

## 5. SUBTRACTION OF SIGNED NUMBERS

*every subtraction problem is an addition problem in disguise*  
*opposites*

In this section, we'll study problems like  $-3 - (-5)$ ; that is, problems of the form  $x - y$ . The good news is that every subtraction problem is an addition problem in disguise! In one easy step, every subtraction problem is changed to an addition problem, which you already know how to solve.

To understand subtraction, you must understand *opposites*. Recall that opposites are the same distance from zero, but on opposite sides of zero. Thus, the opposite of 3 is  $-3$ , the opposite of  $-5$  is 5, and the opposite of 0 is 0.

### EXERCISES

1. Find the opposites:
  - a. the opposite of  $-3$  is \_\_\_\_\_
  - b. the opposite of 2 is \_\_\_\_\_
  - c. the opposite of  $-\frac{1}{2}$  is \_\_\_\_\_
  - d. the opposite of 0 is \_\_\_\_\_

*recognizing subtraction problems*

It's important that you can recognize subtraction problems, and read them aloud correctly. There are several things that you should notice as you study the examples below:

- when a negative number is being subtracted, it goes inside parentheses
- the subtraction sign is read as 'minus'
- a negative number like  $-3$  is read as 'negative three'
- even though the same symbol ' $-$ ' is used both for subtraction and for finding the opposite of a number, it is read in different ways

EXAMPLES:

$3 - 5$ : the number being subtracted is 5; read aloud as 'three minus five'

$2 - (-3)$ : the number being subtracted is  $-3$ ; read aloud as 'two minus negative three'

$-1 - 6$ : the number being subtracted is 6; read aloud as 'negative one minus six'

$-2 - (-7)$ : the number being subtracted is  $-7$ ; read aloud as 'negative two minus negative seven'

### EXERCISES

2. Identify the number being subtracted, and state how the problem should be read aloud:
  - a.  $-1 - 3$
  - b.  $1 - (-3)$
  - c.  $1 - 3$
  - d.  $-1 - (-3)$

*to subtract a number, add its opposite*

To subtract a number, you add its opposite. To subtract 3, you add  $-3$ . To subtract  $-3$ , you add 3.

That is,  $x - y = x + (-y)$  for all real numbers  $x$  and  $y$ . In words, ' $x$  minus  $y$  equals  $x$  plus the opposite of  $y$ '. Here's a diagram of the sentence:

$$x \quad \underbrace{- y}_{\text{to subtract } y}, \quad = \quad x \quad \underbrace{+ (-y)}_{\text{add the opposite of } y}$$

three steps  
in a  
subtraction problem

There are three steps in a subtraction problem. These steps are illustrated using the example  $-3 - (-5)$ :

- (1) identify the number being subtracted (Answer:  $-5$ )
- (2) find the opposite of the number being subtracted (Answer:  $5$ )
- (3) rewrite the subtraction problem as addition of the opposite (Answer:  $-3 - (-5) = -3 + 5 = 2$ )

**EXERCISES**

SAMPLE:

Consider the subtraction problem  $3 - (-6)$ :

What number is being subtracted? Answer:  $-6$

What is the opposite of the number being subtracted? Answer:  $6$

To subtract the number  $-6$ , instead add the number  $6$ .

Thus,  $3 - (-6) = 3 + 6 = 9$ .

3. Consider the subtraction problem  $2 - (-3)$ :

What number is being subtracted? \_\_\_\_\_

What is the opposite of the number being subtracted? \_\_\_\_\_

To subtract the number \_\_\_\_\_ instead add the number \_\_\_\_\_.

Thus,  $2 - (-3) =$  \_\_\_\_\_.

4. Consider the subtraction problem  $-2 - 3$ :

What number is being subtracted? \_\_\_\_\_

What is the opposite of the number being subtracted? \_\_\_\_\_

To subtract the number \_\_\_\_\_ instead add the number \_\_\_\_\_.

Thus,  $-2 - 3 =$  \_\_\_\_\_.

more examples

For a familiar problem like  $5 - 1 = 4$ , you certainly won't need to use the process outlined above, but you should understand that it still works.

Consider  $5 - 1$ :

- (1) identify the number being subtracted (Answer:  $1$ )
- (2) find the opposite of the number being subtracted (Answer:  $-1$ )
- (3) rewrite the subtraction problem as addition of the opposite (Answer:  $5 - 1 = 5 + (-1) = 4$ )

more than  
two numbers

Here are a couple problems with more than two numbers. Notice that every subtraction is turned into an addition in the first step.

$$\begin{aligned} -3 - 5 + (-2) - (-7) + 4 &= -3 + (-5) + (-2) + 7 + 4 \\ &= -10 + 11 \\ &= 1 \end{aligned}$$

$$\begin{aligned} 2 - (-3) - 1 + 5 + (-4) &= 2 + 3 + (-1) + 5 + (-4) \\ &= 10 + (-5) \\ &= 5 \end{aligned}$$

**EXERCISES***web practice*

5. Go to <http://fishcaro.crosswinds.net> and follow the links to the practice problems for section 5. Here, you'll get unlimited practice with subtraction of signed numbers. There is also practice with more complicated problems that combine both addition and subtraction. For your convenience, there are worksheets provided in this section. Additional worksheets can be produced at the web site.

## SOLUTIONS TO EXERCISES: SUBTRACTION OF SIGNED NUMBERS

1. a. the opposite of  $-3$  is  $3$   
b. the opposite of  $2$  is  $-2$   
c. the opposite of  $-\frac{1}{2}$  is  $\frac{1}{2}$   
d. the opposite of  $0$  is  $0$
2. a.  $-1 - 3$ : the number being subtracted is  $3$ ; read aloud as 'negative one minus three'  
b.  $1 - (-3)$ : the number being subtracted is  $-3$ ; read aloud as 'one minus negative three'  
c.  $1 - 3$ : the number being subtracted is  $3$ ; read aloud as 'one minus three'  
d.  $-1 - (-3)$ : the number being subtracted is  $-3$ ; read aloud as 'negative one minus negative three'
3. Consider the subtraction problem  $2 - (-3)$ :  
What number is being subtracted? Answer:  $-3$   
What is the opposite of the number being subtracted? Answer:  $3$   
To subtract the number  $-3$  instead add the number  $3$ .  
Thus,  $2 - (-3) = 2 + 3 = 5$ .
4. Consider the subtraction problem  $-2 - 3$ :  
What number is being subtracted? Answer:  $3$   
What is the opposite of the number being subtracted? Answer:  $-3$   
To subtract the number  $3$  instead add the number  $-3$ .  
Thus,  $-2 - 3 = -2 + (-3) = -5$ .