COURSE OBJECTIVES LIST: ALGEBRA I

COURSE DESCRIPTION:

Algebra involves the representation and manipulation of mathematical information using variables.

One theme of Algebra I is the development of tools needed to work with a wide variety of mathematical expressions. To this end, we explore functions, sets, number line concepts, order of operations, percents, ratios, radicals, properties of exponents, scientific notation, rational expressions, and factoring.

A second theme is the introduction of basic tools for solving equations and inequalities. Central to this task is the study of the addition and multiplication properties of equality.

Finally, students must develop the wisdom to differentiate between renaming expressions and solving equations. A thorough study of lines helps to illustrate the interconnections among the themes. Algebra I is a prerequisite for further study in mathematics.

The course objectives are elaborated as follows. The order in which the objectives are listed is not necessarily the order in which they will be taught.

MANIPULATING AND RENAMING EXPRESSIONS

- differentiate between *expressions* (the “nouns” of mathematics) and *mathematical sentences* (the “complete thoughts” of mathematics; e.g., equations and inequalities)
- review of arithmetic with signed numbers (as needed)
- basic properties of real numbers: the real numbers ($\mathbb{R}$) and the real number line; positive, negative, nonnegative, nonzero, even, odd; opposites; whole numbers and integers; density property of $\mathbb{R}$; consecutive integers
- size (distance from zero) versus order (left/right positioning)
- average of two numbers as the number exactly halfway between
- the phrases “at least” and “at most”; translating into inequalities
- special properties of 0 and 1; unit conversion as an application of multiplication by 1; why division by zero is not allowed
- absolute value: $|x|$ as the distance from x to 0; solving simple sentences like $|x| = 2$, $|x| > 2$ and $|x| \le 2$ using distance concepts
- Order of operations: be able to key every expression type studied in the course into the calculator correctly. PEMDAS (Please Excuse My Dear Aunt Sally) can be introduced initially, but order of operations concepts should be continually revisited as new types of expressions are introduced.

- Understand the difference between *exact* and *approximate* answers. For example, $\sqrt{3}$ is an exact solution of the equation $x^2 = 3$, whereas 1.732050808 is an approximate solution. (Students often think that when they write down every digit their calculator gives them, then they're getting an exact solution.) Whenever possible, exact answers should be given first, and approximate solutions thereafter. Thus, a preferred format for the solution of the equation $x^2 = 3$ would read as follows:

$$\begin{aligned} x^2 &= 3 \\ x &= \pm\sqrt{3} \\ x &\approx \pm 1.7321 \end{aligned}$$

- distributive, commutative, and associative properties
- sets and set notation: set concept; members/elements of a set; finite versus infinite sets; list method; sentence type "$x \in S$"; interval notation; subsets
- Numerical expressions as representing a sequence of operations. For example, the expression "$2x - 3$" reflects the sequence of operations: take a number, multiply by 2, then subtract 3.
- working with exponents: exponent laws; negative, zero, and rational exponents (like $8^{2/3}$ or $\left(\frac{16}{81}\right)^{-3/4}$)
- working with radicals: square, cube and n^{th} roots; relationship between radicals and rational exponents; basic arithmetic with radicals
- percents; percent increase/decrease
- ratios and proportions
- scientific notation, as a convenient way to represent numbers that are very close to zero, or very far from zero; arithmetic with scientific notation
- combining like terms, FOIL, more advanced use of the distributive law
- rational expressions; arithmetic with rational expressions
- factoring: greatest common factor; factoring trinomials; difference of squares
- the zero factor law; solving equations like $3x(2x - 3)(1 - x) = 0$

BASIC TOOLS FOR SOLVING MATHEMATICAL SENTENCES

- differentiating between "simplifying an expression" and "solving a (mathematical) sentence"; between "equality of expressions" and "equivalence of sentences"
- addition and multiplication properties of equality
- addition and multiplication properties of inequality
- Be able to solve any linear equation/inequality in one variable, particularly those involving fractions and decimals.
- Re-name any sentence with an equivalent sentence in a desired form (e.g., solve a sentence for a desired variable)

- compound inequalities of types "$a < x < b$" and "$x < a$ or $x > b$" (also using other inequality symbols); the mathematical sentence connectives AND and OR
- solving quadratic equations that are factorable over the integers

GRAPHING BASICS

- coordinate plane terminology: ordered pairs; origin; quadrants; x and y axes; x and y coordinates; independent/dependent variables
- plotting points
- the Pythagorean theorem; distance between two points
- midpoint formula
- graphical understanding of solutions of a sentence in one variable (numbers) versus solutions of a sentence in two variables (ordered pairs of numbers)
- understand that the graph of a sentence in two variables is a picture of its solution set
- Be able to solve simple sentences (like $x = 2$, $y = 2$, and $x > 2$) viewed as sentences in two variables.

FUNCTION CONCEPT

- function concept: each input has exactly one corresponding output
- function notation: distinguish between the name of the function (f), the input (x), and the output from the function f when the input is x ($f(x)$)
- Given the description of a function using function notation (like $f(x) = 2x + 3$), describe the corresponding sequence of operations: the function f takes an input, multiplies it by 2 , then adds 3 .
- Given a sequence of operations (like "multiply by 2 , then add 3"), represent the sequence using function notation. Recognize that there are choices: the name for the function, and the name for the typical input. Thus, both $f(x) = 2x + 3$ and $g(t) = 2t + 3$ are correct answers.
- introduction to domain and range of a function; determining domain from a formula (e.g., What is the domain of $f(x) = \frac{1}{x}$? What is the domain of $g(t) = \sqrt{t-3}$?)

LINEAR FUNCTIONS/EQUATIONS/GRAPHS

- write the equation of lines, given sufficient information
- graph lines, given sufficient information
- $y = mx + b$ (slope-intercept) form
- $y - y_1 = m(x - x_1)$ (point-slope) form
- Find the slope of a line: from an equation, two points, or a graph. In particular, know that the slope of a horizontal line is 0 ; a vertical line has no slope.
- know the characterizations for parallel and perpendicular lines

SAMPLE FINAL EXAM QUESTIONS: ALGEBRA I

The purpose of these sample questions is to clarify the course objectives, and also to illustrate the level at which objectives should be mastered.

These sample questions are freely available to both instructors and students. They may be used throughout the year for homework, quizzes, and tests.

These sample questions have been carefully created to have the following properties:

- They do a good job of assessing achievement of the course objectives.
- They have enough inherent variability that their use cannot be construed as "teaching to the test."

This question assesses achievement of many of the MANIPULATING AND RENAMING EXPRESSION objectives.

1. For each expression given below, rename the expression as requested. If the requested name is not possible, so state. A few samples are done for you.

EXPRESSION	RENAME IN THIS FORM	PUT YOUR ANSWER HERE
(sample) 12	a sum of even integers	$2 + 10$ or $4 + 8$ etc.
(sample) 12	xy, where x and y are integers, with $x < 0$	$(-3)(-4)$ or $(-2)(-6)$ etc.
(sample) 12	2^x, where $x \in \{0, 1, 2, 3, \dots\}$	not possible
$\dfrac{1}{\sqrt{2}}$	a fraction with no radical in the denominator	
7	a quotient of integers, where the numerator is greater than 10	
23,070,000	in scientific notation	
7	$x - y$, where x and y are NOT integers	
7	$\frac{1}{2} \cdot x$	
$(x - 2)(x + 3)$	as a sum (i.e., multiply out)	
$x^2 - y^2$	as a product (i.e., factor)	
$\frac{1}{2}$	$\frac{3}{x}$	
$\dfrac{1}{x} - \dfrac{2}{3 + x}$	as a single fraction	
0.25	as a percent	
$\dfrac{x^4 x^{-1}}{(x^2)^3 x}$	x^k	
300 ft/sec	x mph (there are $5,280$ feet in one mile)	

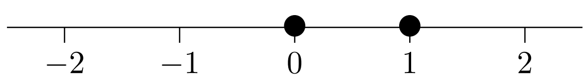
The next two questions assess sentence-solving and graphing skills, including the zero factor law, absolute value as distance, and the mathematical words 'and' and 'or'.

2. Solve each equation/inequality in one variable. Write a list of equivalent sentences, ending with one that can be solved by inspection. Get EXACT answer(s), not decimal approximations. Graph each solution set on a number line. A sample is done for you.

(sample) $x^2 = x$

Solution:

$$
\begin{gathered}
x^2 = x \\
x^2 - x = 0 \\
x(x-1) = 0 \\
x = 0 \ \text{ or } \ x - 1 = 0 \\
x = 0 \ \text{ or } \ x = 1
\end{gathered}
$$



(a) $3x(1-5x)(x^2-16) = 0$

(b) $\frac{1}{2}x - 7 = 3x + \frac{x}{5}$

(c) $|x-3| > 1$

(d) $2 < |x| < 3$

(e) $1 - 2x < 3$

(f) $x^2 = x + 2$

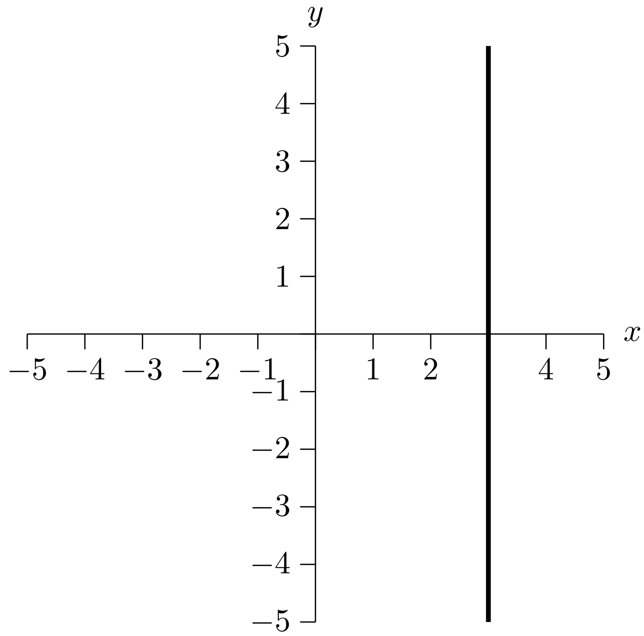
(g) $1 < x$ or $x \leq -1$

(h) $0.005(x - 0.01) = 0.003 + 0.4x$

3. Graph each of the following equations/inequalities, where each sentence is viewed as a sentence in two variables. (That is, $x = 3$ should be viewed as $x + 0y = 3$.) A sample is done for you.

(sample) $x = 3$

Solution:



(a) $x > 3$

(b) $2y - 3 = 0$

(c) $x = 3$ and $y = 2$

(d) $x = 3$ or $y = 2$

(e) $y = 2x - 1$

(f) $y = \sqrt{x}$

(g) $|x| = 2$

(h) $y \leq 2$

(i) $\frac{y-2}{3} = 2x - 1$

This question assesses order of operation and calculator skills.

4. Estimate each of the following numbers on your calculator. For full credit, each answer must be correct to five decimal places.

(a) $\dfrac{1+\sqrt{2}}{\sqrt[3]{5}-7}$

(b) $3x^2 - 5x + 1$, where $x = -1.8$

(c) $|1-2x|$, where $x = \sqrt{3}$

(d) $(2.03 \times 10^{-9})(-4.1 \times 10^{7})$

The next three questions assess understanding of the function concept, function notation, and graphing calculator skills.

5. (a) What is the domain of the function $f(x) = \dfrac{1-3x}{x-2}$?

(b) Use your graphing calculator to graph the function f in the window $-1 < x < 3$ and $-15 < y < 10$.

(c) Find the x-intercept of the graph.

(d) Use your calculator to estimate a value for x for which $f(x) = 5$. (Zoom, as necessary, to get f(x) within 0.01 of 5.)

6. Write an expression (using the variable x) to represent each sequence of operations.

(a) take a number, multiply by 2, then subtract 3

(b) take a number, subtract 3, then multiply by 2

(c) take a number, multiply it by 2, cube the result, add 1, then divide by the original number

Write the sequence of operations that is being described by each expression.

(d) $3x - 1$

(e) $2(x+1)^3 - 5$

(f) $\dfrac{x-3}{7} - 1$

7. Let $f(x) = x^2 - 2x + 1$. Evaluate each of the following expressions.

(a) $f(0)$

(b) $f(1) - 2$

(c) $f(f(-1))$

The last two questions assess achievement of the LINEAR FUNCTIONS/EQUATIONS/GRAPHS objectives.

8. Write the equation of the line, in $y = mx+b$ form, that satisfies the given conditions.

(a) slope 3 , passing through the point $(2, -1)$

(b) the horizontal line that crosses the y-axis at 2

(c) the line that is perpendicular to $x - 3y = 5$ and passes through the point $(0, 3)$

9. A certain telephone company charges a \$4.95 monthly fee, for which the customer gets 100 minutes of calls anywhere in the United States. Each additional minute costs 7 cents. Carol always talks more than 100 minutes. Let $x > 100$ denote the number of minutes that Carol talks in a month, and let $C(x)$ denote the dollar amount owed to the telephone company. Find a formula for $C(x)$.