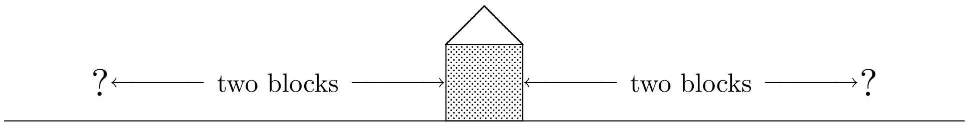
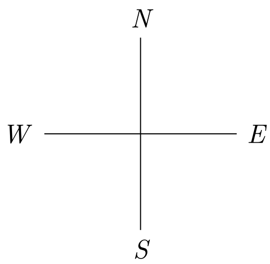
14. I LIVE TWO BLOCKS WEST OF YOU

introduction: position of one object relative to another

Suppose you're running errands, and meet someone who recently moved to the street where you live. During the conversation, your 'new neighbor' comments on their home's location relative to yours:

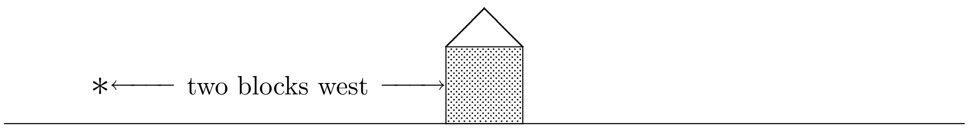
- Oh, we live only two blocks from you!





Do you know where they live? Not exactly. They could live at either of the places shown above: two blocks to the west, or two blocks to the east. However, suppose they were to instead say:

- Oh, we live two blocks west of you!



Now, their location is uniquely determined. Such information about the position of one object relative to another is commonplace in English, as further illustrated by the following phrases:

- The math book you want is three from the left on that shelf.
- She's two inches shorter than you are.
- The exit you need is the second one after the underpass.

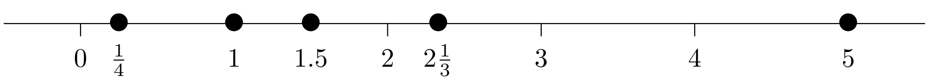
This type of information finds its mathematical counterpart in the concept of *order*.

a natural ordering of the real numbers

Suppose you're given a collection of numbers (all different), and asked to 'order them'. If all the numbers are positive, then you'd probably order them according to their size: smallest (closest to zero) to largest (farthest from zero). For example, the numbers 1, $\frac{1}{4}$, $2\frac{1}{3}$, 5, and 1.5 could be ordered like this:

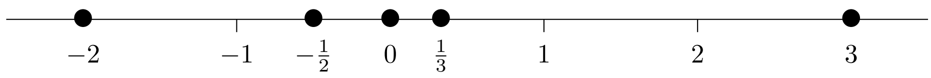
$$\frac{1}{4} \qquad 1 \qquad 1.5 \qquad 2\tfrac{1}{3} \qquad 5$$

Observe that if you were to walk from left to right along the number line, you'd first encounter $\frac{1}{4}$; then 1; then 1.5; then $2\frac{1}{3}$; then 5.



ordering an arbitrary collection of real numbers

If the collection contains negative numbers, then the most natural ordering occurs by listing them as they would be encountered in walking from left to right along the number line. Using this idea, the numbers -2, 0, $-\frac{1}{2}$, 3, and $\frac{1}{3}$ would be ordered as



This scheme provides a natural way to order any collection of (different) real numbers. Now, let's begin to make this idea precise.

comparing any two real numbers

Given any two real numbers x and y, exactly one of the following three situations exists:

- x equals y (that is, x and y live at the same place on a real number line);
- x lies to the left of y on a number line; or
- x lies to the right of y on a number line.

EXERCISES

Let x, y, and z be real numbers.

1. Suppose that x lies to the left of y, and y lies to the left of z. What (if anything) can be said about the relationship between x and z?
2. Suppose that x lies to the left of y, and z also lies to the left of y. What (if anything) can be said about the relationship between x and z?
3. Suppose that x and y are both positive, and x lies to the left of y on the number line. What (if anything) can be said about the relationship between $-x$ (the opposite of x) and $-y$ (the opposite of y)?
4. Suppose that x and y are both negative, and x lies to the right of y. Which number is farther from zero? Closer to zero?

mathematical sentences to describe order relationships

There are four mathematical sentences that make it easy to talk about the order relationships between any two real numbers:

$$x < y \qquad x > y \qquad x \le y \qquad x \ge y$$

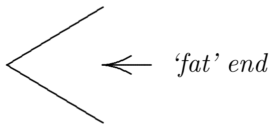
As with all mathematical sentences, you should know *the correct way to read each of these sentences*, and *the condition(s) under which each is true or false*. This is the next topic of discussion.

sentences: $x < y$, $x > y$, $x \le y$, $x \ge y$

sentence	how to read	truth of sentence
$x < y$	x is less than y	TRUE when x lies to the left of y on a number line; FALSE otherwise
$x > y$	x is greater than y	TRUE when x lies to the right of y on a number line; FALSE otherwise
$x \le y$	x is less than or equal to y	TRUE when $x < y$ or $x = y$; FALSE otherwise
$x \ge y$	x is greater than or equal to y	TRUE when $x > y$ or $x = y$; FALSE otherwise

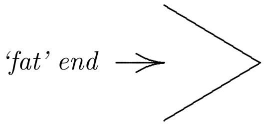
memory devices

The following memory devices may be useful:



- In any *true* sentence, the 'fat' end of the verb opens to the number that lies farthest to the right on the number line. (Perhaps imagine the fat end 'gobbling up' the right-most number.)

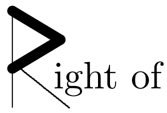
$x > y$	$x < y$
'fat' end opens towards x	'fat' end opens towards y
x lies farthest to the right	y lies farthest to the right
	



- If you let the verb < 'fall', then it looks like the letter 'L', as in '**L**ess than' or 'to the **L**eft of'. So if you want to determine if the sentence '$x < y$' is true, think to yourself:

Is $x < y$?
Is x Less than y?
Does x lie to the Left of y on a number line?

- The symbol > is easily made into a capital letter 'R', as in 'greate**R** than', or 'to the **R**ight of':

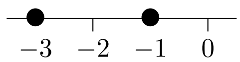
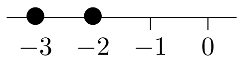
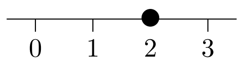
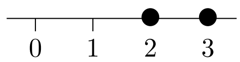
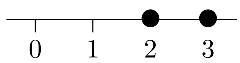


EXAMPLES

Whenever you come across a sentence of the form '$x < y$', think to yourself: does x lie to the left of y on a number line?

Whenever you come across a sentence of the form '$x > y$', think to yourself: does x lie to the right of y on a number line?

The following examples illustrate the thought process you could go through to determine whether a given sentence is true or false:

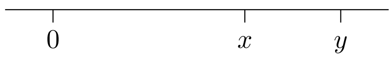
sentence	how to read aloud	thought process	
$-1 > -3$	'negative one is greater than negative three'	Does -1 lie to the right of -3? Yes! The sentence is true.	
$-2 < -3$	'negative two is less than negative three'	Does -2 lie to the left of -3? No! The sentence is false.	
$2 \geq 2$	'2 is greater than or equal to 2'	Is 2 greater than 2, or is 2 equal to 2? Yes: 2 is equal to 2. The sentence is true.	
$3 \geq 2$	'3 is greater than or equal to 2'	Is 3 greater than 2, or is 3 equal to 2? Yes: 3 is greater than 2. The sentence is true.	
$3 \leq 2$	'3 is less than or equal to 2'	Is 3 less than 2, or is 3 equal to 2? No: both considerations are false. The sentence is false.	

CAUTION!
Do NOT read
'$x < y$'
like this . . .

DO NOT read the sentence '$x < y$' as 'x is smaller than y'. The correct way to read it is: 'x is less than y'. Being 'smaller than' and being 'less than' are two *different* ideas:

- 'Smaller than' means closer to zero.
- 'Less than' means farther to the left on a number line.

For *positive* numbers, the two ideas coincide nicely: x is smaller (closer to zero) than y, precisely when x lies to the left of y:



However, consider a more general situation. The sentence '$-5 < 1$' is true, since -5 lies to the left of 1. But would you really want to say that -5 is 'smaller' than 1? (No!)

The same caution applies to the sentence '$x > y$'. Be sure to read it as 'x is greater than y', NOT 'x is bigger than y'.

EXERCISES

5. State how you would read each of the following sentences. Then, state whether the sentence is (always) true, (always) false, or ST/SF:

(a) $1 < 3$	(e) $x \le 1$
(b) $2 \le 2$	(f) $x \ge -1$
(c) $-1 > -3$	(g) $-1 \ge 1$
(d) $-1 < -3$	(h) $x \ge x$

6. Fill in the blanks:

Being 'bigger than' has to do with being ____________________ .

Being 'greater than' has to do with being ____________________ .

★
the mathematical word
'OR'

Precisely: For all real numbers x and y,

$$x \le y \iff (x < y) \text{ OR } (x = y) .$$

The mathematical word 'OR' is defined via the following truth table:

A	B	A OR B
T	T	T
T	F	T
F	T	T
F	F	F

Thus, an 'OR' sentence is true if at least one of the subsentences is true.

The mathematical words 'AND' and 'OR', and the symbol $\iff$, will be discussed in later sections.

greatest

If you are asked for the *greatest* number in some collection, then you're being asked for the number that lives *farthest to the right* on a number line. (Don't confuse this with 'biggest', which means farthest from zero.)

least

If you are asked for the *least* number in some collection, then you're being asked for the number that lives *farthest to the left* on a number line. (Don't confuse this with 'smallest', which means closest to zero, but not equal to zero.)

Every finite set of numbers will have a greatest and a least member. An infinite set of numbers may or may not have a greatest/least member.

EXERCISES

7. Consider the set $S = \{4, 3, 2, 1\}$. What is the greatest member? The least?

8. Consider the set $S = \{1, 2, 3, \dots\}$. Does S have a greatest member? A least member?

9. Consider the set of negative integers, $\{-1, -2, -3, \dots\}$. Does this set have a greatest member? A least member?

10. Consider the set of positive real numbers, $(0, \infty)$. Does this set have a greatest member? A least member?

11. Consider the set $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$. Does this set have a greatest member? A least member?

the phrases: 'at least' 'at most'

There are two phrases that commonly appear, both in mathematics and in life: 'at least' and 'at most'. Consider the following examples:

- I want at least five pieces of candy.
- The most I can afford to spend is \$10.

'at least'

The sentence 'x is at least 5' means that the *least* x is allowed to be is 5; it can be 5, or any number greater than 5. So, the phrase 'x is at least 5' means '$x \ge 5$'.

5 ← *least* x is allowed to be

'at most'

The sentence 'x is at most 10' means that the *most* x is allowed to be is 10; it can be 10, or any number less than 10. So, the phrase 'x is at most 10' means '$x \le 10$'.

most x is allowed to be → 10

EXERCISES

12. Remember that mathematical sentences are often read in slightly different ways, depending on their context. How do you suppose you would read the sentence '$x \ge 5$' in each of the following contexts?

(a) For all $x \ge 5$ ...

(b) Let $x \ge 5$.

13. Translate each of the phrases into a mathematical sentence:

(a) t is at most 2

(b) t is at least 2

(c) y is at most -2

(d) y is at least -2

14. Translate each of the following mathematical sentences into an English phrase using the words 'at least' or 'at most':

(a) $t \le 4$

(b) $t \ge 4$

(c) $y \le -4$

(d) $y \ge -4$

an entire class of sentences, each having the same form

The author would like to make a definition that covers a *whole class of sentences*, including all the following particular instances:

x is at least 2	x is at least -2
x is at least $\frac{1}{2}$	x is at least -3.5
y is at least 4	y is at least $-\frac{1}{3}$
z is at least 7	z is at least -7
...	...

Each of these sentences has a similar form:

some variable is at least *some specific number*

Given a sentence of this form, we're interested in the values of the *variable* that make the sentence true; the 'specific number' is fixed—constant—not allowed to change—within the sentence.

'x is at least k'

To accomplish the feat of describing this *whole class* of sentences in one fell swoop, people often use the letter k (as in the incorrect spelling 'konstant') to denote the number to be held constant, thus yielding the sentence

'x is at least k'.

Notice that the sentence 'x is at least k' has two variables: x and k. The variable x has universal set $\mathbb{R}$. The variable k has universal set $\mathbb{R}$. However, *the two variables serve different purposes*:

- x is allowed to vary *within* a given sentence;
- k is fixed *within* a given sentence, and only allowed to vary from sentence to sentence.

How do we know which variable is which?

In the sentence 'x is at least k', how do we know which variable is which? That is, which variable is allowed to vary *within* the sentence, and which is being *held constant* for a given sentence? Mathematical conventions! The variable k is conventionally used to denote something that is to be held constant in a particular sentence. Also, letters from the beginning of the alphabet (like a, b, and c) frequently denote constants.

This idea of 'varying within a sentence' versus 'varying from sentence to sentence' is difficult. Here's some practice:

EXERCISES

15. Give three sentences of the form 'x is at most k'. (Each sentence should use the variable x, but not k.)

16. Give three sentences of the form '$x = k$'. (Each sentence should use the variable x, but not k.)

17. Give three sentences of the form '$ax + by = c$'. (Assume x and y vary within the sentence; a, b and c are constant within a given sentence.)

sentences that are completely interchangeable

We have seen that the sentences

x is at most 10

and

$$x \le 10$$

have the same meaning. They're completely interchangeable. We can use whichever sentence is most convenient to use in a particular situation. If one sentence is true, so is the other. If one sentence is false, so is the other.

equivalence of sentences

This idea of 'looking different, but having the same truth value' is made precise by the mathematical idea of *equivalence*. *Equivalence of sentences* will be discussed in detail in a future section. For now, there are several things that you must know:

- Although the words 'equal' and 'equivalent' are used almost interchangeably in English, they have VERY different uses in mathematics!
- Generally, we talk about EXPRESSIONS being EQUAL:

  When two NUMBERS are EQUAL, this means that they live at the same place on a real number line.

  When two SETS are EQUAL, this means that they have precisely the same members.
- Generally, we talk about SENTENCES being EQUIVALENT: this has to do with the sentences having the same truth values.

summary: 'x is at least k ' 'x is at most k '

The sentences 'x is at least k' and 'x is at most k' are summarized in the following table. The column labeled 'equivalent sentence' gives a form of the sentence that *looks different*, but that has the same truth values as the original sentence.

sentence	meaning of sentence	equivalent sentence
x is at least k	the least x is allowed to be is k: so, x can equal k, or x can be greater than k	$x \ge k$
x is at most k	the most x is allowed to be is k: so, x can equal k, or x can be less than k	$x \le k$

equation

An **equation** (ee-KWEY-shun) is a mathematical sentence that uses the verb '$=$' . Here are some examples of equations:

$3 = 4$	a false equation
$x + 1 = 1 + x$	an equation that is always true
$x + 1 = x + 2$	an equation that is always false (Why? Keep reading!)
$x = 3$	an equation that is sometimes true/sometimes false

inequality

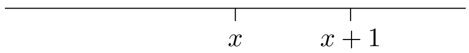
An **inequality** (in–ee–KWAL–i–tee) is a mathematical sentence that uses one of the four verbs: $<$, $\le$, $>$, or $\ge$. Observe that the prefix 'in' is frequently used in English to negate something: *in*ability, *in*complete, *in*decisive, *in*ept. Thus, *in*equality has to do, roughly, with being 'not equal'. Here are some examples of inequalities:

$-1 < -3$	a false inequality
$-1 > -3$	a true inequality
$x < x + 1$	an inequality that is always true (Why? Keep reading!)
$x - 1 > x$	an inequality that is always false (Why? Keep reading!)
$x \ge 1$	an inequality that is sometimes true/sometimes false

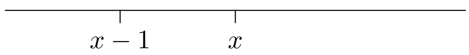
If we know where x lives, where do $x+1$ and $x-1$ live?

Some of the previous examples merit special attention:

- Consider the inequality '$x < x+1$'. Let x be any real number. Then, $x+1$ lives one unit to the right of x on a number line. Therefore, x *always* lies to the left of $x+1$; so the sentence '$x < x+1$' is always true.



- Consider the inequality '$x-1 > x$'. Let x be any real number. Then, $x-1$ lives one unit to the left of x on a number line. Therefore, $x-1$ *never* lies to the right of x, so the sentence '$x-1 > x$' is always false.



EXERCISE

18. Classify each of the following sentences as an *equation* or an *inequality*. In each case, state whether the sentence is (always) true, (always) false, or sometimes true/sometimes false. Think in terms of position on a number line: choose x, and then determine where the other numbers live relative to x.

(a) $x+1 = x$
(b) $x-1 < x$
(c) $x+3 \geq x+2$
(d) $x-1 < x+1$

sentence in one variable

A mathematical sentence that uses only one variable is called a *sentence in one variable*. The variable may appear *any number of times*; the key idea is that *only one distinct letter* appears in the sentence.

Consider the following examples:

- '$3x - 1 = x + 2$' is an *equation in one variable*; here, the variable is x.
- '$t + 2t - \frac{3}{t} > 4$' is an *inequality in one variable*; here, the variable is t.
- '$x+y = 2$' is an *equation in two variables*. It is an *equation* because of the '=' sign; it is an equation *in two variables* because two different variables (x and y) appear.

EXERCISES

19. Classify each of the following as an equation/inequality in n variables.

(a) $x + 2x = 5 - x$
(b) $x + y + z > 0$
(c) $\dfrac{2x-1}{x+4} \leq 5x$
(d) $a + b = 2b - 3 - 5a$

20. Why do you suppose the author chose the letter n (as opposed to, say, t) to represent the number of variables in the instructions to the previous problem?

When is a sentence TRUE?

People are very often interested in knowing when a mathematical sentence is *true*. The following definitions are useful in this context:

DEFINITION *solution of a sentence in one variable; solution set*	Let S denote a sentence in one variable. A number that makes S true is called a *solution* of the sentence. The set of all number(s) that make S true is called the *solution set* of the sentence.

a variable can be used to represent a sentence

Notice that a variable can be used to represent a *sentence*, just as easily as it can be used to represent an expression!

EXERCISE	21. What is the universal set for the variable S in the previous definition?

EXAMPLES

Some mathematical sentences in one variable are investigated next. Notice how correct set notation has been used in reporting each solution set.

sentence	equation or inequality?	solution(s)	solution set
$x = 0$	equation	0 is the only solution	$\{0\}$
$x - 1 = 0$	equation	1 is the only solution	$\{1\}$
$x(x-1) = 0$	equation	0 is a solution 1 is a solution (Why? Keep reading!)	$\{0, 1\}$
$x(x-1)(x+2) = 0$	equation	0 is a solution 1 is a solution -2 is a solution (Why? Keep reading!)	$\{0, 1, -2\}$
$x > 1$	inequality	all numbers to the right of 1 on a number line are solutions	$(1, \infty)$
$x \geq 1$	inequality	the number 1, together with all numbers to the right of 1, are solutions	$[1, \infty)$

equations of the form $xy = 0$

A very important type of equation has been introduced in the previous example:

$$(\textit{something})(\textit{something}) = 0$$

This type of equation has zero on one side, and *things being multiplied* on the other side. How can a sentence of this form be true? To answer this question, consider the following:

Suppose I were to say to you:

I'm thinking of two numbers. When I multiply these numbers together, I get zero.

Can you tell me anything about the numbers I'm thinking of? Indeed! The only way that numbers can multiply to give zero is if at least one of the numbers is equal to zero:

$$3 \cdot 0 = 0 \qquad 0 \cdot \tfrac{1}{2} = 0 \qquad 0 \cdot 0 = 0$$

That is, in order for the sentence '$xy = 0$' to be true, either x must equal 0, or y must equal zero (or both).

reconsider the equation $x(x-1)=0$

With this idea in mind, reconsider the equation:

$$x(x-1)=0$$

The things being multiplied on the left-hand side are:

$$x \qquad \text{and} \qquad x-1$$

In order for the equation to be true, either:

$$x=0 \qquad \text{or} \qquad x-1=0$$

Consequently, the only numbers that make the equation true are 0 and 1 .

EXERCISE

22. What does the phrase '*at least one* of the numbers is equal to zero' mean?

23. Suppose that the sentence '$xyz = 0$' is true. What (if anything) can be said about x, y and z?

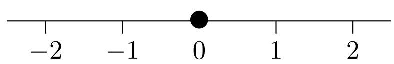
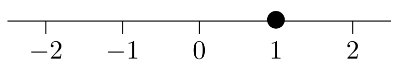
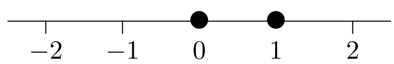
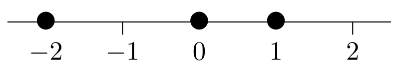
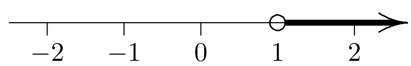
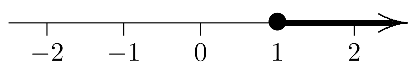
24. Suppose that the sentence '$(x+1)(x+2)(x-1)x = 0$' is true. What (if anything) can be said about x?

graph of a sentence

A *graph* is a pictorial display of information: often, a picture can convey information more effectively than other methods. *Graphs* take on a variety of forms, depending on the information that is to be displayed.

The *graph of a sentence* refers to a 'picture' illustrating the choice(s) that make the sentence true. Particularly when a sentence has *lots* of solutions, it is often easier to understand them via a 'picture'.

The *graph of a sentence in one variable* is a picture, on a number line, of the number(s) that make the sentence true: it is a picture of the number(s) in the solution set. The previous example is now repeated, this time giving both the solution set and the graph of each sentence:

line #	sentence	solution set	graph of sentence
1	$x=0$	$\{0\}$	
2	$x-1=0$	$\{1\}$	
3	$x(x-1)=0$	$\{0,1\}$	
4	$x(x-1)(x+2)=0$	$\{0,1,-2\}$	
5	$x>1$	$(1,\infty)$	
6	$x\ge 1$	$[1,\infty)$	

nature of solution sets for equations versus inequalities

In the examples above, notice that *equations* seem to have a finite number of choices that make them true, whereas *inequalities* seem to have entire *interval(s)* of numbers that make them true. This difference in the nature of the solution sets is one primary reason that people distinguish between the two broad categories of sentences: equations versus inequalities.

graphs aren't needed in simple cases

Most people wouldn't bother graphing the sentences in lines 1–4 above. (The verb '*graphing*' refers to the process of making a graph.) The solution sets are so simple, that there's no insight gained from looking at the 'dots' on a number line, versus just looking at the solution set. Graphs are typically used to organize much larger amounts of information.

EXERCISE

25. For each sentence below, do the following:
- State how you might read the sentence aloud.
- Give the solution set, using correct set notation.
- Graph each sentence. That is, show, on a number line, the number(s) that make each sentence true.

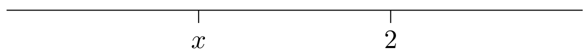
(a) $x < 2$
(b) $2 > x$
(c) $x \geq 2$
(d) $2 \leq x$

reading a sentence 'backwards'

In the previous exercise, the choices for x that make (a) and (b) true are identical. The choices for x that make (c) and (d) true are also identical. This is not a coincidence:

- If you read the sentence '$x < 2$' in the normal way (from left to right) it is read as 'x is less than 2'. This is true whenever x lies to the left of 2.
- If you read the sentence '$x < 2$' 'backwards' (i.e., from right to left) then it becomes '$2 > x$'. This is true whenever 2 lies to the right of x.

However, x lies to the left of 2 precisely when 2 lies to the right of x. Therefore, the solution sets are the same.



Even though the sentences '$x < 2$' and '$2 > x$' are read differently, and certainly look different, *their truth values are always the same*. Study the chart below:

x	substitution into '$x < 2$'	substitution into '$2 > x$'
1	$1 < 2$ (true)	$2 > 1$ (true)
3	$3 < 2$ (false)	$2 > 3$ (false)
2	$2 < 2$ (false)	$2 > 2$ (false)
1.8	$1.8 < 2$ (true)	$2 > 1.8$ (true)

The sentences are true at the same time, and false at the same time. If one sentence is true, so is the other. If one sentence is false, so is the other. They are completely interchangeable, with respect to their truth values!

EXERCISE

26. Compare the truth values of the sentences ‘ $x \geq 2$ ’ and ‘ $2 \leq x$ ’ by filling in the chart below. The first one is done for you.

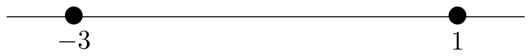
x	substitution into ‘ $x \geq 2$ ’	substitution into ‘ $2 \leq x$ ’
3	$3 \geq 2$ (true)	$2 \leq 3$ (true)
4		
2		
1		
2.3		

EXERCISES
web practice

27. Go to **http://fishcaro.crosswinds.net** and follow the links to the practice problems for section 14. Here you will practice determining if inequalities (with and without variables) are true or false, practice with the phrases ‘at least’ and ‘at most’, and solve sentences of the form $xy = 0$. For your convenience, there are also worksheets provided in this text on the following pages. Additional worksheets can be produced at the web site.

END-OF-SECTION EXERCISES

Write a mathematical sentence that is true for each number in the given set:

(sample) 

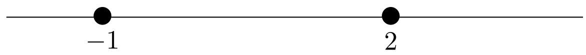
ANS: $(x-1)(x+3) = 0$

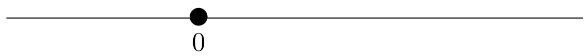
27. 

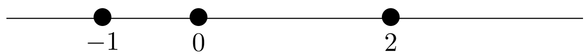
28. 

29. 

30. 

31. 

32. 

33. 

Write a mathematical sentence that is true for each number in the given set:

(sample) positive real numbers ANS: $x > 0$

34. negative real numbers
35. nonnegative real numbers
36. nonpositive real numbers
37. nonzero real numbers

Shade the specified sets of numbers on a number line:

38. all real numbers less than 1
39. all real numbers whose distance from zero is less than 1
40. all real numbers whose distance from zero is equal to 1
41. all real numbers greater than 1
42. all real numbers whose distance from zero is greater than 1

SECTION SUMMARY

I LIVE TWO BLOCKS WEST OF YOU

NEW IN THIS SECTION	HOW TO READ	MEANING
order		*Order* refers to a natural left/right ordering on a number line. Given numbers x and y, either x equals y; or x lies to the right of y; or x lies to the left of y.
$x < y$	'ex is less than wye'	true when x lies to the left of y on a number line; false otherwise
$x > y$	'ex is greater than wye'	true when x lies to the right of y on a number line; false otherwise
$x \le y$	'ex is less than or equal to wye'	true when $x < y$ or $x = y$; false otherwise
$x \ge y$	'ex is greater than or equal to wye'	true when $x > y$ or $x = y$; false otherwise
greatest		lying farthest to the right on a number line
least		lying farthest to the left on a number line
at least		'x is at least k' is equivalent to '$x \ge k$'
at most		'x is at most k' is equivalent to '$x \le k$'
equation		a mathematical sentence that uses the verb '$=$'
inequality		a mathematical sentence that uses one of the four verbs: $<$, $\le$, $>$, $\ge$
sentence in one variable		a mathematical sentence that uses only one variable
sentence in n variables		a mathematical sentence that uses n variables
solution of a sentence in one variable		Let S denote a sentence in one variable. A number that makes S true is called a *solution* of the sentence.
solution set		Let S denote a sentence in one variable. The set of all number(s) that make S true is called the *solution set* of the sentence.

NEW IN THIS SECTION	HOW TO READ	MEANING
equations of the form: $xy = 0$		In order for the equation '$xy = 0$' to be true, either x must equal zero, or y must equal zero (or both).
graph		A pictorial display of information. Graphs take on a variety of forms, depending on the information that is to be displayed.
graph of a sentence		A 'picture' illustrating the choice(s) that make the sentence true. That is, a 'picture' of the solution set of the sentence.
graph of a sentence in one variable		A picture, on a number line, of the number(s) that make the sentence true.