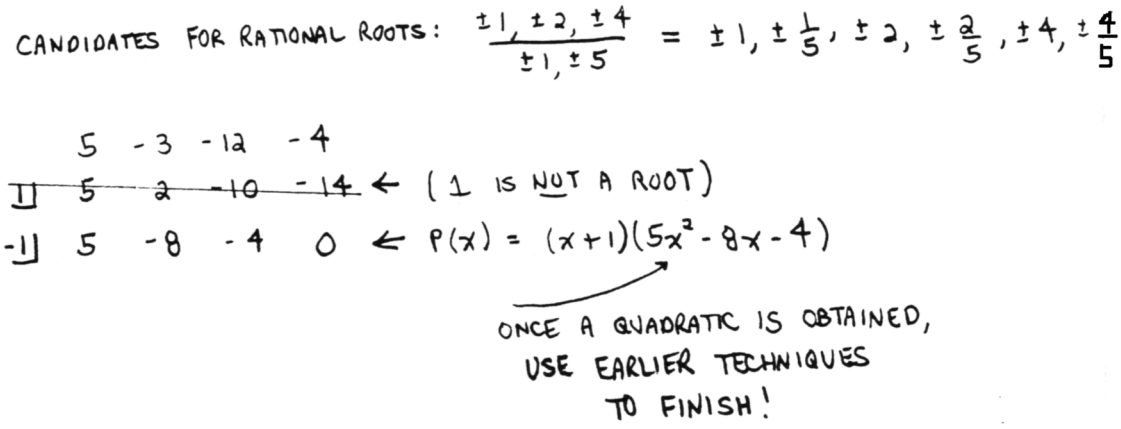
¬(A∨B) and (¬A)∧(¬B) ALWAYS HAVE THE SAME TRUTH VALUES!

EXERCISE 4.

1. $Q(x) = x^4 - 2$ and $T(x) = x^4 + x^3 + x^2 + x - 2$ will have precisely the same candidates for rational roots.

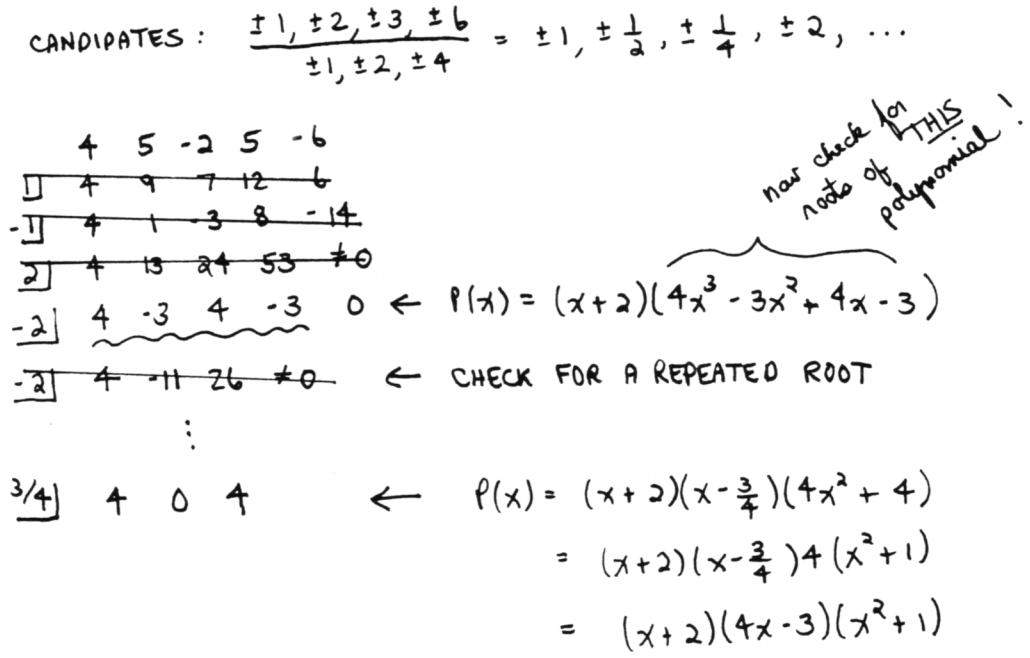
2. $\underbrace{5x^3 - 3x^2 - 12x - 4}_{:=P(x)} = (x-2)(x+1)(5x+2)$

   Here is one correct approach to obtaining this factorization:



3. $\underbrace{4x^4 + 5x^3 - 2x^2 + 5x - 6}_{:=P(x)} = (x+2)(4x-3)(x^2+1)$

   Here is one correct approach:



4. $\overbrace{3x^4 - x^3 + 12x^2 - 4x}^{:=P(x)} = x(3x-1)(x^2+4)$

Here is one correct approach:

CANDIDATES: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3} = \pm 1, \pm \frac{1}{3}, \pm 2, \ldots$

$P(x) = x(\underbrace{3x^3 - x^2 + 12x - 4}_{\tilde{P}(x)})$

	3	-1	12	-4
1	3	2	14	10
-1	3	-4	16	-20
1/3	3	0	12	0 ←

$\tilde{P}(x) = (x - \frac{1}{3})(3x^2 + 12) = (x - \frac{1}{3})3(x^2+4)$

$= (3x-1)(x^2+4)$

So $P(x) = x(3x-1)(x^2+4)$

EXERCISE 5.

$P(x) = x^4 - 2x^2 - 3x - 2$

	1	0	-2	-3	-2	
1	1	1	-1	-4	-6	$P(1) = -6$
-1	1	-1	-1	-2	0	$P(-1) = 0$
2	1	2	2	1	0	$P(2) = 0$
-2	1	-2	2	-7	12	

END-OF-SECTION EXERCISES:

1. $P(x) = 2x^3 - 3x^2 - 3x - 5 = (x^2 + x + 1)(2x - 5)$
2. $P(x) = 2x^6 - 4x^5 + 3x^4 - 2x^3 + x^2 = x^2(2x^2 + 1)(x - 1)^2$
3. $P(x) = x^4 - 5x^2 + 6 = (x - \sqrt{2})(x + \sqrt{2})(x - \sqrt{3})(x + \sqrt{3})$
4.. $P(x) = x^3 + x^2 - x = x(x + \dfrac{1 - \sqrt{5}}{2})(x + \dfrac{1 + \sqrt{5}}{2})$