4. ADDITION OF SIGNED NUMBERS

if it's easy for you, then jump to the exercises!

Problems like $(-2)+(-3)=-5$ and $(-3)+5=2$ are easy for some people and hard for others. If they're easy for you, then just skim quickly through this section and jump right to the exercises. However, if problems like this confuse you, then this section is for you!

signed numbers

The phrase 'signed numbers' refers to numbers that can be either positive (like 5) or negative (like -5). That is, 'signed numbers' are allowed to have a minus sign.

at first, it may seem a bit complicated

At first glance, it may seem that the process of adding signed numbers is a bit complicated. Keep in mind that writing down a process often makes it looks more complicated than it really is. (Think about writing the instructions for tying a shoe!) After lots of practice, though, the process will become second nature, and you'll be whizzing through problems with total confidence.

real numbers: position versus movement

Every real number can be interpreted in two ways:

- as a **position** on the number line;
- as a **movement**.

You will see that both of these interpretations—position and movement—are used when adding and subtracting signed numbers.

numbers as position

NUMBERS AS POSITION:

The number 3 can mean: go to position 3 on the number line.

The number -3 can mean: go to position -3 on the number line.

numbers as movement

NUMBERS AS MOVEMENT:

Positive numbers can indicate movement to the right. For example, 3 can mean 'move 3 units to the right'.

Negative numbers can indicate movement to the left. For example, -3 can mean 'move 3 units to the left'.

EXERCISES

1. Give both the *position* and *movement* interpretations for each of these numbers: 2, $\frac{1}{2}$, -3.5, and 0.

recognizing the numbers being added

You must be able to recognize an addition problem, and identify the numbers that are being added:

In the addition problem $2+5+(-4)$, there are three numbers being added: 2, 5, and -4.

In the addition problem $-3+(-1)+6$, there are three numbers being added: -3, -1, and 6.

Notice that when you add a negative number, you should put it in parentheses, unless it comes first.

EXERCISES

2. Identify the numbers that are being added in each addition problem below:
   a. $5 + (-3) + 8 + (-4)$
   b. $-7 + 2 + (-3)$
3. Write the addition problem:
   a. add, in this order: -4, 8, and -9
   b. add, in this order: 7, 1, and -5

SIZE and SIGN

Every number has a SIZE (its distance from zero). Every nonzero number has a SIGN (positive or negative). Recall that the number 0 is 'neutral'—it is not positive, and not negative.

The number 3 : its SIZE is 3, and its SIGN is positive.

The number -3 : its SIZE is 3, and its SIGN is negative.

In the movement interpretation of a real number, the SIZE tells us how far to move, and the SIGN tells us which direction to move.

EXERCISES

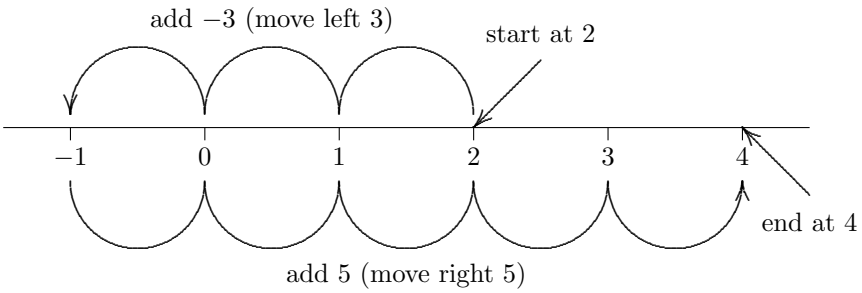
4. Give both the SIZE and SIGN for each of these numbers, if possible: 5 , $\frac{1}{2}$, -4 , and 0 .

an addition problem:
first number indicates position;
subsequent numbers indicate movement

Now we're ready to combine the position and movement ideas in an addition problem. The process is illustrated first with an example:

Consider the problem $2 + (-3) + 5$.

- The first number, 2 , indicates a position. Go to 2 on the number line.
- Adding a negative number indicates movement to the left. Thus, adding -3 says to move 3 units to the left.
- Adding a positive number indicates movement to the right. Thus, adding 5 says to move 5 units to the right.
- You end up at position 4 . Thus, $2 + (-3) + 5 = 4$.



number line interpretation of: $2 + (-3) + 5 = 4$

EXERCISES

5. Interpret the problem $-3 + (-1) + 5$ in terms of position and movement on the number line.
6. Interpret the problem $6+(-2)+(-3)+1$ in terms of position and movement on the number line.

alternate approach:

start at zero, and all numbers indicate movement

Here's an alternate approach to interpreting the addition problem $2+(-3)+5$.

You can always start at zero! That is, write $2+(-3)+5$ as $0+2+(-3)+5$. Start at 0, move 2 to the right, 3 to the left, and 5 to the right, ending up at 4.

You should understand both interpretations, but in practice you can use whichever is more natural to you. The 'start at zero' interpretation is used in the following discussion.

EXERCISES

7. Interpret the problem $6+(-2)+(-3)+1$ using the 'start at zero' interpretation.

efficiency in the process

You probably don't want to be drawing number lines every time you need to do an addition of signed numbers problem. The good news is that every problem boils down to either an addition problem or a subtraction problem, which can be done efficiently in your head. Keep reading!

adding numbers with the same sign: an addition problem

When you add numbers with the same signs (both positive or both negative), then in your head you do an addition problem. Here are two examples:

$2+3$: Start at zero. Move to the right 2, then to the right 3. End up at 5. Thus $2+3=5$.

$-2+(-3)$: Start at zero. Move to the left 2, then to the left 3. The total distance moved is $2+3=5$. You moved to the left, so you end up at -5. Thus, $-2+(-3)=-5$.

Notice that in both of these problems, you do an *addition* problem in your head; this addition problem gives the total distance moved. If you always move to the right, the final answer is positive. If you always move to the left, the final answer is negative.

EXERCISES

8. Indicate whether the numbers being added have the *same sign* or *different signs*:
   a. $2+7$
   b. $-3+(-8)$
   c. $7+(-4)$
   d. $-6+5$

adding numbers with different signs: a subtraction problem

When you add numbers with different signs (one positive, one negative), then in your head you do a subtraction problem. Here are two examples:

$2 + (-3)$: Start at zero. Move 2 to the right and 3 to the left. You moved more to the left—how much more? $3 - 2 = 1$. So you end up at -1. Thus, $2 + (-3) = -1$. The mental process is this:

Once you recognize that you're adding numbers with different signs, throw away (for the moment) all the signs, take the bigger number, and subtract the smaller number. This gives you the net distance traveled. If you move farther to the right, your answer is positive. If you move farther to the left, your answer is negative. Notice that when you add numbers with different signs, in your head you do a subtraction problem.

another example

Here's another example:

$3 + (-2)$: Start at zero. Move 3 to the right and 2 to the left. You moved more to the right—how much more? $3 - 2 = 1$. So you end up at 1. Thus, $3 + (-2) = 1$.

EXERCISES

9. For each problem below, indicate whether you will be doing an *addition* or *subtraction* problem in your head:
   a. $2 + 7$
   b. $-3 + (-8)$
   c. $7 + (-4)$
   d. $-6 + 5$
10. In the problem $3 + (-2)$, the bigger number is positive, so the answer is positive.

    In the problem $2 + (-3)$, the bigger number is negative, so the answer is negative.

    Decide if each answer below will be positive or negative:
    a. $5 + (-7)$
    b. $7 + (-5)$
    c. $-4 + 6$
    d. $6 + (-4)$
11. Try to come up with a memory device to help you remember that when you have the same sign, you add; when you have different signs, you subtract.

a five step process for adding signed numbers

When you add two signed numbers, you can follow this five step process. As you read through these steps, think of applying these questions to the problem $2 + (-3)$:

step 1: what numbers are added?

(1) What numbers are being added? (Answer: 2 and -3)

step 2: same or different signs?

(2) Do these numbers have the same sign or different signs? (Answer: different signs)

step 3: addition or subtraction problem?

(3) In your head, will you be doing an addition or subtraction problem? (Answer: subtraction problem)

step 4: do the arithmetic

(4) Do the appropriate addition or subtraction problem. (Answer: Throw away the signs, leaving you with 2 and 3. Subtract the smaller from the larger: $3 - 2 = 1$)

step 5: answer positive or negative?

(5) Is your answer positive or negative? (Answer: The bigger number is negative. So $2 + (-3) = -1$)

Don't worry! This will become automatic!

Although this seems like a lot of work, don't worry! After a lot of practice, it will become completely automatic.

more examples

Here are more examples of the five-step process.

EXAMPLE: $-5 + (-3)$

- The numbers being added are -5 and -3.
- They have the same sign. Both numbers are negative.
- In your head, you'll be doing an addition problem.
- $5 + 3 = 8$
- The numbers being added are negative, so the result is negative.
  $-5 + (-3) = -8$

EXAMPLE: $-6 + 4$

- The numbers being added are -6 and 4.
- They have different signs: -6 is negative; 4 is positive
- In your head, you'll be doing a subtraction problem.
- Throw away the signs, leaving you with 6 and 4. Subtract the smaller from the larger: $6 - 4 = 2$
- The bigger number is negative, so the answer is negative.
  $(-6) + 4 = -2$

more than two numbers

If you have more than two numbers being added, just turn it into a two-number problem in the first step, by combining the positive and negative numbers separately, like this:

$$\begin{aligned} -3 + 5 + (-2) + 1 + 4 + (-6) &= \overbrace{-3 + (-2) + (-6)}^{\text{combine negative}} + \overbrace{5 + 1 + 4}^{\text{combine positive}} \\ &= -11 + 10 \\ &= -1 \end{aligned}$$

EXERCISES
web practice

Go to my homepage **http://onemathematicalcat.org** and navigate to my Algebra I course, which has about 170 sequenced lessons. It can be used as a complete year-long high school course, or one semester in college. You're currently looking at the pdf version—you'll see that the HTML version has unlimited, randomly-generated, online and offline practice in every section. It's all totally free. Enjoy!

SOLUTIONS TO EXERCISES:
ADDITION OF SIGNED NUMBERS

1. the number 2:

position: go to position 2 on the number line

movement: move 2 units to the right

the number $\frac{1}{2}$:

position: go to position $\frac{1}{2}$ on the number line

movement: move $\frac{1}{2}$ unit to the right

the number -3.5:

position: go to position 3.5 on the number line

movement: move 3.5 units to the right

the number 0:

position: go to position 0 on the number line

movement: do not move (i.e., stay at current location)

2. a) 5, -3, 8, and -4

b) -7, 2, and -3

3. a) $-4+8+(-9)$

b) $7+1+(-5)$

4. the number 5: SIZE is 5, SIGN is positive

the number $\frac{1}{2}$: SIZE is $\frac{1}{2}$, SIGN is positive

the number -4: SIZE is 4, SIGN is negative

the number 0: SIZE is 0, no sign

5. start at position -3; move 1 unit to the left; move 5 units to the right; end up at 1

6. start at position 6; move 2 units to the left; move 3 more units to the left; move 1 unit to the right; end up at 2

7. start at 0; move 6 units to the right; move 2 units to the left; move 3 more units to the left; move 1 unit to the right; end up at 2

8. a) same signs (both positive)

b) same signs (both negative)

c) different signs

d) different signs

9. a) addition (numbers have the same sign; both positive)

b) addition (numbers have the same sign; both negative)

c) subtraction (numbers have different signs)

d) subtraction (numbers have different signs)

10. a) negative (the bigger number is -7, which is negative)

b) positive (the bigger number is 7, which is positive)

c) positive (the bigger number is 6, which is positive)

d) positive (the bigger number is 6, which is positive)

11. Same Sign, Sum. Different Signs, Difference.