

SOLUTIONS TO EXERCISES: TRANSFORMING TOOL #1 (the Addition Property of Equality)

IN-SECTION EXERCISES:

1. A ‘theorem’ is a mathematical result that is both TRUE and IMPORTANT (★ that has been proved).
2. The universal set for each variable is \mathbb{R} (the set of real numbers). The phrase ‘For all *real numbers* ...’ gives us this information.
3. ‘ a equals b (*slight pause*) is equivalent to (*slight pause*) a plus c equals b plus c ’;
or
‘ a equals b if and only if a plus c equals b plus c ’
4. In this context, ‘at least one’ means ‘one, or more than one’.
5. (a) TRUE. In an addition problem, the grouping of the numbers does not affect the result.
(b) FALSE. For example, choose $x = 1$ and $y = 2$. Then, the sentence ‘ $1 - 2 = 2 - 1$ ’ is false.
(c) TRUE. You can commute the numbers in a multiplication problem, without changing the result. This fact is formally referred to as the **Commutative Property of Multiplication**.
(d) FALSE. For example, choose $x = 1$ and $y = 2$. Then, the sentence ‘ $\frac{1}{2} = \frac{2}{1}$ ’ is false.
6. (a) Add 4 to both sides. (To undo ‘subtract 4’, apply the transformation ‘add 4’.)
(b) Subtract 7 from both sides. (To undo ‘add 7’, apply the transformation ‘subtract 7’.)
(c) Add 7 to both sides. (To undo ‘subtract 7’, apply the transformation ‘add 7’.)
(d) Subtract 7 from both sides. (To undo ‘add 7’, apply the transformation ‘subtract 7’.)
(e) Subtract 3 from both sides. (To undo ‘add 3’, apply the transformation ‘subtract 3’.)
(f) Subtract 4 from both sides. (To undo ‘add 4’, apply the transformation ‘subtract 4’.)

7. (a)

(1)	(2)	(3)	(4)	(5)
$x - 1 = 4$	$x - 1 = 4$	$x - 1 = 4$	$x - 1 = 4$	For all real numbers x ,
$x - 1 + 1 = 4 + 1$	$+ 1 \quad + 1$	$x = 5$	$x = 5$	$x - 1 = 4 \iff x = 5.$
$x = 5$	<hr style="width: 50%; margin: 0 auto;"/> $x = 5$	$\left. \begin{array}{l} x - 1 = 4 \\ x = 5 \end{array} \right\} \text{add 1}$		

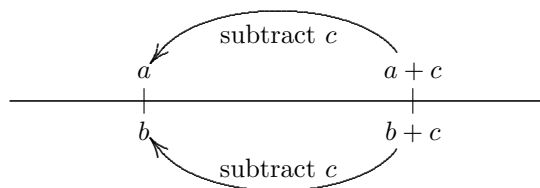
(b)

(1)	(2)	(3)	(4)	(5)
$5 = 3 + x$	$5 = 3 + x$	$5 = 3 + x$	$5 = 3 + x$	For all real numbers x ,
$5 - 3 = 3 + x - 3$	$- 3 \quad - 3$	$2 = x$	$2 = x$	$5 = 3 + x$
$2 = x$	<hr style="width: 50%; margin: 0 auto;"/> $2 = x$	$\left. \begin{array}{l} 5 = 3 + x \\ 2 = x \end{array} \right\} \text{subtract 3}$	$x = 2$	$\iff 2 = x$
$x = 2$	$x = 2$			$\iff x = 2.$

8. (a)

- Suppose that ' $a + c = b + c$ ' is true. Then, $a + c$ and $b + c$ live at the same place on a number line.
- Subtracting c from both numbers moves them both by the same amount.
- Since they started at the same place, they'll end up at the same place. Thus, ' $a = b$ ' is also true.

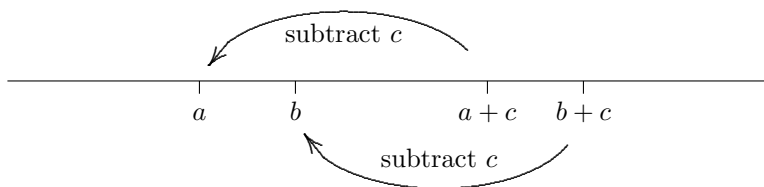
The sketch below assumes that c is a positive number:



(b)

- Suppose that ' $a + c = b + c$ ' is false. Then, $a + c$ and $b + c$ live at different places on a number line.
- Subtracting c from both numbers moves them both by the same amount.
- Since they started at different places, they'll end up at different places. Thus, ' $a = b$ ' is also false.

The sketch below assumes that c is a positive number:



END-OF-SECTION EXERCISES:

For problems 9—12, only one format is illustrated for each solution.

9.

$$\begin{aligned} 4x &= 5 + 3x \\ 4x - 3x &= 5 + 3x - 3x \\ x &= 5 \end{aligned}$$

CHECK:

$$\begin{aligned} 4(5) &\stackrel{?}{=} 5 + 3(5) \\ 20 &\stackrel{?}{=} 5 + 15 \\ 20 &= 20 \end{aligned}$$

10.

$$\begin{aligned} x + 4 &= 2x - 1 \\ 4 &= x - 1 && \left. \begin{array}{l} \text{subtract } x \\ \text{add } 1 \end{array} \right\} \\ 5 &= x \\ x &= 5 \end{aligned}$$

CHECK:

$$\begin{aligned} 5 + 4 &\stackrel{?}{=} 2(5) - 1 \\ 9 &\stackrel{?}{=} 10 - 1 \\ 9 &= 9 \end{aligned}$$

11.

$$\begin{aligned} 3t - 7 + 2t &= 4t + 5 \\ 5t - 7 &= 4t + 5 \\ t - 7 &= 5 \\ t &= 12 \end{aligned}$$

CHECK:

$$\begin{aligned} 3(12) - 7 + 2(12) &\stackrel{?}{=} 4(12) + 5 \\ 36 - 7 + 24 &\stackrel{?}{=} 48 + 5 \\ 29 + 24 &\stackrel{?}{=} 53 \\ 53 &= 53 \end{aligned}$$

12.

$$\begin{aligned} 1 &= 8 - t \\ 1 + t &= 8 && \left. \begin{array}{l} \text{add } t \\ \text{subtract } 1 \end{array} \right\} \\ t &= 7 \end{aligned}$$

CHECK:

$$\begin{aligned} 1 &\stackrel{?}{=} 8 - 7 \\ 1 &= 1 \end{aligned}$$