

TABLE OF CONTENTS

| | |
|--|----|
| 1. THE LANGUAGE OF MATHEMATICS | 1 |
| expressions versus sentences; conventions; definitions | |
| 2. BRUSHING UP ON BASIC ARITHMETIC SKILLS | 12 |
| multiplication concept; multiplication and addition practice; divisibility tests | |
| 3. THE REAL NUMBERS | 17 |
| real number line; positive and negative; nonnegative; nonzero; whole numbers; consecutive; size versus order; greater than, less than; big versus small; opposites; integers; density property; equality | |
| 4. ADDITION OF SIGNED NUMBERS | 30 |
| signed numbers; numbers as position; numbers as movement; every number has a size and a sign; rules for addition of signed numbers | |
| 5. SUBTRACTION OF SIGNED NUMBERS | 36 |
| rewriting a subtraction problem as an addition problem; reading subtraction problems aloud | |
| 6. AVERAGE | 39 |
| the average of two different numbers is exactly halfway between; converting fractions with a denominator of 2 to decimal form; subscript notation; the average of more than 2 numbers as a balancing point; the average always lies between the greatest and least numbers | |
| 7. THE BASE TEN NUMBER SYSTEM | 43 |
| concept of base and place value; exponent notation for place values; reading base ten num- bers aloud; multiplying by powers of ten | |
| 8. DECIMALS | 51 |
| extending place values to the right of the ones place; the decimal point; reading decimals aloud; renaming decimals as fractions; multiplying and dividing by powers of ten; percents; changing percents to decimals; changing decimals to percents | |
| 9. MATHEMATICIANS ARE FOND OF COLLECTIONS | 57 |
| definition of set; members of a set; finite versus infinite sets; list method; the empty set; interval notation; subsets | |
| 10. HOLDING THIS, HOLDING THAT | 71 |
| letters used to represent other objects—variables; variables ‘hold’ objects; variables are used to state a general principle, represent a sequence of operations, represent an ‘unknown’ quantity; solving a sentence | |
| 11. INTRODUCTION TO FRACTIONS | 90 |
| fraction concept—part of a whole; locating fractions on a number line; $\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$; every subtraction problem is an addition problem in disguise; fractions involving zero | |
| 12. MULTIPLYING FRACTIONS | 96 |
| concept for multiplying fractions; reciprocals; dividing fractions; every fraction has three signs; simplifying fractions with signed numbers | |

| | | |
|-----|---|-----|
| 13. | ADDING FRACTIONS | 105 |
| | adding fractions with a common denominator; multiples; least common multiple; the list method for finding the least common multiple; renaming fractions with a desired denominator; factors; prime numbers; finding the least common multiple from the prime factorizations; subtracting fractions | |
| 14. | I LIVE TWO BLOCKS WEST OF YOU | 113 |
| | order properties of the real numbers—position of one number relative to another; mathematical sentences used to describe order relationships; the phrases ‘greatest’, ‘least’, ‘at least’, ‘at most’; describing a whole class of sentences in one fell swoop; equations versus inequalities; solution of a sentence in one variable; graph of a sentence; reading a sentence backwards | |
| 15. | NUMBERS HAVE LOTS OF DIFFERENT NAMES! | 131 |
| | different names can reveal various properties that a number has; theorems are to a mathematician as tools are to a carpenter; the power of adding zero and multiplying by one; simplifying an expression versus solving a sentence | |
| 16. | EXACT VERSUS APPROXIMATE | 147 |
| | equal versus approximately equal; simplest form of a fraction; cancelling; simplifying a fraction; rational and irrational numbers; pi; writing rational numbers as decimals; writing decimals as rational numbers | |
| 17. | ROUNDING | 155 |
| | rounding to n decimal places; choosing the nearest candidate; rounding up versus rounding down; what to do if the two candidates are equidistant; efficient implementation of the rounding ideas | |
| 18. | UNIT CONVERSION | 159 |
| | all unit conversion problems are accomplished by multiplying by 1 in an appropriate form; whenever two quantities are equal, we get two names for the number 1; one-step conversions (multiplying by 1 once); unit conversion information to be memorized—units of length, time, volume, and weight/mass; using a calculator to compute expressions of the form $\frac{ab}{cd}$ | |
| 19. | MORE UNIT CONVERSIONS | 168 |
| | multi-step conversions (multiplying by 1 more than once); units of volume like ft^3 ; develop good calculator skills—get exact answer first, key in without writing down intermediate results, approximate only at the last step; rates and rate problems | |
| 20. | EXPONENT NOTATION | 177 |
| | exponent notation: base, exponent, power; positive integer exponents: repeated multiplication; $x^0 = 1$ for nonzero x ; negative integer exponents: $x^{-n} = \frac{1}{x^n}$; order of operation considerations; $(-2)^4$ versus -2^4 ; $(\frac{a}{b})^{-1} = \frac{b}{a}$ | |
| 21. | EXPONENT LAWS | 186 |
| | the exponent laws: motivations and examples; exponent laws offer different orders of operation; rules can be used left-to-right and right-to-left; reason why $x^0 = 1$; reason why $x^{-1} = \frac{1}{x}$ | |
| 22. | RADICALS | 196 |
| | undoing powers; the cube root undoes a cube; odd roots; properties of odd roots; reading roots aloud; even roots—two problems; $\sqrt{4}$ denotes the <i>nonnegative</i> number which, when squared, equals 4; even roots; radicals; approximating radicals on a calculator | |

| | |
|---|-----|
| 23. RATIONAL EXPONENTS | 207 |
| writing radicals with rational exponents; order of operation considerations; opposite versus subtraction; making sense of $x^{\frac{p}{q}}$; problems that can arise with rational exponents; evaluating arbitrary radicals on a calculator | |
| 24. THE DISTRIBUTIVE LAW | 213 |
| products involving signed variables, like $(-a)b$; the distributive law; understanding the distributive law in terms of area; problems like $-a(b - c)$; a negative sign in front of a group; simplifying expressions using the distributive law in one step | |
| 25. REVISITING EXPONENTS | 222 |
| raising a number and its opposite to the same even power gives the same result; raising a number and its opposite to the same odd power gives opposites as the result; simplifying expressions like $(2x)^3$, $(-3xy)^2$, and $-(-x)^2(-2x)^3$ —figure out the sign (plus or minus) first, then the number part, then the variable part | |
| 26. FOIL AND MORE | 232 |
| extending the distributive law to cover situations like $a(b + c + d)$, $(a + b)(c + d)$, and $(a + b + c)(d + e + f)$; terms; identifying terms; a term includes its sign; numerical coefficients; constant terms; like terms; combining like terms; the ‘treat it as a singleton’ technique; FOIL (First Outer Inner Last) | |
| 27. THESE SENTENCES CERTAINLY LOOK DIFFERENT | 241 |
| comparing expressions versus comparing sentences; equality of expressions versus equivalence of sentences; connectives; truth tables; synonyms for ‘is equivalent to’; the mathematical words ‘and’ and ‘or’ | |
| 28. INTRODUCTION TO FACTORING | 259 |
| precise statement of the Zero Factor Law and translation; the forward and reverse directions of ‘ $S1$ if and only if $S2$ ’; recognizing products—the last operation is multiplication; recognizing sums—the last operation is addition; identifying factors in products, and terms in sums; using the distributive law to factor simple expressions of the form $ab + ac$ | |
| 29. GREATEST COMMON FACTOR | 266 |
| the list method for finding the greatest common factor; listing the factors of a number in an efficient way; the prime factorization method for finding the greatest common factor; finding the greatest common factor of variable expressions; factoring out the greatest common factor | |
| 30. TRANSFORMING TOOL #1 (the Addition Property of Equality) | 277 |
| the Addition Property of Equality; facts versus commands; ‘for all’ sentences; translating a theorem; preferred formats for solving a sentence; why the Addition Property of Equality is true; preferred format for checking solutions | |
| 31. TRANSFORMING TOOL #2 (the Multiplication Property of Equality) | 293 |
| the Multiplication Property of Equality; what goes wrong with multiplying by zero; undoing the last operation first; why the Multiplication Property of Equality is true | |
| 32. SOLVING LINEAR EQUATIONS IN ONE VARIABLE | 308 |
| classifying families of sentences; the standard form tells you what any member of the family looks like; linear equations in one variable; procedure for finding the unique solution using simplification and the Addition and Multiplication Properties of Equality; using your calculator to check a solution; solving equations involving fractions; putting fractions in simplest form on your calculator; solving equations involving decimals | |

33. SOLVING LINEAR INEQUALITIES IN ONE VARIABLE 318
linear inequalities in one variable; tools for solving inequalities; if you multiply or divide by a negative number, you must change the direction of the inequality symbol; solving linear inequalities; linear inequalities have infinitely many solutions; ‘spot-checking’ your solutions; solving inequalities involving fractions and decimals; the compound inequality ‘ $a < x < b$ ’

MORE TO COME AS THE YEAR PROGRESSES!!!