COURSE OBJECTIVES LIST: GEOMETRY

PREREQUISITES: All skills from Algebra I are assumed. A prerequisites test may be given during the first week of class to assess knowledge of these prerequisite skills and to locate deficiencies.

COURSE BOOK DESCRIPTION:

There are several ideas that intertwine in geometry: the study of properties of geometric figures (such as lines, triangles, quadrilaterals and circles); the maturation of inductive and deductive logical skills; the development of area and volume concepts; the exploration of the beautiful interaction between geometry and algebra. The Geometer's Sketchpad is used to give extensive hands-on experience with the mathematical concepts and to encourage experimentation. An honors section more deeply explores the rigors of mathematical proof.

The course objectives are elaborated as follows. The order in which the objectives are listed is not necessarily the order in which they will be taught. Note that different texts may use different notation.

PROPERTIES OF GEOMETRIC FIGURES:

GEOM1.	intuitive understanding of points, lines, and planes; collinear and coplanar; concept of intersection
GEOM2.	notation for points, lines, and planes: points (capital roman, A); lines (lowercase script ℓ, or $\overleftrightarrow{AB}$); planes (uppercase script $\mathcal{P}$, or three noncollinear points ABC)
GEOM3.	space (the collection of all points); geometric figure (a subset of space)
GEOM4.	dimension concept: one, two, and three-dimensional; concept of perimeter for two-dimensional objects
GEOM5.	distinguish between CONGRUENCY (same size and shape) and SIMILARITY (same shape, but not necessarily the same size); use proportions or scaling factors to find measurements in similar figures

LINE SEGMENTS, LINES and RAYS:

LINE1.	distinguish between a line segment ($\overline{AB}$) and the length of the line segment (AB). A line segment is a geometric figure; its length is a number.
LINE2.	perpendicular bisector of a segment: definition (cuts the segment into two equal pieces and is perpendicular to the segment); important property as the set of all points equidistant from the endpoints
LINE3.	concept of distance between a point and a line; distance between parallel lines
LINE4.	definition of a ray; notation for a ray ($\overrightarrow{AB}$)
LINE5.	terminology for lines: parallel, perpendicular, skew, transversal

LINE6. lines cut by a transversal: interior and exterior angles; alternate interior angles; corresponding angles; consecutive interior angles

LINE7. (test for parallel lines) The following are equivalent:

- two lines are parallel
- a pair of alternate interior angles are congruent
- a pair of alternate exterior angles are congruent
- a pair of consecutive interior angles are supplementary
- a pair of corresponding angles are congruent

ANGLES:

ANGLE1. definition of angle; notation for angles ($\angle ABC$)

ANGLE2. terminology for angles: vertex; interior; exterior; acute, obtuse, and right angles; degree measure; adjacent angles; vertical angles (and congruency theorem); linear pair; complementary and supplementary; angle bisector

ANGLE3. angle bisector: definition (cuts the angle into two angles of equal measure); important property as the set of all points equidistant from the sides of the angle

TRIANGLES:

TRI1. terminology for triangles: vertex (vertices); sides; right triangle (legs, hypotenuse); isosceles; equilateral; scalene; median; altitude

TRI2. the base angles of an isosceles triangle are congruent

TRI3. The sum of the (measures of the) angles in any triangle is 180° .

TRI4. Exterior Angle Theorem for a triangle

TRI5. congruent triangles: CPCTC (Corresponding Parts of Congruent Triangles are Congruent);
SSS, SAS, ASA, AAS congruency theorems;
there is *not* a SSA congruence theorem—be able to draw a counterexample

TRI6. inequalities for sides and angles of a triangle: Let s_1 and s_2 be the lengths of two sides in a triangle, with opposite angles θ_1 and θ_2 respectively. Then, $s_2 > s_1$ iff $m(\theta_2) > m(\theta_1)$.

TRI7. Triangle Inequality Theorem: the sum of the lengths of any two sides in a triangle is greater than the length of the third side. In other words; the shortest distance between two points is a straight line.

TRI8. additional exploration of triangles, to provide interest, practice, and depth: e.g., similarity theorems for triangles; or, the "Hinge" Theorem (inequalities involving two triangles; given two triangles with two congruent sides, the side opposite the larger included angle is longer than the side opposite the smaller included angle)

TRI9. the Pythagorean Theorem and its proof (this is also covered in Algebra I; try to give a different proof than they saw in the previous course)

TRI10. lengths of sides in two special triangles: the 30°-60°-90° triangle and the 45°-45°-90° triangle.

TRI11. right triangle definitions of sine, cosine, and tangent, to approximate lengths of sides in right triangles; calculator usage, with awareness of angle mode

TRI12. calculator usage of $\cos^{-1}$, $\sin^{-1}$, and $\tan^{-1}$ to approximate unknown angles in triangles

QUADRILATERALS:

QUAD1. quadrilaterals (4-sided polygons): parallelogram; rectangle; rhombus; square; trapezoid

QUAD2. concept of Venn diagram: the diagram summarizing the relationship between quadrilaterals, parallelograms, rhombi, rectangles, and squares may be useful

QUAD3. (test for parallelograms) The following are equivalent for a quadrilateral. (That is, given a quadrilateral, if one of the following statements is true, then they are all true; if one is false, then they are all false.)

- it is a parallelogram
- both pairs of opposite sides are congruent
- both pairs of opposite angles are congruent
- the diagonals bisect each other
- one pair of opposite sides is both parallel and congruent

QUAD4. additional exploration of quadrilaterals, to provide interest, practice, and depth: e.g., the median property of trapezoids: its length is the average of the length of the bases

CIRCLES:

CIRCLE1. terminology for circles: center; radius (radii); diameter; chord; arc; central angle; semicircle; tangent to a circle; secant line; sector

CIRCLE2. making pie charts (circle graphs)

CIRCLE3. the irrational number π as the ratio of the circumference to the diameter in any circle; common approximations to π as a fraction ($\frac{22}{7}$) and decimal (3.14)

CIRCLE4. formula for the circumference of a circle

CIRCLE5. relationship between inscribed angles and intercepted arcs

CIRCLE6. A line is tangent to a circle at point P if and only if it is perpendicular to the radius at P. Some additional theorems concerning tangent and/or secant lines should be investigated, to give depth to the coverage of circles. However, the instructor may pick and choose as desired.

CIRCLE7. algebraic description of a circle; writing equations of circles; graphing circles that are already in the form $(x-h)^2+(y-k)^2=r^2$

POLYGONS:

POLY1. terminology for polygons: triangle, quadrilateral, pentagon, hexagon; regular polygons; convex figures

POLY2. additional exploration of polygons, to provide interest, practice, and depth: e.g., perimeter/area formulas for regular polygons; or, formula for the sum of interior and exterior angles in convex polygons

LOGICAL SKILLS: The study of logic and mathematical language issues that was begun in Algebra I is firmed up and extended. The logic and math language concepts from Algebra I (those developed in "One Mathematical Cat, Please!") are assumed.

Students will apply their logical skills to a variety of proofs involving geometric figures. An Honors section will do more proofs, and proofs of higher complexity. The Sample Questions illustrate the difference in level of proof required for Honors and non-Honors sections.

LOGIC1. inductive reasoning: conjecture (educated guess); making conjectures; counterexample (a specific example that shows that a conjecture is not true)

LOGIC2. deductive reasoning: the two-column proof is easier for beginners; Honors sections may also work with the paragraph-style proof

LOGIC3. truth tables (i.e., the definitions) and verbalizations for the basic sentence connectives: A and B, A or B, $A \Rightarrow B$, $A \iff B$, not A.

LOGIC4. equivalent forms of conditional sentences (implications):

- If A, then B (or: B, if A)
- A implies B
- $A \Rightarrow B$
- Whenever A, B (or: B, whenever A)

hypothesis; conclusion; vacuously true; converse; contrapositive

LOGIC5. (equivalences for implications and their verbalizations) The following are equivalent:

- $A \Rightarrow B$
- not $B \Rightarrow$ not A
- (not A) or B

Direct proof of an implication; proof of an implication by contrapositive.

LOGIC6. NEGATING SENTENCES (in a nice way):

sentence	negation
$a = b$	$a \neq b$
$a > b$	$a \leq b$
$a \leq b$	$a > b$, etc.
$A \Rightarrow B$	A and (not B)
A and B	(not A) or (not B)
A or B	(not A) and (not B)

In particular: note that the negation of an implication is *not* an implication.

AREA AND VOLUME CONCEPTS:

AREA1. basic area concept: What is a square unit? Be able to show specified amounts of area, e.g., one square inch, 3 cm^2, one square blah (where 'blah' is some unit of length)

AREA2. formulas for areas of: rectangle, parallelogram, triangle, circle, trapezoid

AREA3. terminology: simple closed curve, non-simple closed curve, cylinder, prism, polyhedron, sphere, cone, pyramid

AREA4. formula: surface area of a right circular cylinder

AREA5. formulas: volume of a cylinder; volume of a cone

AREA6. formulas: volume and surface area of a sphere

ADDITIONAL EXPLORATION:

To provide interest, practice, and depth, time should be spent exploring a topic of interest to teacher/students. For example:

- INTERACTION BETWEEN GEOMETRY AND ALGEBRA: Choose one conic section (ellipse, hyperbola, or parabola) and explore the relationship between its geometric (defining) properties, and its algebraic representation once a coordinate system is introduced.
- TRANSFORMATIONS: Explore translations, rotations, reflections, and dilations.
- NON-EUCLIDEAN GEOMETRIES

GEOMETER'S SKETCHPAD:

GSK1. Students should be comfortable working with the Geometer's Sketchpad. It should be used regularly throughout the course.

CONSTRUCTIONS and SKETCHING SKILLS:

CONST1. Draw a box, showing hidden lines and giving the illusion of three dimensions.

Only a compass and straightedge are used for these constructions. Students must be able to explain why each construction works.

CONST2. copying a segment

CONST3. bisecting a segment

CONST4. copying an angle

CONST5. bisecting an angle

CONST6. constructing parallel lines

CONST7. dropping a perpendicular from a point to a line

Other constructions are optional.

SAMPLE FINAL EXAM QUESTIONS: GEOMETRY

The purpose of these sample questions is to clarify the course objectives, and also to illustrate the level at which objectives should be mastered. Each Geometry final exam will have a part that is common to *all* Geometry sections; this common part will consist of problems that are similar in format to these Sample Final Exam Questions. The remainder of the final exam will be created by the individual instructor.

These sample questions are freely available to both instructors and students. They may be used throughout the year for homework, quizzes, and tests.

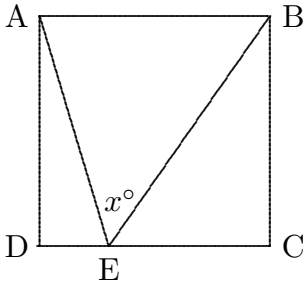
These sample questions have been carefully created to have the following properties:

- They do a good job of assessing achievement of the course objectives.
- They have enough inherent variability that their use cannot be construed as "teaching to the test."

There are many problems that incorporate algebra into geometry in a superficial way, while testing important properties of geometric figures. Here are some examples. *Any* required properties could be tested here.

1. (a) In $\triangle PQR$, $\angle Q \cong \angle R$. If $PQ = 10x - 14$, $PR = 2x + 50$, and $RQ = 4x - 30$, find the value of x.

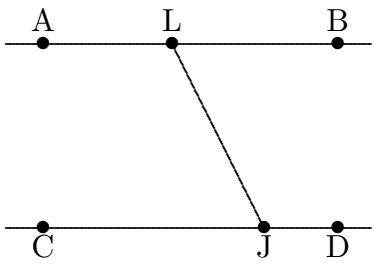
   (b) In the accompanying diagram, $ABCD$ is a rectangle, E is a point on $\overline{CD}$, $m\angle DAE = 20°$, and $m\angle CBE = 30°$. What is $m\angle x$?



(not to scale)

   (c) In $\triangle ABC$, $m\angle A = 3x + 40$, $m\angle B = 8x + 35$, and $m\angle C = 10x$. What is the longest side of the triangle?

   (d) In the accompanying diagram, A, L and B are collinear; C, J, and D are collinear. If $m\angle JLB = 6x - 7$ and $m\angle LJD = 7x + 5$, find the value of x.



Problems (2), (3) and (4) help to distinguish between the level of proof expected in Honors classes and non-Honors classes. Problems (2) and (3) are for all classes; problem (4) is for Honors classes only.

2. Next to each letter, write a valid conclusion that can be deduced from each set of true statements. If no valid conclusion can be deduced, write "no conclusion."

   (a) A or B
   not A _____

   (b) $P \Rightarrow Q$
   P _____

   (c) $A \Rightarrow B$
   B _____

   (d) (not A) $\Rightarrow$ (not B)
   B _____

3. Negate each of the following sentences. Write the negation as simply as possible.

sentence	negation
$x < 3$	
Carol has red hair and is short.	
$P \Rightarrow Q$	
$x = 3$ and $y \geq 4$	

4. Write a two-column proof.

   GIVEN: If Fred does not waste time in class, then he does well in Geometry.

   If Fred is absent from class, then his grades will go down.

   Either Fred does not waste time in class, or Fred is absent from class.

   Fred's grades do not go down.

   Let T represent "Fred wastes time in class."

   Let A represent "Fred is absent from class."

   Let S represent "Fred's grades go down."

   Let B represent "Fred does well in Geometry."

   PROVE: Fred does well in Geometry.

Questions may certainly draw on prerequisite skills: e.g., finding slopes of lines and distances between points, as illustrated in the following problem:

5. Quadrilateral $ABCD$ has vertices $A(-3,6)$, $B(6,0)$, $C(9,-9)$, and $D(0,-3)$. Prove that ABCD is a parallelogram, but *not* a rhombus.

6. (a) Write the equation of the circle with center $(1,-3)$ and radius 5.

   (b) Is the point $(3,0)$ on the circle described above? Justify your answer.

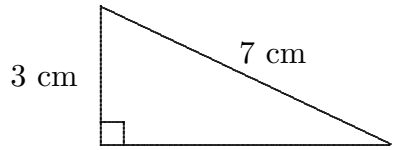
7. Find the area of each of the following geometric figures. Be sure to include correct units.

   (a) the circle through the points $(0,0)$ and $(3,4)$

   exact area: _____

   area rounded to the nearest hundredth: _____

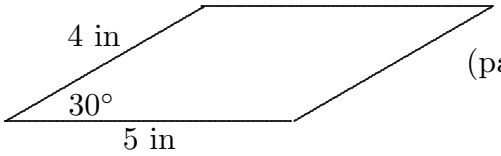
   (b)



   exact area: _____

   area rounded to the nearest hundredth: _____

   (c)



(parallelogram)

   exact area: _____

   area rounded to the nearest hundredth: _____

8. Find the requested volumes. Be sure to include correct units.

(a) A can of tennis balls has the shape of a right circular cylinder, as shown below. It contains three spherical tennis balls, each having radius 1.5 inches. Find the volume of the container.

volume, rounded to the nearest hundredth: _____

(b)

exact volume: _____

(c) The circumference of the Earth is approximately 25,000 miles. Find the volume of the Earth.

volume, rounded to the nearest hundredth: _____

Here are two ways that understanding of basic vocabulary can be tested: by asking for definitions, or by asking for examples:

9. Write a definition for each of the following:

(a) a geometric figure

(b) an isosceles triangle

(c) a regular hexagon

(d) a cylinder

(e) skew lines

(f) an acute angle

10. Make a sketch illustrating each of the following sets of requirements. If not possible, so state.

(a) a rectangle that is not a square

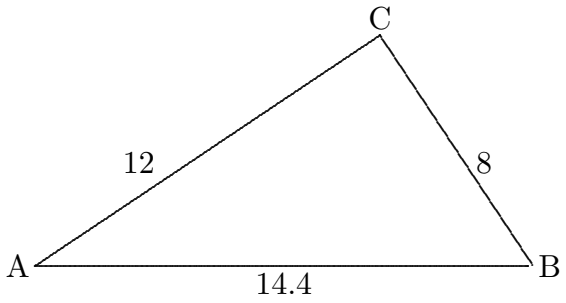
(b) a non-convex two-dimensional object

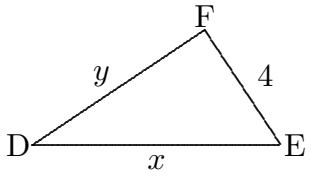
(c) a chord of a circle that is not a diameter

(d) a right triangle

Certainly the concepts of similarity and congruency must be addressed:

11. In the accompanying diagram, $\triangle ABC$ is similar to $\triangle DEF$, $AB = 14.4$, $BC = 8$, $CA = 12$, $DE = x$, and $EF = 4$. Find x and y.





12. (a) What does it mean to say that two geometric figures (in a plane) are *congruent*?

(b) What does it mean to state that two triangles are congruent by SSS?

(c) Is there a SSA congruency theorem for triangles? If not, give a counterexample.

(d) In the diagram below, $\angle A \cong \angle E$, and C is the midpoint of $\overline{AE}$. Correctly state two triangles that are congruent, and justify by citing an appropriate congruency theorem.