COURSE OBJECTIVES LIST: ALGEBRA II

Algebra II Honors is offered.

PREREQUISITES: All skills from Algebra I and Geometry are assumed. A prerequisites test may be given during the first week of class to assess knowledge of these prerequisite skills and to locate deficiencies.

COURSE DESCRIPTION:

Algebra II develops the tools introduced in the previous algebra course, while introducing many more concepts and exploring real-life applications. Topics include recursion, probability and statistics, systems, matrices, polynomials, rational functions, exponential and logarithmic functions, graphical transformations, periodic functions, and parametric equations. One central issue is the relationship between the algebraic and the graphical representations of information; the graphing calculator is used extensively in exploring this interplay. An honors section encourages more creative, critical and in-depth study of these topics.

The course objectives are elaborated as follows. The order in which the objectives are listed is not necessarily the order in which they will be taught.

- matrix notation: matrix (matrices); dimensions of a matrix; element; equality of matrices
- arithmetic with matrices: adding/subtracting; multiplying by a constant; matrix multiplication
- the set of complex numbers ($\mathbb{C}$): $a+bi$ form; plotting points in the complex plane; arithmetic with complex numbers (adding/subtracting, multiplying); complex conjugate
- completing the square technique (apply to graphing circles)
- logarithms: the number $\log_b x$ should be understood from two points of view. Firstly, it is the *exponent* that b must be raised to, in order to get x. Secondly, the function $\log_b x$ "undoes" the function b^x.
- logarithm terminology: common logs, natural logs
- change of base formula for logarithms
- properties of logarithms: for positive numbers x and y, $\ln xy = \ln x + \ln y$, $\ln \frac{x}{y} = \ln x - \ln y$, $\ln x^y = y \ln x$
- basic concept of probability: a probability is a number between 0 and 1 that represents the likelihood of occurrence of something; calculating simple probabilities
- combinations versus permutations; distinguishing between the two; formulas for each
- sequences: recursive versus nonrecursive; arithmetic and geometric; function and subscript notation; loans and investment applications; compound interest formula; graphing sequences

- statistics: mean, median, mode; measures of spread (variance and standard deviation)
- parametric equations: can be used to generate a graph swept out in time; converting from parametric to non-parametric form; applications
- functions: review concept; review function notation
- Graph of a function: a picture of all its (input,output) pairs. By convention, the inputs are on the horizontal axis, and the outputs on the vertical axis. The label "$y = f(x)$" states that each y-value is the output from the function f when the input is x.
- recognize functions from a formula, verbal descriptions, tables, graphs
- Understand the phrase: "y is a function of x". Recognize when y is a function of x; when x is a function of y (from words, tables, graphs)
- composition of functions: evaluation (given $f(x)$ and $g(x)$, find $f(g(x))$)
- Understand the equivalence of two common requests:

  Graph the equation $y = x^2$. That is, show all the points that make the equation true: $(x, y) = (x, x^2)$.

  and

  Graph the function $f(x) = x^2$. That is, show all the input/output pairs: $\big(x, f(x)\big) = (x, x^2)$.
- familiarity with these "basic models": $y = x$, $y = x^2$, $y = x^3$, $y = |x|$, $y = \sqrt{x}$, $y = \frac{1}{x}$, and $y = k$. Given the equation, know the shape of the graph. Given the graph, identify the (probable) equation.
- Explain the graphical interpretation of the solution set of sentences of the form $f(x) = 0$, $f(x) > 0$, etc. For example, the solution set of $f(x) = 0$ is the set of x-intercept(s) for the graph of f.
- Explain the graphical interpretation of the solution set of sentences of the forms $f(x) = g(x)$, $f(x) > g(x)$, etc. For example, the solution set of $f(x) = g(x)$ is the set of x-value(s) of the intersection point(s) of the graphs of f and g.
- horizontal and vertical translations: going from $y = f(x)$ to $y = f(x \pm c)$ and $y = f(x) \pm c$
- vertical scaling: going from $y = f(x)$ to $y = kf(x)$
- reflection about the x-axis: going from $y = f(x)$ to $y = -f(x)$
- Apply the techniques discussed above to graph a wide variety of functions/equations without the use of a graphing calculator. (However, a graphing calculator may be used to verify results.) For example, be able to graph $y = -\sqrt{x-7} + 3$ without a calculator.
- solve any quadratic equation $ax^2 + bx + c = 0$; the quadratic formula
- solve exponential equations like $2^x = 5$
- solve logarithmic equations like $\log_2 x = 5$
- review lines and slope

- linear systems of equations in 2 variables: solution of a system by both the substitution and elimination techniques; graphical interpretations
- writing a linear system in matrix form
- solving simple nonlinear systems of equations/inequalities in 2 variables, like $y = x^2$, $y = x + 1$
- quadratic functions: $y = ax^2 + bx + c$ and $y = a(x - h)^2 + k$ forms; graph as parabolas; vertex
- polynomials: definition; degree
- review properties of exponents
- exponential functions: allowable bases; growth versus decay
- logarithmic functions: allowable bases; shapes of graphs for different bases
- rational functions: horizontal and vertical asymptotes;

  Use notation like: as $x \to -1^+$, $y \to -\infty$.

  Distinguish between puncture points (holes) and vertical asymptotes.
- periodic functions: periodicity; brief exposure to the cosine and sine functions as x and y values of points on the unit circle. Show the relationship to the right triangle definitions that were studied in geometry.

SAMPLE FINAL EXAM QUESTIONS: ALGEBRA II

The purpose of these sample questions is to clarify the course objectives, and also to illustrate the level at which objectives should be mastered. Each Algebra II final exam will have a part that is common to *all* Algebra II sections; this common part will consist of problems that are similar in format to these Sample Final Exam Questions. The remainder of the final exam will be created by the individual instructor.

These sample questions are freely available to both instructors and students. They may be used throughout the year for homework, quizzes, and tests.

These sample questions have been carefully created to have the following properties:

- They do a good job of assessing achievement of the course objectives.
- They have enough inherent variability that their use cannot be construed as "teaching to the test."

1. For each expression given below, rename the expression as requested. If the requested name is not possible, so state. A few samples are done for you.

EXPRESSION	RENAME IN THIS FORM	ANSWER
(sample) 12	a sum of even integers	$2+10$ or $4+8$ etc.
(sample) 12	xy, where x and y are integers, with $x<0$	$(-3)(-4)$ or $(-2)(-6)$ etc.
(sample) 12	2^x, where $x \in \{0,1,2,3,\dots\}$	not possible
x^2+2x+3	involving a perfect square, $(x+k)^2$	
$\lvert 2x+3\rvert$, for $x<-\frac{3}{2}$	without absolute values	
$2\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix} - \begin{bmatrix}3 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 4\end{bmatrix}$	$\begin{bmatrix}a & b\\ c & d\end{bmatrix}$	
$2+3i-i^2+(1-i)(3+4i)$, $(i=\sqrt{-1})$	$a+bi$	
$\dfrac{x^2-x-1}{x-3}$	$Q(x)+\frac{R(x)}{D(x)}$	
$\log_7 5$	involving the natural log	
$\dfrac{4\log 10^x}{3}$	without logarithms	
$\ln x^4 - \ln x^2 + \ln(x^2+1)$	a single logarithm	
$(x-2y)^4$	expanded form (Hint: use Pascal's triangle)	
$(-\infty,-2]\cap(-4,5]$	as a single interval	
$\{x \mid x \ge -2\}$	using interval notation	
${}_5C_2$	as a whole number	
${}_5P_2$	as a whole number	

2. Solve each equation/inequality/system. For full credit, each solution must display the following:

(1) a graph that clearly illustrates the solution set

(2) a numerical approximation of the solution(s), if needed (round to 4 decimal places)

(3) the (exact) solution(s)

A sample is done for you.

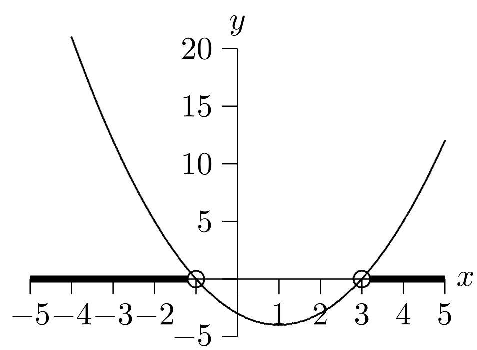
(sample) $x^2 - 2x > 3$

Solution: Rewrite:

$$x^2 - 2x - 3 > 0$$
$$(x-3)(x+1) > 0$$

(graph $y = (x-3)(x+1)$; see where graph lies above x-axis and read off solution set)

$$x < -1 \quad \text{or} \quad x > 3$$



(a) $|2x-3| > 5$

(b) $2x - 3x^2 \le -1$

(c) $3^{2x-1} = 10$

(d) $\log_3(x^2 - 1) = -2$

(e) $\sqrt{3x^2 + 5x - 3} = x$

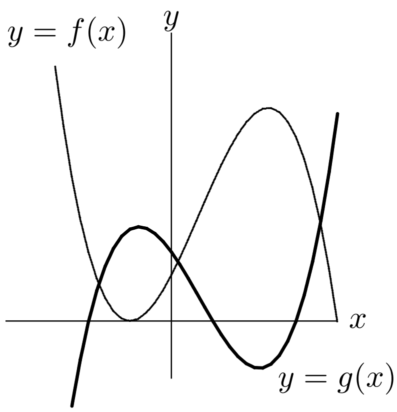
(f) $y = x^2 + 1$ and $y = 2x + 4$

(g) Let $f(x) = \begin{cases} x+2\,, & \text{if } x < 0 \\ 2\,, & \text{if } 0 \le x < 1 \\ x-1 & \text{if } x \ge 1 \end{cases}$. Solve the equation $f(x) = 1$.

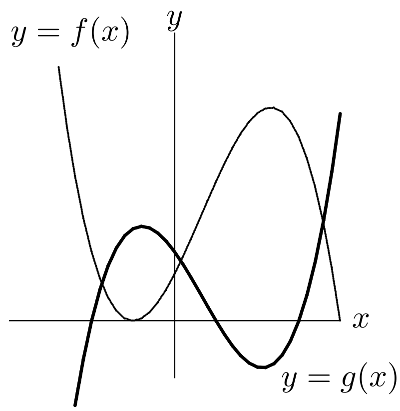
3. (a) Let $f(x) = x^2 - 2x + 1$ and $g(x) = 1 - 3x$. Find both $g(f(x))$ and $f(g(x))$.

(b) Find functions f and g such that $f(g(x)) = \sqrt[3]{x^2 - 1}$.

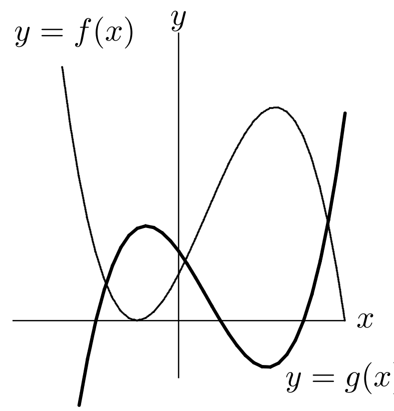
4. On each graph below, clearly shade the solution set on the x-axis.



$f(x) = g(x)$



$f(x) \geq g(x)$



$f(x) < g(x)$

5. Write a list of transformations that takes the graph of $y = f(x)$ to the graph of $y = 5 - 3|f(x+1)|$. There may be more than one correct answer.

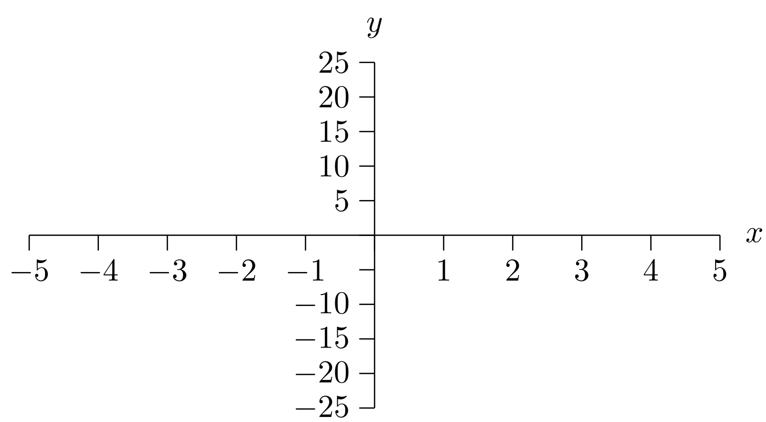
EQUATION	TRANSFORMATION
$y = f(x)$	(starting place)

6. Starting with the equation $y = x^2 - 2x + 1$, apply the specified sequence of transformations.

EQUATION	TRANSFORMATION
$y = x^2 - 2x + 1$	(starting place)
	up 1
	left 3
	reflect about the x-axis
	vertical scale by a factor of 2

7. Find the equation of a polynomial P satisfying the following properties: $P(-3) = 0$, 1 is a zero of P, the graph of P crosses the x-axis at $x = 2$, P has degree 5, and $P(0) = 7$.

8. Graph the rational function $g(x) = \dfrac{x^2 - 1}{(2x-1)(x+3)}$ in the space below.



If any of the following do not exist, so state:

x-intercept(s): ____________________

y-intercept(s): ____________________

Equation(s) of any horizontal asymptote(s): ____________________

Equation(s) of any vertical asymptote(s): ____________________

Equation(s) of any slant asymptote(s): ____________________

Fill in the blank: as $x \to \infty$, $y \to$ ____________________

Fill in the blank: as $x \to -3^+$, $y \to$ ____________________