

## SOLUTIONS TO EXERCISES: NUMBERS HAVE LOTS OF DIFFERENT NAMES!

### IN-SECTION EXERCISES:

1. STEP 1:  $\overbrace{0}^{0 \text{ pets}} + 2 = 2$ ; write down the number  $\textcircled{2}$ , and circle it.

Since you own fewer than 2 pets, go to STEP 2.

STEP 2:  $2 - \overbrace{0}^{0 \text{ pets}} = 2$ ;  $2 \cdot \textcircled{2} = \boxed{4}$ ; write down the number  $\boxed{4}$ , and put a box around it. Go to STEP 4.

STEP 4:  $\overbrace{0}^{0 \text{ pets}} \cdot \overbrace{0}^{0 \text{ pets}} = 0$ ;  $0 + \boxed{4} = 4$

2. STEP 1:  $\overbrace{7}^{7 \text{ pets}} + 2 = 9$ ; write down the number  $\textcircled{9}$ , and circle it.

Since you own more than 2 pets, go to STEP 3.

STEP 3:  $\overbrace{7}^{7 \text{ pets}} - 2 = 5$ ; opposite is  $-5$ ;  $(-5) \cdot \textcircled{9} = \boxed{-45}$ . Write down the number  $\boxed{-45}$  and put a box around it.

STEP 4:  $\overbrace{7}^{7 \text{ pets}} \cdot \overbrace{7}^{7 \text{ pets}} = 49$ ;  $49 + (\boxed{-45}) = 4$

3.  $2 \cdot 8 + 5 \cdot 4$ : give 2 pieces of candy to each of 8 kids, and 5 pieces of candy to each of 4 kids; OR  
give 2 pieces to each of 8 kids, and 4 pieces to each of 5 kids; OR  
give 8 pieces to each of 2 kids, and 5 pieces to each of 4 kids.

$5 \cdot 7 + 1$ : give 5 pieces of candy to each of 7 kids, with 1 piece left over.

4.

name for 60

information revealed by name

$6 \cdot 10$	give 6 pieces of candy to each of 10 kids; or give 10 pieces of candy to each of 6 kids
$3 \cdot 20$ or $20 \cdot 3$	60 pieces of candy can be evenly distributed among 3 kids, by giving 20 pieces to each
$7 \cdot 8 + 4$ or $8 \cdot 7 + 4$	give 7 pieces of candy to each of 8 kids, with 4 pieces left over
$16 \cdot 3 + 2 \cdot 6$	give 16 pieces to each of 3 kids, and 2 pieces to each of 6 kids; OR give 3 pieces to each of 16 kids, and 6 pieces to each of 2 kids; OR give 16 pieces to each of 3 kids, and 6 pieces to each of 2 kids; OR give 3 pieces to each of 16 kids, and 2 pieces to each of 6 kids.
$\frac{1}{3}(180)$	give one-third of a piece to each of 180 kids

5. The universal set for  $x$  is  $\mathbb{R}$  because the theorem says ‘For all real numbers  $x \dots$ ’.

6. You can add zero to any real number, and this doesn’t change the identity of the number. Adding zero gives a new *name* for a number, but doesn’t change where it *lives* on a real number line. Consequently, the number 0 is often given the fancy name ‘additive identity’.

7. The universal set for  $x$  is  $\mathbb{R}$  because the theorem says ‘For all real numbers  $x \dots$ ’.

8. You can multiply any real number by 1, and this doesn't change the identity of the number. Multiplying by 1 gives a new *name* for a number, but doesn't change where it *lives* on a real number line. Consequently, the number 1 is often given the fancy name 'multiplicative identity'.

9. (a)  $0 = 5 + (-5) = (-5) + 5 = 5 - 5$

(b)  $0 = \frac{1}{2} + (-\frac{1}{2}) = (-\frac{1}{2}) + \frac{1}{2} = \frac{1}{2} - \frac{1}{2}$

(c)  $0 = 3.2 + (-3.2) = (-3.2) + 3.2 = 3.2 - 3.2$

(d)  $0 = (-7) + 7 = 7 + (-7)$

10. (a)  $1 = \frac{5}{5} = 5 \cdot \frac{1}{5} = \frac{1}{5} \cdot 5$

(b)  $1 = \frac{1/2}{1/2} = \frac{1}{2} \cdot 2 = 2 \cdot \frac{1}{2}$

(c)  $1 = \frac{3.2}{3.2} = 3.2 \cdot \frac{1}{3.2} = \frac{1}{3.2} \cdot 3.2$

(d)  $1 = \frac{-7}{-7} = -7 \cdot \frac{1}{-7} = \frac{1}{-7} \cdot (-7)$  When a negative number comes after a centered dot, it is customary to put the negative number inside parentheses, because  $\frac{1}{-7} \cdot -7$  can look somewhat confusing.

11. (a) true

(b) Since ' $2 + 3 = 5 + 1$ ' is false, the entire sentence ' $2 + 3 = 1 + 5 = 6$ ' is false. Students sometimes 'string' things together with equal signs as they work through a calculation, using '=' to mean something like 'I'm going on to the next step'. **DON'T DO THIS! BE CAREFUL!**

(c) true:  $1 + 2 + 3 = 1 + 5$  is true, and  $1 + 5 = 6$  is true.

(d) true for all nonzero real numbers  $t$ ; not defined if  $t = 0$

(e) true

(f)  $1 + (2 + 3) + 4 = 5$  is false;  $5 = 1 + 5$  is false;  $1 + 5 = 6 + 4$  is false;  $6 + 4 = 10$  is true. The entire sentence is FALSE because there is at least one 'piece' that is false. (Indeed, in this case, three of the four subsentences are false!)

12.  $a = b = c = d$  is shorthand for:  $a = b$  and  $b = c$  and  $c = d$

END-OF-SECTION EXERCISES:

13. EXP (simplest name: 36)

14. SEN, true

15. SEN, false (Don't use '=' to mean that you're going on to the next step!)

16. SEN, always true

17. SEN, true