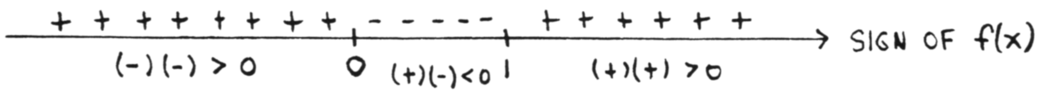
CHAPTER 5. USING THE INFORMATION GIVEN BY THE DERIVATIVE

Section 5.1 Increasing and Decreasing Functions

Quick Quiz:

1. See page 276.
2. Zeroes of f: $f(x) = 0 \iff (x = 0 \text{ or } x = 1)$. Choose the test points -1, $\frac{1}{2}$, and 2. The information is summarized below.



3. TRUE
4. TRUE

END-OF-SECTION EXERCISES:

1. Positive: $(-\infty, -2) \cup (1, \infty)$
   Negative: $(-2, 1)$
3. Positive: $(-\infty, -1) \cup (3, \infty)$
   Negative: $(-1, 3)$
5. Positive: $(-\infty, \frac{1}{3}) \cup (\frac{3}{4}, \infty)$
   Negative: $(\frac{1}{3}, \frac{3}{4})$
7. Positive: $(0, \infty)$
   Negative: $(-\infty, -1) \cup (-1, 0)$
9. Positive: $(-4, -1) \cup (1, \infty)$
   Negative: $(-\infty, -4) \cup (-1, 1)$
11. Positive: $(0, \infty)$
    Negative: $(-\infty, 0)$
13. Positive: $(1, \infty)$
    Negative: $(\frac{1}{2}, 1)$
15. The function f increases on $(-\infty, -2) \cup (1, \infty)$ and decreases on $(-2, 1)$.
17. The function f decreases on $(-\infty, -1)$ and increases on $(-1, \infty)$.
19. The function f decreases on $(0, \frac{1}{e})$, and increases on $(\frac{1}{e}, \infty)$.
21. b) 2278
    c) 3870
23. c) $1 + 2 + 2^2 + 2^3 + 2^4 = 31$
    d) $2^6 + \cdots + 2^{10} = 1984$

Section 5.2 Local Maxima and Minima—Critical Points

Quick Quiz:

1. The point $(c, f(c))$ must be a critical point. Thus, either it is an endpoint of the domain of f, or $f'(c) = 0$, or $f'(c)$ does not exist.
2. NO! There are critical points that are not local extreme points.
3. The 'critical points' for a function f are the CANDIDATES for the local extreme points of f.
4. NO! When $A \Rightarrow B$ is true, $B \Rightarrow A$ may be either true or false.
5. Since f is differentiable, it is also continuous. By the First Derivative Test, there is a maximum at $x = a$; a minimum at $x = c_1$; a maximum at $x = c_2$; and a minimum at $x = b$.

END-OF-SECTION EXERCISES:

3. TRUE
5. TRUE

7. TRUE 9. FALSE 11. TRUE 13. TRUE

15. FALSE 17. TRUE 19. TRUE

Section 5.3 The Second Derivative—Inflection Points

Quick Quiz:

1. The second derivative of a function tells us the rate of change of the slopes of the tangent lines. This information is referred to as the *concavity* of the function.
2. f is concave up on I if and only if $f''(x) > 0$ for every $x \in I$
3. The converse is: If $x^2 = 1$, then $x = 1$.

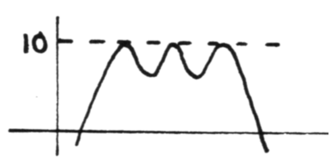
   The sentence is false. Choose x to be -1. Then the hypothesis '$(-1)^2 = 1$' is true, but the conclusion '$-1 = 1$' is false.
4. By the Second Derivative Test, the point $(c, f(c))$ is a local maximum point for f.
5. $f'(x) = 3(x-1)^2$, $f''(x) = 6(x-1)$, so $f''(1) = 6(1-1) = 0$

END-OF-SECTION EXERCISES:

1. local minima at $x = 0$ and $x = 1$; local maximum at $x = \frac{1}{2}$
3. $f(x)$ is positive on $(-\infty, -2.5) \cup (-2, \infty)$
   $f(x)$ is negative on $(-2.5, -2)$
5. f is concave up on $(-2, 2)$
   f is concave down on $(-3, -2) \cup (2, \infty)$
7. $\mathcal{D}(f') = \mathbb{R} - \{-4, -3, -2\}$
9. $\{x \mid f(x) > 10\} = (-2, -1.5)$
11. $\{x \mid f''(x) < 0\} = (-3, -2) \cup (2, \infty)$
13. $\lim_{t \to -2} f(t)$ does not exist
15. The critical points are: $\{(x, 4) \mid x \in (-\infty, -4)\}$, $(0, 2)$, $(-4, 4)$ and $(-3, 8)$
17. $\{x \in \mathcal{D}(f) \mid f \text{ is not differentiable at } x\} = \{-4, -3\}$
19. $\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = f'(0) = 0$

Section 5.4 Graphing Functions—Some Basic Techniques

Quick Quiz:

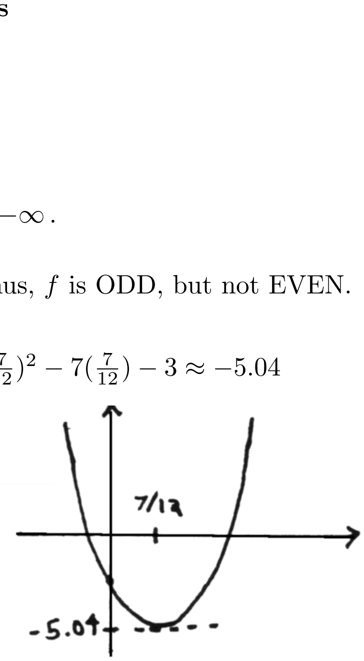
1. 

2. For $x \gg 0$ and $x \ll 0$, $P(x) \approx -6x^7$. So as $x \to \infty$, $P(x) \to -\infty$.
   As $x \to -\infty$, $P(x) \to \infty$.
3. $f(-x) = (-x)^5 - (-x) = -x^5 + x = -(x^5 - x) = -f(x)$. Thus, f is ODD, but not EVEN.
4. $f'(x) = 12x - 7$; $f'(x) = 0 \iff x = \frac{7}{12}$

   There is a horizontal tangent line at $(\frac{7}{12}, f(\frac{7}{12}))$; $f(\frac{7}{12}) = 6(\frac{7}{12})^2 - 7(\frac{7}{12}) - 3 \approx -5.04$

   $f''(x) = 12$, so $f''(x) > 0$ for all x

   Also, $f(x) = 0$
   $\iff (2x-3)(3x+1) = 0$
   $\iff x = \frac{3}{2}$ OR $x = -\frac{1}{3}$



Section 5.5 More Graphing Techniques

Quick Quiz:

1. Find A and B for which $AB = (3)(-8) = -24$ and $A + B = -2$; take $A = -6$ and $B = 4$. Then:

$$\begin{aligned} 3x^2 - 2x - 8 &= 3x^2 - 6x + 4x - 8 \\ &= 3x(x-2) + 4(x-2) \\ &= (3x+4)(x-2) \end{aligned}$$

2. First, solve $3x^2 - 2x - 8 = 0$ using the Quadratic Formula:

$$x = \frac{2 \pm \sqrt{4 - 4(3)(-8)}}{6} = \frac{2 \pm 10}{6} = 2,\ -\frac{4}{3}$$

Then:

$$\begin{aligned} 3x^2 - 2x - 8 &= 3(x-2)(x + \frac{4}{3}) \\ &= (x-2)(3x+4) \end{aligned}$$

3. CANDIDATES: $\frac{\pm 1, \pm 2}{\pm 1} = \pm 1,\ \pm 2$
4. not(A and B) $\iff$ (not A) or (not B)
5. $P(1) = -1$; the remainder upon division by $x - 1$ equals -1

END-OF-SECTION EXERCISES:

1. $P(x) = 2x^3 - 3x^2 - 3x - 5 = (x^2 + x + 1)(2x - 5)$
3. $P(x) = x^4 - 5x^2 + 6 = (x - \sqrt{2})(x + \sqrt{2})(x - \sqrt{3})(x + \sqrt{3})$

Section 5.6 Asymptotes—Checking Behavior at Infinity

Quick Quiz:

1. An *asymptote* is a curve (often a line) that a graph gets close to as x approaches $\pm\infty$, or some finite number.
2. $\lim_{x \to c^-} f(x) = -\infty \quad \iff \quad \forall\ M < 0\ \ \exists\ \delta > 0$ such that if $x \in (c - \delta, c)$, then $f(x) < M$
3. VERTICAL: $x = -2$

   HORIZONTAL: $y = 3$
4. Both individual limits (the 'numerator' limit and the 'denominator' limit) must exist. Also, the 'denominator' limit cannot equal zero.