

18. UNIT CONVERSION

*multiplying by 1
in an
appropriate form*

You have seen ‘multiplying by 1 in an appropriate form’ used to get a common denominator when adding fractions; this is only one of a multitude of uses for this important technique. One of the most common applications of ‘multiplying by 1 in an appropriate form’ occurs in the context of unit conversion, which is the subject of this section and the next.

unit conversion

Often, in life, you’re required to convert a quantity from one unit to another. For example, you might need to convert centimeters to inches; miles to feet; tablespoons to teaspoons; or feet/second (read as ‘feet per second’) to miles/hour. In such cases, you have a quantity of interest, but are seeking a *new name* for that quantity. The process of finding this new name is called *unit conversion*.

All unit conversion problems can be accomplished by multiplying by 1 in an appropriate form.

*in a fraction $\frac{N}{D}$,
 N is called
the NUMERATOR;
 D is called
the DENOMINATOR*

Recall that in any fraction $\frac{N}{D}$, the quantity ‘upstairs’ (N) is called the **numerator** (NEW-mer-ā-tor). The quantity ‘downstairs’ (D) is called the **denominator** (dee-NAHH-mi-nā-tor). Read $\frac{N}{D}$ as ‘ N divided by D ’, or ‘ N over D ’.

*whenever two quantities
are equal, we get
two names for
the number 1*

Let’s re-visit the true sentence $\frac{x}{x} = 1$ (for nonzero real numbers x). This fact says that *whenever you have a fraction where the numerator and denominator are equal, then the fraction represents the number 1*:

- Since 1 meter = 1000 millimeters,

$$\frac{1 \text{ meter}}{1000 \text{ millimeters}} = 1 \quad \text{and} \quad \frac{1000 \text{ millimeters}}{1 \text{ meter}} = 1 .$$

- Since 1 tablespoon = 3 teaspoons,

$$\frac{1 \text{ tablespoon}}{3 \text{ teaspoons}} = 1 \quad \text{and} \quad \frac{3 \text{ teaspoons}}{1 \text{ tablespoon}} = 1 .$$

- Since 1 mile = 5280 feet,

$$\frac{1 \text{ mile}}{5280 \text{ feet}} = 1 \quad \text{and} \quad \frac{5280 \text{ feet}}{1 \text{ mile}} = 1 .$$

EXERCISE

1. Give two names for the number 1 that are a consequence of the following facts:

- (a) 1 pint = 2 cups
- (b) 1 m = 100 cm (m = meter; cm = centimeter)
- (c) 1 bleep = 3.4 blops (made-up units!)

*working with fractions:
a reminder*

Here's a reminder of the fraction skills you'll need when doing unit conversion problems:

- To multiply fractions, you multiply 'across':

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

For example,

$$\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$$

- A number sitting 'next to' a fraction can be moved into the numerator:

$$a \cdot \frac{b}{c} = \frac{a \cdot b}{1 \cdot c} = \frac{ab}{c}$$

- When multiplying fractions involving numbers with units:
 - Any unit that appears in *both* the numerator and denominator can be 'cancelled'—it disappears.
 - Group together all the numbers, and write these first. Then, group together any units that haven't been cancelled, and write these last.
 - Simplify the numerical part.

Here's an example:

$$\begin{aligned} \frac{(12 \text{ ft})(1 \text{ yd})}{3 \text{ ft}} &= \frac{(12 \cancel{\text{ft}})(1 \text{ yd})}{3 \cancel{\text{ft}}} \\ &= \frac{12 \cdot 1}{3} \text{ yd} \\ &= 4 \text{ yd} \end{aligned}$$

Cancel any units that appear in both the numerator and denominator.

Group together all the numbers, and write these first. Write any surviving unit(s) last.

Simplify the numerical part.

These ideas are put together in the following examples.

some 1-step conversions

First, some ‘one-step’ conversions. (These only require multiplying by the number 1 *once*.)

Any unit of length can be converted to any other unit of length. For example,

$$42 \text{ in} = 42 \cancel{\text{ in}} \cdot \overset{\text{multiply by 1}}{\underbrace{\frac{1 \text{ ft}}{12 \cancel{\text{ in}}}}} = \frac{42 \cdot 1}{12} \text{ ft} = 3.5 \text{ ft}$$

Here, the name $\frac{1 \text{ ft}}{12 \text{ in}}$ was used for the number 1.

However, if feet are being converted to inches, then a different name for the number 1 is needed:

$$3.5 \text{ ft} = 3.5 \cancel{\text{ ft}} \cdot \overset{\text{multiply by 1}}{\underbrace{\frac{12 \text{ in}}{1 \cancel{\text{ ft}}}}} = \frac{3.5 \cdot 12}{1} \text{ in} = 42 \text{ in}$$

Although these examples have shown factors of 1 after grouping together the numbers, in practice you don’t need to show them. Then, the previous two problems look a bit simpler:

$$42 \text{ in} = 42 \cancel{\text{ in}} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{ in}}} = \frac{42}{12} \text{ ft} = 3.5 \text{ ft}$$

$$3.5 \text{ ft} = 3.5 \cancel{\text{ ft}} \cdot \frac{12 \text{ in}}{1 \cancel{\text{ ft}}} = 3.5 \cdot 12 \text{ in} = 42 \text{ in}$$

You can show the factors of 1, or not—whichever is most comfortable for you.

Resist the temptation to modify your original expression!

When you’re converting units, as in the previous examples, don’t modify the original expression after you’ve written it. That is, you should write:

$$\overset{\text{Don't modify this!}}{\underbrace{42 \text{ in}}} = 42 \cancel{\text{ in}} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{ in}}} = \frac{42}{12} \text{ ft} = 3.5 \text{ ft} ,$$

NOT

$$42 \cancel{\text{ in}} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{ in}}} = \frac{42}{12} \text{ ft} = 3.5 \text{ ft} .$$

When you look back at your work, you’ll want to remember what you had started with: if you’ve modified your ‘starting expression’ then this will not be easy to see.

EXERCISES

2. The web exercises give you unlimited one-step conversion problems. Here are a few to get you started. Even though you may be able to do these in your head, write down all the steps to get practice with the correct format. (In the next section, when the conversions get more complicated, you won’t be able to do the work in your head.) Use abbreviations for the units.
 - a. Convert 17 feet to inches.
 - b. Convert 13.5 feet to yards.
 - c. Convert 2 weeks to days.
 - d. Convert 3 days to hours.

*unit conversion
information
that you should
memorize*

Any unit of length can be converted to any other unit of length.
Any unit of time can be converted to any other unit of time.
Any unit of volume can be converted to any other unit of volume.
Any unit of mass or weight can be converted to any other unit of mass or weight.
The following tables give the information that you should memorize. In particular, you must be able to do the following:

- Given a unit, identify it as a unit of length, time, volume, or mass/weight. (★ Weight is mass times acceleration due to gravity.)
- Know the names of units, and their common abbreviations.
- Be able to identify units that are metric.
- Know when two quantities are *equal* (like 1 foot equals 12 inches), and when two quantities are only *approximately equal* (like 1 centimeter is about 0.4 inches). Whenever two quantities are only approximately equal, more precise conversion information is available if needed. For example, a more accurate comparison of centimeters and inches is that 1 centimeter is about 0.3937 inches. However, all conversion problems in this text and in the web exercises will use the approximations given.
- Know all the conversion information. Many of these facts (like 1 min = 60 sec) will be familiar to the majority of readers. However, there are likely many pieces of information that you will have to purposefully commit to memory.

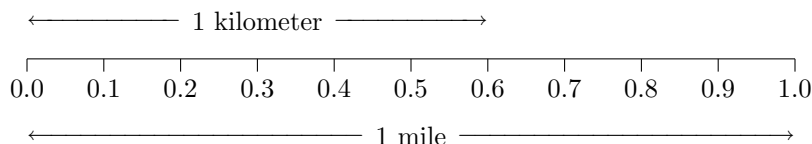
*a question you must
be able to answer:
which is longer?*

Glancing at the unit conversion tables, you'll notice information like this:

$$1 \text{ km} \cong 0.6 \text{ mi}$$

That is, 1 kilometer is about 0.6 miles. The question is: which is longer, a kilometer or a mile?

To answer this question, study the diagram below:



From the diagram, it is clear that a mile is longer than a kilometer. The problem for many people is that when you look at the sentence $1 \text{ km} \cong 0.6 \text{ mi}$, the bigger number (1) is attached to the shorter length (km), and this seems to cause confusion. So be careful about this!

EXERCISES

3.
 - a. $1 \text{ cm} \cong 0.4 \text{ in}$; which is longer, a centimeter or an inch?
 - b. $1 \text{ liter} \cong 1.1 \text{ quart}$; which is more, a liter or a quart?
 - c. Suppose that 'bleep' and 'blop' are units of length, and that $0.9 \text{ bleep} = 1 \text{ blop}$; which is longer, a bleep or a blop?

UNIT CONVERSION INFORMATION

Abbreviations may be used for either singular or plural usage:

1 foot is abbreviated as **1 ft**

2 feet is abbreviated as **2 ft**

UNITS OF LENGTH

NAME OF UNIT	COMMON ABBREVIATION	CONVERSION INFORMATION
millimeter (metric)	mm	
centimeter (metric)	cm	1 cm is about 0.4 in a centimeter is shorter than an inch
meter (metric)	m	1 m is about 1.1 yd a meter is a little longer than a yard 1 m = 1000 mm 1 m = 100 cm
kilometer (metric)	km	1 km is about 0.6 mi a kilometer is shorter than a mile a kilometer is a little more than half a mile 1 km = 1000 m
inch	in	1 in is about 2.5 cm
foot	ft	1 ft = 12 in
yard	yd	1 yd = 36 in 1 yd = 3 ft
mile	mi	1 mi is about 1.6 km a mile is longer than a kilometer a mile is a little more than one and a half kilometers 1 mi = 5,280 ft

UNITS OF TIME

NAME OF UNIT	COMMON ABBREVIATION	CONVERSION INFORMATION
second	sec	
minute	min	1 min = 60 sec
hour	hr	1 hr = 60 min
day		1 day = 24 hr
week	wk	1 wk = 7 days
month		1 month is about 30 days
year	yr	1 yr = 365 days
century		1 century = 100 yr

UNITS OF VOLUME

NAME OF UNIT	COMMON ABBREVIATION	CONVERSION INFORMATION
liter (metric)		1 liter is about 1.1 qt 1 liter is a little more than a quart 1 liter = 1000 ml
milliliter (metric)	ml	
gallon	gal	1 gal = 4 qt
quart	qt	1 qt = 4 cups
pint	pt	1 pt = 2 cups
cup		1 cup = 8 fl oz
fluid ounce	fl oz	
tablespoon	tb	1 tb = 3 tsp
teaspoon	tsp	

UNITS OF MASS AND WEIGHT

NAME OF UNIT	COMMON ABBREVIATION	CONVERSION INFORMATION
kilogram (metric)	kg	1 kg = 1000 g
gram (metric)	g	
ton		1 ton = 2000 lb
pound	lb	1 lb = 16 oz
ounce	oz	

calculator skills:
computing expressions
of the form
 $\frac{ab}{cd}$

Unit conversion problems end up with fractions of the form $\frac{ab}{cd}$, and you must be able to compute these correctly on your calculator.
Here's the key to success. First think of rewriting $\frac{ab}{cd}$ as

$$ab \cdot \frac{1}{c} \cdot \frac{1}{d}$$

Then, since multiplying by $\frac{1}{c}$ is the same as dividing by c (and the same with d) rewrite again:

$$\frac{ab}{cd} = a \cdot b \div c \div d$$

This makes it clear that factors in the numerator require the multiplication key, and factors in the denominator require the division key.

EXAMPLE To key in the fraction $\frac{(2)(5280)}{(14)(60)}$ on an algebraic calculator (like the TI-83), you can use these keystrokes:

$$2 \times 5280 \div 14 \div 60$$

and the answer you should get, rounded to the thousandths place, is 12.571.

using parentheses keys Alternately, if your calculator has parentheses keys, you can key it in like this:

$$2 \times 5280 \div (14 \times 60)$$

EXERCISES

4. Key the following into your calculator. Round your answers to four decimal places. Try keying them in both ways described above, with and without using parentheses.

a. $\frac{(60)(24)}{(1.1)(5280)}$

b. $\frac{(2.54)(2000)}{(36)(0.4)}$

c. $\frac{(0.6)(365)}{(24)(100)}$

d. $\frac{(1.6)(1000)}{(36)(3)}$

EXERCISES

web practice

Go to my homepage <http://onemathematicalcat.org> and navigate to my Algebra I course, which has about 170 sequenced lessons. It can be used as a complete year-long high school course, or one semester in college. You're currently looking at the pdf version—you'll see that the HTML version has unlimited, randomly-generated, online and offline practice in every section. It's all totally free. Enjoy!

SOLUTIONS TO EXERCISES: UNIT CONVERSION

1. a. $1 = \frac{1 \text{ pint}}{2 \text{ cups}} = \frac{2 \text{ cups}}{1 \text{ pint}}$

b. $1 = \frac{1 \text{ m}}{100 \text{ cm}} = \frac{100 \text{ cm}}{1 \text{ m}}$

c. $1 = \frac{1 \text{ bleep}}{3.4 \text{ blop}} = \frac{3.4 \text{ blop}}{1 \text{ bleep}}$

2. a. $17 \text{ ft} = 17 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 17 \cdot 12 \text{ in} = 204 \text{ in}$

b. $13.5 \text{ ft} = 13.5 \text{ ft} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{13.5}{3} \text{ yd} = 4.5 \text{ yd}$

c. $2 \text{ wk} = 2 \text{ wk} \cdot \frac{7 \text{ days}}{1 \text{ wk}} = 2 \cdot 7 \text{ days} = 14 \text{ days}$

d. $3 \text{ days} = 3 \text{ days} \cdot \frac{24 \text{ hr}}{1 \text{ day}} = 3 \cdot 24 \text{ hr} = 72 \text{ hr}$

3. a. an inch is longer

b. a liter is a little more than a quart

c. a bleep is longer than a blop

4. a. $\frac{(60)(24)}{(1.1)(5280)} \cong 0.2479$

b. $\frac{(2.54)(2000)}{(36)(0.4)} \cong 352.7778$

c. $\frac{(0.6)(365)}{(24)(100)} \cong 0.0913$

d. $\frac{(1.6)(1000)}{(36)(3)} \cong 14.8148$