ABBREVIATED SOLUTIONS TO QUICK QUIZ QUESTIONS and ODD-NUMBERED END-OF-SECTION EXERCISES

CHAPTER 1. ESSENTIAL PRELIMINARIES

Section 1.1 The Language of Mathematics—Expressions versus Sentences

Quick Quiz:

1. a mathematical expression
2. numbers, functions, sets
3. $x = \frac{x}{2} + \frac{x}{2}$ (many others are possible)
4. $\sqrt{x} > 2$ and $4 - 3 = 7$ are sentences

End-of-Section Exercises:

1. EXP
3. SEN, T
5. SEN, F
7. SEN, T
9. SEN, ST/SF
11. SEN, T
13. EXP
15. SEN, T
17. SEN, ST/SF
19. SEN, T
21. SEN, F
23. SEN, T
25. SEN, ST/SF
27. EXP
29. SEN, T
31. SEN, T
33. SEN, ST/SF
35. SEN, F
37. Commutative Property of Addition
39. Distributive Property
41. If $x = 1$ and $y = 3$: $1 - 3 = 1 + (-3) = -2$
    If $x = 1$ and $y = -3$: $1 - (-3) = 1 + (-(-3)) = 1 + 3 = 4$
43. The expression xyz is not ambiguous; if one person computes this as $(xy)z$ and another as $x(yz)$, the same results are obtained.

Section 1.2 The Role of Variables

Quick Quiz:

1. The variables are x and y; the constants are A, B, and C.
2. With universal set $\mathbb{R}$, $x^2 = 3$ has solution set $\{\sqrt{3}, -\sqrt{3}\}$. With universal set $\mathbb{Z}$, the solution set is empty.
3. To 'solve' an equation means to find all choices (from some universal set) that make the equation true.
   Three solutions of $x + y = 4$: $(0, 4)$, $(4, 0)$, and $(2, 2)$. There are an infinite number of solution pairs!
4. The equation $x^2 \geq 0$ is (always) true. The equation $x > 0$ is conditional; it is true for $x \in (0, \infty)$, and false otherwise.
5. Choose two from the following list:
   - variables are used in mathematical expressions to denote quantities that are allowed to vary (like in the formula $A = \pi r^2$);
   - variables are used to denote a quantity that is initially unknown, but that one would like to know (for example, 'solve $2x + 3 = 5$');
   - variables are used to state a general principle (like the commutative law of addition).

End-of-Section Exercises:

1. EXP
3. SEN, F
5. SEN, T
7. SEN, F

9. SEN, F

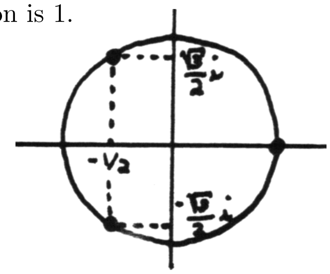
11. SEN, T

13. EXP

15. $\mathbb{R}$: the only solution is 1;
    the rational numbers: the only solution is 1;
    the integers: the only solution is 1.

17. $\mathbb{R}$: setting each factor to 0, the real number solutions are 1, $-\pi$, and $\frac{3}{2}$;
    the rational numbers: the only rational solutions are 1 and $\frac{3}{2}$;
    the integers: the only integer solution is 1.

19. a) The points are plotted at right:



b) Since $(-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 = \frac{1}{4} + \frac{3}{4} = 1$, the point $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ lies on the unit circle. Same for the remaining point.

c) Clearly, the number 1 satisfies the equation. To see that $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$ satisfies $x^3 = 1$, observe that:

$$\begin{aligned}
(-\frac{1}{2} + \frac{\sqrt{3}}{2}i)^3 &= (-\frac{1}{2} + \frac{\sqrt{3}}{2}i)(-\frac{1}{2} + \frac{\sqrt{3}}{2}i)(-\frac{1}{2} + \frac{\sqrt{3}}{2}i) \\
&= (\frac{1}{4} - \frac{\sqrt{3}}{2}i - \frac{3}{4})(-\frac{1}{2} + \frac{\sqrt{3}}{2}i) \\
&= (-\frac{1}{2} - \frac{\sqrt{3}}{2}i)(-\frac{1}{2} + \frac{\sqrt{3}}{2}i) \\
&= \frac{1}{4} + \frac{3}{4} \\
&= 1
\end{aligned}$$

Similarly for the remaining number.

d) The equation $x^3 - 1 = 0$ has the same solutions as the equation $x^3 = 1$, so the problem has already been solved.

Section 1.3 Sets and Set Notation

Quick Quiz:

1. F; the set has only 3 members
2. F
3. T
4. F
5. $105 = 5 \cdot 3 \cdot 7$; F

End-of-Section Exercises:

1. EXP; this is a set
3. SEN, T
5. SEN, F
7. SEN, C. The truth of this sentence depends upon the set S and the element x.
9. SEN, C. The truth depends on x. If x is 1, 2, or 3, then the sentence is true. Otherwise, it is false.
11. EXP; this is a set

13. SEN, T
15. SEN, C. The only number that makes this true is 1.
17. SEN, T. No matter what real number is chosen for x, both component sentences '$|x| \geq 0$' and '$x^2 \geq 0$' are true.
19. SEN, T. The two elements are both sets: $\{1\}$ and $\{1, \{2\}\}$
21. SEN, F. The number $\frac{3}{7}$ is in reduced form; the denominator has factors other than 2's and 5's.

Section 1.4 Mathematical Equivalence

Quick Quiz:

1. F; when x is -2, the first sentence is false, but the second is true.
2. THEOREM: For all real numbers a, b and c:

$$a = b \quad \iff \quad a + c = b + c$$

3. THEOREM: For all real numbers a, b and c:

$$a > b \quad \iff \quad a + c > b + c$$

4. $\{(x, y) \mid x \neq 3 \text{ and } y \neq 0\}$
5. equivalent
6. expressions; sentences

End-of-Section Exercises:

3. SEN, T. Both sentences have the same implied domain, and the same solution set, $\{4\}$.
5. SEN, T. Both sentences have the same implied domain, and the same solution set, $\{0\}$.
7. EXP
9. SEN, T. Both sentences have the same implied domain, and the same solution set, $\{-2\}$.
11.

$$\begin{aligned} 5x - 7 = 3 \quad &\iff \quad 5x = 10 \quad \text{(add 7)} \\ &\iff \quad x = 2 \quad \text{(divide by 5)} \end{aligned}$$

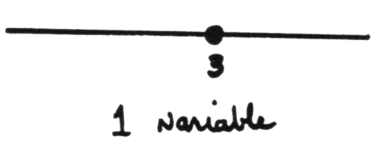
13.

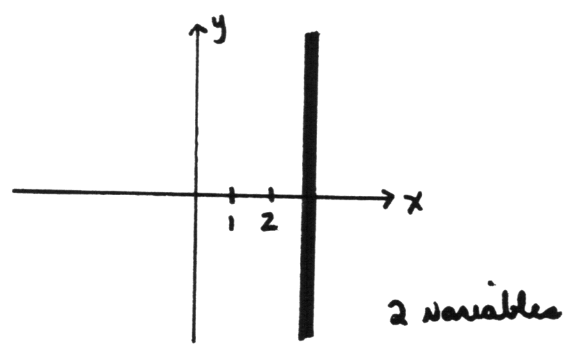
$$\begin{aligned} 3x < x - 11 \quad &\iff \quad 2x < -11 \quad \text{(subtract } x\text{)} \\ &\iff \quad x < -\frac{11}{2} \quad \text{(divide by the positive number 2)} \end{aligned}$$

Section 1.5 Graphs

Quick Quiz:

1.





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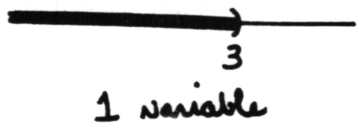
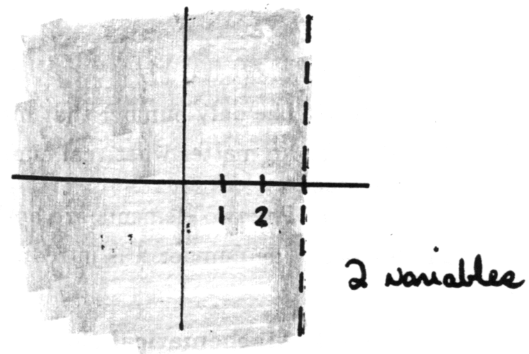
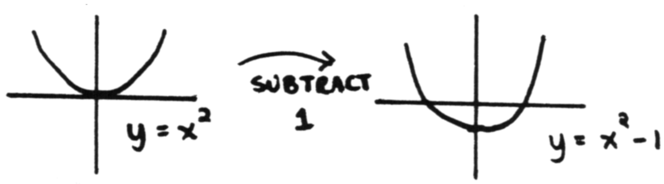
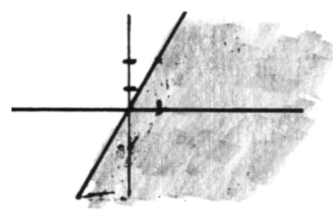




3. $y - x^2 + 1 = 0 \iff y = x^2 - 1$;

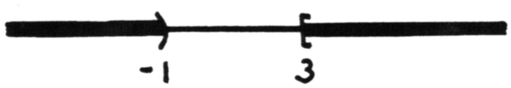


4. First graph the boundary, $y = 2x$. We want all points on or below this line.



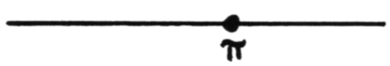
5. TRUE

6.

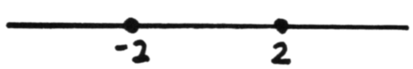


End-of-Section Exercises:

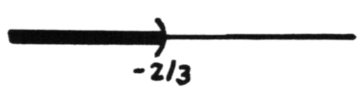
1. $x = \pi$



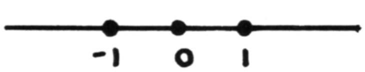
3. $|x| = 2 \iff x = 2$ or $x = -2$



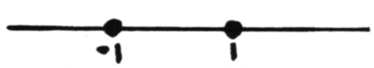
5. $3x < -2 \iff x < -\frac{2}{3}$



7. $x = 0$ or $|x| = 1$



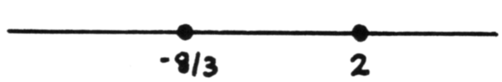
9. $x = 1$ or $|x| = 1$



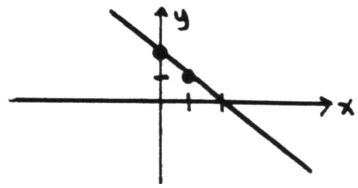
11. The critical observation here is that there are TWO numbers whose absolute value is 7: 7, and -7. Thus:

$$\begin{aligned} |3x+1| = 7 &\iff 3x + 1 = 7 \ \text{ or } \ 3x + 1 = -7 \\ &\iff 3x = 6 \ \text{ or } \ 3x = -8 \\ &\iff x = 2 \ \text{ or } \ x = -\frac{8}{3} \end{aligned}$$

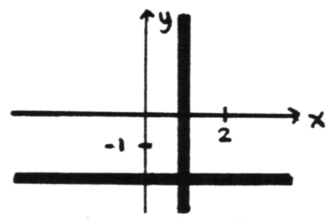
The solution set is $\{2, -\frac{8}{3}\}$.



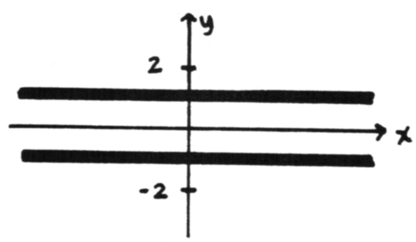
13. $x + y = 2 \iff y = -x + 2$. The graph is the line that crosses the y-axis at 2, and has slope -1.



15. The graph of '$x = 1$ or $y = -2$' is the set of all points with x-coordinate 1, together with all points with y-coordinate -2. The graph is shown below.



17. The solution set of $|y| = 1$, viewed as an equation in two variables, is $\{(x, y) \mid x \in \mathbb{R}, |y| = 1\}$. Thus, we seek all points with y-coordinates 1 or -1. See the graph below.



19.

$$\begin{aligned} |x + y| = 1 \quad &\iff \quad x + y = 1 \ \text{ or } \ x + y = -1 \\ &\iff \quad y = -x + 1 \ \text{ or } \ y = -x - 1 \end{aligned}$$

The graph is the two lines shown below.