

NAME _____

SAMPLE TEST, worth 100 points, Chapter 6

Show all work that leads to your answers. Good luck!

8 pts

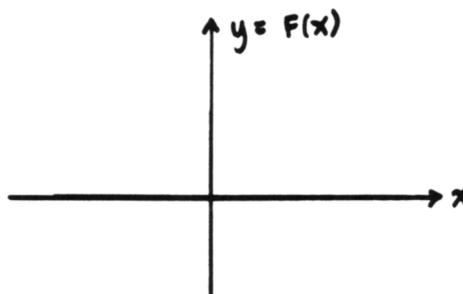
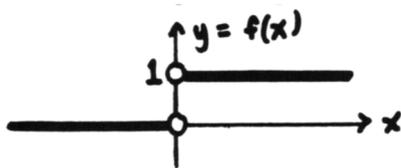
TRUE or FALSE. Circle the correct response. (2 points each)

- T F $F(x) = x \ln x - x + 2$ is an antiderivative of $f(x) = \ln x$.
- T F If $f'(x) = g'(x)$ for all $x \in \mathbb{R}$, then f and g differ by at most a constant.
- T F $\int t^2 dt = t^3 + C$
- T F An antiderivative of f is a function with derivative f .

6 pts

The graph of a function f is given below. In the space provided, graph a function F satisfying:

- $F'(x) = f(x) \quad \forall x \in \mathcal{D}(f)$;
- F is continuous at 0 ; and
- $F(-1) = 2$.



13 pts

(6 pts) These questions have to do with the indefinite integral $\int f(x) dx$.

Fill in the blanks:

The symbol \int is called the _____.

The function f being integrated is called the _____.

The process of finding $\int f(x) dx$ is called _____.

(There are two possible correct answers here.)

(2 pts) Rewrite the integral $\int x^2 dx$ using a different dummy variable.

(5 pts) What is meant by the phrase 'the linearity of the integral'?

6 pts

Classify each entry as an EXPRESSION or a SENTENCE.

If a *sentence*, state whether it is TRUE, FALSE, or CONDITIONAL.

(2 pts) a) $\int f(t) dt + \int g(t) dt$

(2 pts) b) $\int x dx = \frac{1}{2} \int 2x dx$

(2 pts) c) $f(1) = 2$

32 pts

Evaluate the following indefinite integrals. Be sure to write complete mathematical sentences. Use any appropriate methods.

(8 pts) a) $\int \left(\frac{2}{x} + e^{3x} - 1 \right) dx$

(8 pts) b) $\int \ln(x - 1) dx$

(8 pts) c) $\int \frac{t}{\sqrt[3]{t^2 - 1}} dt$

(8 pts) d) $\int \frac{3x}{2x + 1} dx$

6 pts

Find a function g satisfying $g'(x) = \frac{1}{\sqrt{2x - 1}}$ and $g(1) = 2$.

4 pts

Give an antidifferentiation 'counterpart' to the differentiation formula:

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

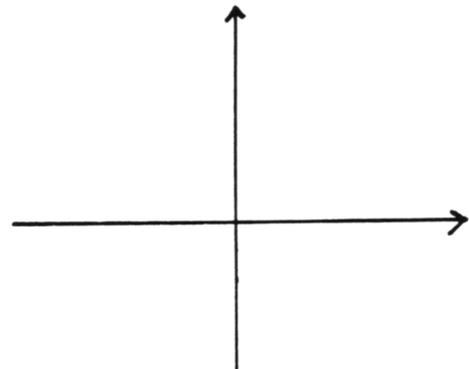
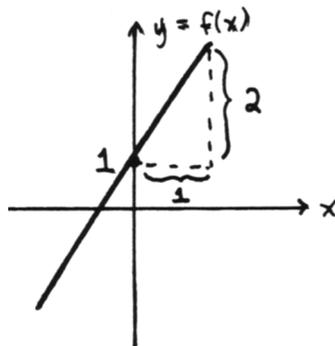
9 pts

(5 pts) Fill in a reason for each step below:

$$\begin{aligned} \int (2x+1)^3 dx &= \int (2x+1)^3 \cdot \frac{2}{2} dx && (\quad) \\ &= \frac{1}{2} \int (2x+1)^3 2 dx && (\quad) \\ &= \frac{1}{2} \int u^3 du && (\quad) \\ &= \frac{1}{2} \cdot \frac{u^4}{4} + C && (\quad) \\ &= \frac{1}{8} (2x+1)^4 + C && (\quad) \end{aligned}$$

(4 pts) Now, CHECK this antidifferentiation problem.

8 pts

A function f is graphed below. Find all the antiderivatives of f . Graph two of these antiderivatives in the space provided.

8 pts

A function f is graphed below. On the same graph, graph another function that has the same derivative as f , and has a nonremovable discontinuity at $x = 1$.