SOLUTIONS TO EXERCISES: TRANSFORMING TOOL #2 (the Multiplication Property of Equality)

IN-SECTION EXERCISES:

1. A 'theorem' is a mathematical result that is both TRUE and IMPORTANT (★ that has been proved).

2. The universal set for both a and b is $\mathbb{R}$ (the set of real numbers). The universal set for c is all nonzero real numbers.

3. 'a equals b (*slight pause*) is equivalent to (*slight pause*) a times c equals b times c'

or

'a equals b if and only if a times c equals b times c'

4. You can multiply or divide both sides of an equation by any *nonzero* number, and this won't change the truth of the equation.

5. Suppose that '$a = b$' is true. Multiplying both sides by 0 gives the equation '$a \cdot 0 = b \cdot 0$', which is also true. Thus, multiplying by zero cannot take a true equation to a false equation.

6. (a) $a = 3\,,\quad b = 0\,,\quad c = 5$

The compound sentence is true:

$$\overbrace{\overbrace{3 = 0}^{\text{false}} \iff \overbrace{3 \cdot 5 = 0 \cdot 5}^{\text{false}}}^{\text{true}}$$

(b) $a = 0\,,\quad b = 0\,,\quad c = 5$

The compound sentence is true:

$$\overbrace{\overbrace{0 = 0}^{\text{true}} \iff \overbrace{0 \cdot 5 = 0 \cdot 5}^{\text{true}}}^{\text{true}}$$

(c) $a = 0\,,\quad b = 0\,,\quad c = 0$

The compound sentence is true:

$$\overbrace{\overbrace{0 = 0}^{\text{true}} \iff \overbrace{0 \cdot 0 = 0 \cdot 0}^{\text{true}}}^{\text{true}}$$

(d) $a = 0\,,\quad b = 1\,,\quad c = 0$

The compound sentence is false:

$$\overbrace{\overbrace{0 = 1}^{\text{false}} \iff \overbrace{0 \cdot 0 = 1 \cdot 0}^{\text{true}}}^{\text{false}}$$

7. (a) If the compound sentence is false, then c must equal 0. The *only* way that the sentences '$a = b$' and '$ac = bc$' can have different truth values, is when $c = 0$.

(b) If the compound sentence is true, then no conclusion can be reached about c. The number c might equal 0, or not.

8. (a) Multiply both sides by 4. (To undo 'divide by 4', apply the transformation 'multiply by 4'.)

(b) Divide both sides by 7. (To undo 'multiply by 7', apply the transformation 'divide by 7'.)

(c) Multiply both sides by 7. (To undo 'divide by 7', apply the transformation 'multiply by 7'.)

(d) Divide both sides by 7. (To undo 'multiply by 7', apply the transformation 'divide by 7'.)

(e) Divide both sides by 3. (To undo 'multiply by 3', apply the transformation 'divide by 3'.)

(f) Divide both sides by 4. (To undo 'multiply by 4', apply the transformation 'divide by 4'.)

9. Other orders of applying the transforming tools are possible. Only one format is illustrated.

(a)

$$2x - 1 = 3 - 4x$$
$$2x = 4 - 4x$$
$$6x = 4$$
$$x = \tfrac{4}{6}$$
$$x = \tfrac{2}{3}$$

CHECK:

$$2(\tfrac{2}{3}) - 1 \overset{?}{=} 3 - 4(\tfrac{2}{3})$$
$$\tfrac{4}{3} - 1 \overset{?}{=} 3 - \tfrac{8}{3}$$
$$\tfrac{4}{3} - \tfrac{3}{3} \overset{?}{=} \tfrac{9}{3} - \tfrac{8}{3}$$
$$\tfrac{1}{3} = \tfrac{1}{3}$$

(b)

$$1 - x = 5x - 3$$
$$1 = 6x - 3$$
$$4 = 6x$$
$$\tfrac{4}{6} = x$$
$$x = \tfrac{2}{3}$$

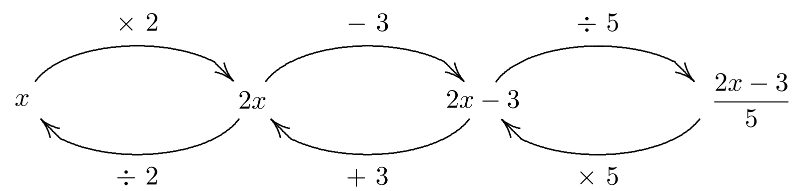
CHECK:

$$1 - \tfrac{2}{3} \overset{?}{=} 5(\tfrac{2}{3}) - 3$$
$$\tfrac{3}{3} - \tfrac{2}{3} \overset{?}{=} \tfrac{10}{3} - \tfrac{9}{3}$$
$$\tfrac{1}{3} = \tfrac{1}{3}$$

YES: For all real numbers x, $2x - 1 = 3 - 4x \iff 1 - x = 5x - 3$. Both equations are true only when $x = \frac{2}{3}$, and are false otherwise.

10. (a) Start with x, multiply by 2, subtract 3, divide by 5.

Undo with: multiply by 5, add 3, divide by 2.



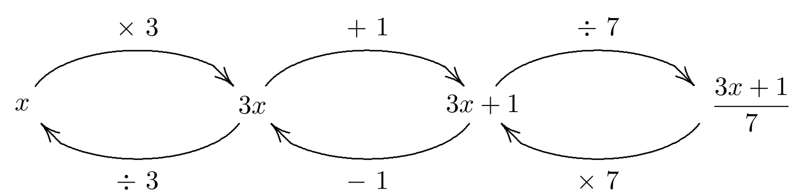
$$\tfrac{2x-3}{5} = 1$$
$$2x - 3 = 5$$
$$2x = 8$$
$$x = 4$$

CHECK:

$$\tfrac{2(4)-3}{5} \overset{?}{=} 1$$
$$\tfrac{5}{5} \overset{?}{=} 1$$
$$1 = 1$$

(b) Start with x, multiply by 3, add 1, divide by 7.

Undo with: multiply by 7, subtract 1, divide by 3.

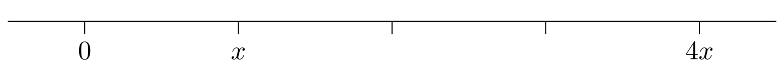


$$2 = \tfrac{3x+1}{7}$$
$$14 = 3x + 1$$
$$13 = 3x$$
$$\tfrac{13}{3} = x$$
$$x = \tfrac{13}{3}$$

CHECK:

$$2 \overset{?}{=} \tfrac{3(\frac{13}{3})+1}{7}$$
$$2 \overset{?}{=} \tfrac{13+1}{7}$$
$$2 = 2$$

11. (a)



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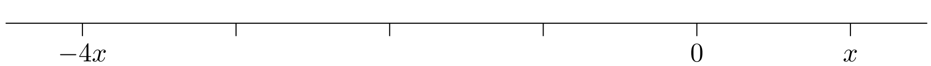
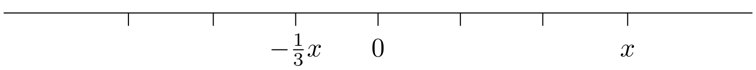
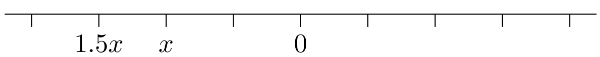
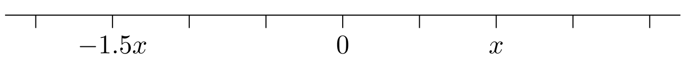
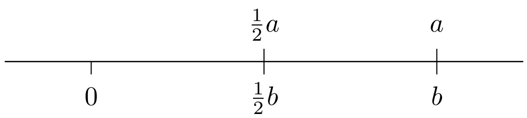


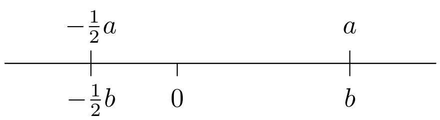
(c) 

(d) 

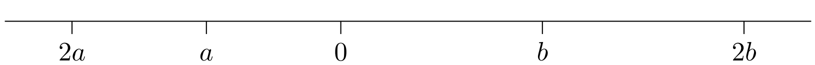
(e) 

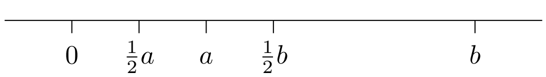
(f) 

12. (a) 

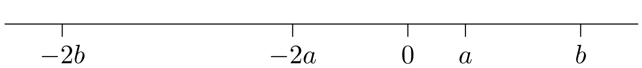
(b) 

(c) 

13. (a) 

(b) 

(c) 

(d) 

14. Suppose that '$ac = bc$' is true. Then, ac and bc live at the same place on a number line. Dividing by c (that is, multiplying by $\frac{1}{c}$), 'stretches' or 'shrinks' the original distance from zero (and causes a flip to the opposite side of zero, whenever c is negative). Since they started at the same place, they'll end up at the same place. Thus, '$a = b$' is also true.

15. Suppose that '$ac = bc$' is false. Then, ac and bc live at different places on a number line. Dividing by c (that is, multiplying by $\frac{1}{c}$), 'stretches' or 'shrinks' the original distance from zero (and causes a flip to the opposite side of zero, whenever c is negative). Since they started at different places, they'll end up at different places. Thus, '$a = b$' is also false.

END-OF-SECTION EXERCISES:

16.

$$\begin{aligned} t - 1 + 3t &= 7 - 2t \\ 4t - 1 &= 7 - 2t \\ 6t - 1 &= 7 \\ 6t &= 8 \\ t &= \tfrac{8}{6} \\ t &= \tfrac{4}{3} \end{aligned}$$

CHECK:

$$\begin{aligned} \tfrac{4}{3} - 1 + 3(\tfrac{4}{3}) &\stackrel{?}{=} 7 - 2(\tfrac{4}{3}) \\ \tfrac{4}{3} - \tfrac{3}{3} + \tfrac{12}{3} &\stackrel{?}{=} \tfrac{21}{3} - \tfrac{8}{3} \\ \tfrac{13}{3} &= \tfrac{21}{3} \end{aligned}$$

17.

$$\begin{aligned} \tfrac{5-2x}{7} &= x \\ 5 - 2x &= 7x \\ 5 &= 9x \\ \tfrac{5}{9} &= x \\ x &= \tfrac{5}{9} \end{aligned}$$

CHECK:

$$\begin{aligned} \frac{5-2(\frac{5}{9})}{7} &\stackrel{?}{=} \tfrac{5}{9} \\ \frac{\frac{45}{9} - \frac{10}{9}}{7} &\stackrel{?}{=} \tfrac{5}{9} \\ \frac{\frac{35}{9}}{7} &\stackrel{?}{=} \tfrac{5}{9} \\ \tfrac{35}{9} \cdot \tfrac{1}{7} &\stackrel{?}{=} \tfrac{5}{9} \\ \tfrac{5}{9} &= \tfrac{5}{9} \end{aligned}$$

18.

$$\begin{aligned} 3x + 1 &= 1 + 3x \\ 3x &= 3x \\ 0 &= 0 \end{aligned}$$

Since the final equation is always true, the original equation is always true. That is: for all real numbers x, $3x + 1 = 1 + 3x$.

19.

$$\begin{aligned} 3x + 1 &= 3x + 2 \\ 1 &= 2 \\ 0 &= 1 \end{aligned}$$

Since the final equation is always false, the original equation is always false. That is, the equation '$3x + 1 = 3x + 2$' is false, for all real numbers x.