EXERCISE

15. Trace the following, to practice writing variables in the correct way:

x x x x x i i i i i l l l l l

y y y y y j j j j j m m m m m

z z z z z k k k k k n n n n n

t t t t t

END-OF-SECTION EXERCISES

For exercises 16–26: Classify each entry as a mathematical expression (EXP), or a mathematical sentence (SEN).

If an expression, state whether it is a number or a set.

Classify the truth value of each sentence: (always) true (T); (always) false (F); or sometimes true/sometimes false (ST/SF).

16. xyz ________
17. $xyz = zyx$ ________
18. $5x + 1$ ________
19. $5x + 1 = 1 + 5x$ ________
20. $[0, 5)$ ________
21. $5 \in [0, 5)$ ________
22. $\frac{x}{4} - 1$ ________
23. $5(0.15) = 0.75$ ________
24. $\{0, 5\}$ ________
25. $5 \in \{0, 5\}$ ________
26. $(1.05)(0.8)(0.7p) = 100$ ________

For each mathematical sentence below, write an English sentence that shows the thought process behind determining when the sentence is true. Then, solve the sentence by inspection.

If there is more than one number that makes the sentence true, then shade the numbers that make the sentence true on a number line.

The first one is done for you.

(sample): $\frac{x}{4} = 5$: What number, when divided by 4, gives 5? Answer: 20

27. $\frac{20}{x} = 5$

28. $20 - x = 2$

29. $3x = 4x$

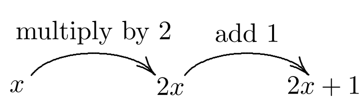
30. $3x \neq 4x$

31. $x + 1 \neq x + 2$

EXERCISES *web practice*	Go to my homepage **http://onemathematicalcat.org** and navigate to my Algebra I course, which has about 170 sequenced lessons. It can be used as a complete year-long high school course, or one semester in college. You're currently looking at the pdf version—you'll see that the HTML version has unlimited, randomly-generated, online and offline practice in every section. It's all totally free. Enjoy!

SECTION SUMMARY

HOLDING THIS, HOLDING THAT

NEW IN THIS SECTION	HOW TO READ	MEANING
variable universal set		A variable is a symbol (usually a letter) used to represent a member of a specified set. This specified set is called the variable's *universal set*.
common uses for variables		to state a general principle; to represent a sequence of operations; to represent an 'unknown'
reading letters aloud	'arr' represents r or R 'ess' represents s or S 'tee' represents t or T 'ex' represents x or X 'wye' represents y or Y 'zee' represents z or Z	'words' used to represent letters in the alphabet, when discussing how to read a mathematical sentence
For all real numbers x and y ... For all $x \in \mathbb{R}$ and $y \in \mathbb{R}$ ...		different ways to say the same thing
$x \in \mathbb{R}$ For all $x \in \mathbb{R}$ Let $x \in \mathbb{R}$	'ex is in arr' 'For all ex in arr' 'Let ex be in arr'	Context will determine the correct way to read '$x \in \mathbb{R}$'.
xy	'ex wye' (preferred) or 'x times y'	a shorthand for $x \cdot y$; when no confusion can result, the centered dot that denotes multiplication can be dropped
$2x$	'two ex' (preferred) or 'two times ex'	whenever a variable is being multiplied by a specific number, write the specific number *first*
mapping diagram 		a diagram that can be used to represent a sequence of operations
$2x+1$	'two ex plus one'	denotes the sequence of operations: take a number, multiply by 2, then add 1
$2(x+1)$	'two times the quantity ex plus one'	denotes the sequence of operations: take a number, add 1, then multiply by 2
solving a sentence		the process of determining when a sentence is *true*

SECTION SUMMARY

HOLDING THIS, HOLDING THAT

NEW IN THIS SECTION	HOW TO READ	MEANING
solving a sentence by inspection		Looking at a sentence, stopping and thinking, and determining when the sentence is true.
lowercase letters (like a, n, x)		*numbers* are usually represented by lowercase letters
lowercase letters from end of alphabet (particularly t, x, y)		a variable with universal set $\mathbb{R}$ (or, any *interval* of real numbers) is most likely to be named with a lowercase letter from the *end* of the alphabet
lowercase letters from middle of alphabet (particularly i, j, k, m, n)		a variable with universal set $\mathbb{Z}$ (or, any subset of $\mathbb{Z}$) is most likely to be named with a lowercase letter from the *middle* of the alphabet
uppercase letters (like A, B, S)		*sets* are usually represented by uppercase letters
hand-writing variables j k l m n	write x, NOT × write y, NOT × write z, NOT z write t, NOT t write i, NOT i write l, NOT l	Try to duplicate an italic typestyle when hand-writing variables, to prevent confusion.

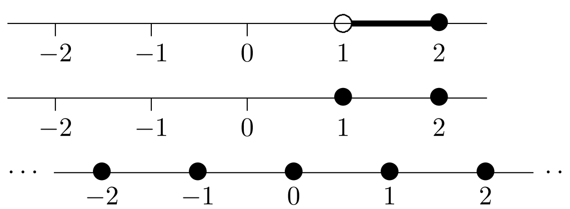
SOLUTIONS TO EXERCISES: HOLDING THIS, HOLDING THAT

IN-SECTION EXERCISES:

1.
$(1,2]$

$\{1,2\}$

$\mathbb{Z}$



2. w can 'hold' any of the numbers in the set $[0,1]$

3. w can 'hold' any of the numbers in the set $[0,2]$

4. The order that you multiply two numbers does not affect the result. That is, you can 'commute' the numbers in a multiplication problem without affecting the result.

5. For all real numbers x , y , and z , $x \cdot (y \cdot z) = (x \cdot y) \cdot z$.

6. For all $x \in \mathbb{R}$ and $y \in \mathbb{R}$, $x \cdot y = y \cdot x$.
For all $x \in \mathbb{R}$, $y \in \mathbb{R}$, and $z \in \mathbb{R}$, $x \cdot (y \cdot z) = (x \cdot y) \cdot z$.

7. ab , $3x$, $5ac$, 12

8a. $3x - 4$
8b. $3(x-4)$
8c. $\frac{x}{2} + 1$
8d. $\frac{x+1}{2}$

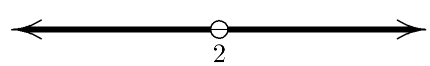
9a. Take a number, multiply by 5 , then subtract 3 .
9b. Take a number, subtract 3 , then multiply by 5 .
9c. Take a number, divide by 4 , then subtract 1 .
9d. Take a number, subtract 1 , then divide by 4 .

10. (a) d (b) t (c) s (d) v

11a. $(0.7)(170) = 119$; you owe \$119
11b. $(0.8)(119) = 95.2$; you owe \$95.20
11c. $(1.05)(95.2) = 99.96$; you owe \$99.96
11d. You'll get 4¢ change!

12a. What number is equal to 5 ? ANS: 5

12b. What numbers are not equal to 2 ? ANS: All real numbers except 2 :



12c. Three times what number gives 12 ? ANS: 4

12d. What number, divided by 3 , gives 4 ? ANS: 12

12e. What number, plus itself, plus itself again, gives 12 ? ANS: 4

12f. Two plus what number is the same as two minus that number? ANS: 0

12g. Fifteen, divided by what number, gives 3 ? ANS: 5

12h. Twelve, minus some number, minus the number again, gives 10 . What is the number? ANS: 1

13a. Since the universal set is $\mathbb{R}$, the best choice is 'Let $x \in \mathbb{R}$'. Read as: 'Let x be in arr' or 'Let x be a real number'.

13b. Since the universal set is $\mathbb{Z}$, the best choice is 'Let $k \in \mathbb{Z}$'. Read as: 'Let k be in zee' or 'Let k be an integer'.

13c. Since $[0, 2]$ is an interval of real numbers, the best choice is 'For all $t \in [0, 2]$'. You could read this as: 'For all real numbers t between 0 and 2 (including the endpoints)'.

13d. Since $\{1, 2, 3, \dots\}$ is a subset of the integers, the best choice is 'For all $i \in \{1, 2, 3, \dots\}$'. You could read this as: 'For all positive integers i'.

14a. number

14b. set

14c. number with universal set $\mathbb{R}$ (or some *interval* of real numbers)

14d. number with universal set $\mathbb{Z}$ (or some subset of $\mathbb{Z}$)

14e. set

14f. number with universal set $\mathbb{R}$ (or some *interval* of real numbers)

END-OF-SECTION EXERCISES:

16. EXP, number

17. SEN, T

18. EXP, number

19. SEN, T

20. EXP, set

21. SEN, F

22. EXP, number

23. SEN, T

24. EXP, set

25. SEN, true

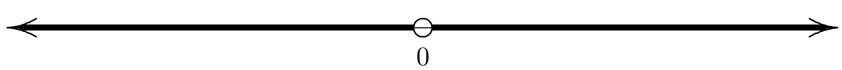
26. SEN, ST/SF

27. Twenty, divided by what number, gives 5? ANS: 4

28. What number, subtracted from 20, gives 2? ANS: 18

29. What number has the property that 3 times it is the same as 4 times it? ANS: 0

30. What number(s) have the property that 3 times them is not the same as 4 times them? ANS: all real numbers except 0



31. What number(s) have the property that when you add one to them, you get something different than when you add two to them? ANS: all real numbers