PREREQUISITES: CALCULUS

Mathematics builds! To be successful in Calculus, there are certain skills that you are expected to already have mastered. These prerequisites are summarized on this sheet. Although some of the topics listed here may be reviewed in Calculus, you are expected to already have some familiarity with them, so that we can quickly move beyond the basics to higher-level discussions. ALGEBRA I, GEOMETRY, ALGEBRA II, and PRECALCULUS are all prerequisites to CALCULUS.

There will be a quiz over this prerequisite material, which will count as part of your grade. "Sample Prerequisite Problems" (with solutions) are available on the web. The Prerequisite Quiz will consist of problems that have a similar format to the Sample Prerequisite Problems.

DON'T PANIC if you're rusty on (or just haven't ever seen!) some of the topics listed on this sheet: math courses at different schools sometimes cover different material. The first few days of class will be devoted to review, and filling in gaps. Also, the Math Department teachers are all available to help you. It's important, however, that you get this material at your fingertips right away, because we'll be drawing on these skills frequently.

Both **Calculus Honors** and **AP Calculus AB** have the same prerequisites, and cover the same material. Students enrolled in AP Calculus AB have the following additional requirements:

- actual AP problems will be a regular part of homework, quizzes, and tests
- students are required to take the Advanced Placement Test
- there is extra class time each week to allow for the exploration of ideas in greater depth than the normal class schedule provides

The **Advanced Placement Program Course Description** for MAY 2002–MAY 2003 gives a concise summary of the prerequisites for Calculus:

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include those that are linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise defined. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions of numbers such as 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{2}$.

In particular, the following are prerequisites for Calculus:

1. RENAMING EXPRESSIONS: set notation (interval, set-builder, union, intersection); factoring; laws of exponents and logarithms; complex numbers; completing the square technique; long division of polynomials; relationship between zeros and factors of polynomials
2. SOLVING EQUATIONS AND INEQUALITIES IN ONE VARIABLE: linear; quadratic; absolute value; exponential; logarithmic; radical; systems; rational; trigonometric; compound inequalities; the zero factor law. Understand extraneous solutions, and when they can arise. Be sure that you can distinguish between *exact* and *approximate* solutions. You should understand the relationship between the algebraic and graphical solutions of sentences.
3. GRAPHING SENTENCES IN TWO VARIABLES: familiarity with these "basic models": $y = x$, $y = x^2$, $y = x^3$, $y = |x|$, $y = \sqrt{x}$, $y = \frac{1}{x}$, $y = k$, $y = \ln x$ (and other bases), $y = e^x$ (and other bases), $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = \sec x$. Be able to graph circles, lines, piecewise-defined functions, and transformations of the "basic models" involving: horizontal and vertical translations; horizontal and vertical scaling; reflection about the x-axis and y-axis; absolute value transformation.
4. BASIC GEOMETRY FORMULAS: perimeters of common figures, including the circumference of a circle. Know AREA formulas for: rectangle, triangle, circle, trapezoid. Know VOLUME formulas for: sphere, right cylinder (familiar base).

SAMPLE PREREQUISITE PROBLEMS: PRECALCULUS

(no calculators allowed)

Multiplication Tables (through 12)

(You will have two minutes to do the following 24 multiplication problems.)

$2 \times 6 =$	$3 \times 2 =$	$4 \times 9 =$	$5 \times 2 =$
$8 \times 8 =$	$9 \times 3 =$	$10 \times 7 =$	$2 \times 4 =$
$5 \times 1 =$	$6 \times 8 =$	$7 \times 9 =$	$8 \times 10 =$
$0 \times 10 =$	$1 \times 11 =$	$7 \times 3 =$	$11 \times 9 =$
$6 \times 4 =$	$7 \times 11 =$	$3 \times 7 =$	$4 \times 5 =$
$9 \times 5 =$	$10 \times 6 =$	$12 \times 10 =$	$9 \times 12 =$

(Be sure that you can easily do problems like these: arithmetic with whole numbers, decimals, fractions; arithmetic with signed numbers)

$\dfrac{0}{7.2} =$	$-\dfrac{(6)(-2)}{-3} =$	$-3-(-2) =$
$1,000 \times 3.47 =$	$\dfrac{248.36}{100} =$	$\dfrac{1}{3} - \dfrac{1}{5} =$
$\dfrac{1}{3} \cdot \dfrac{1}{5} =$	$\dfrac{1}{3} \div \dfrac{1}{5} =$	$126 \times 24 =$

SAMPLE PREREQUISITE PROBLEMS: PRECALCULUS

Problems 1–13 should be done WITHOUT A CALCULATOR.

1. For each expression given below, rename the expression as requested. If the requested name is not possible, so state. A few samples are done for you.

EXPRESSION	RENAME IN THIS FORM	ANSWER
(sample) 12	a sum of even integers	$2+10$ or $4+8$ etc.
(sample) 12	2^x , where $x \in \{0, 1, 2, 3, \dots\}$	not possible
$\dfrac{1}{\sqrt{2}}$	a fraction with no radical in the denominator	
23,070,000	in scientific notation	
$x^2 - y^2$	as a product (i.e., factor)	
$\dfrac{x^4 x^{-1}}{(x^2)^3 x}$	x^k	
300 ft/sec	x mph (there are 5,280 feet in one mile)	
7,036	$x \cdot 10^2 + y \cdot 10^{-1}$	
$8^{-2/3}$	as a simple fraction	
$x^2 + 2x + 3$	involving a perfect square, $(x+k)^2$	
$\|2x+3\|$, for $x < -\frac{3}{2}$	without absolute values	
$2\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$	
$2 + 3i - i^2 + (1-i)(3+4i)$, $(i = \sqrt{-1})$	$a + bi$	
$\dfrac{x^2 - x - 1}{x - 3}$	$Q(x) + \frac{R(x)}{D(x)}$	
$\log_7 5$	involving the natural log	
$\dfrac{4 \log 10^x}{3}$	without logarithms	
$\ln x^4 - \ln x^2 + \ln(x^2+1)$	a single logarithm	
$(x-2y)^4$	expanded form (Hint: use Pascal's triangle)	
$(-\infty, -2] \cap (-4, 5]$	as a single interval	
$\{x \mid x \geq -2\}$	using interval notation	

2. Solve each equation/inequality/system. Get EXACT answers, not decimal approximations. Report each solution set using correct set notation. A sample is done for you.

(sample) $x^2 - 2x > 3$

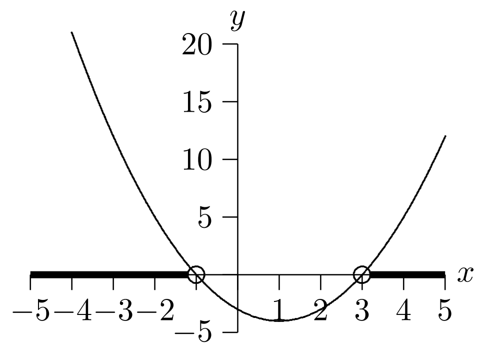
Solution: Rewrite:

$$x^2 - 2x - 3 > 0$$
$$(x-3)(x+1) > 0$$

(graph $y = (x-3)(x+1)$; see where graph lies above x-axis and read off solution set)

$$x < -1 \quad \text{or} \quad x > 3$$

$$\text{Solution set: } (-\infty, -1) \cup (3, \infty)$$



(a) $3x(1-5x)(x^2-16) = 0$

(b) $\frac{1}{2}x - 7 = 3x + \frac{x}{5}$

(c) $|2x - 3| > 5$

(d) $2 < |x| < 3$

(e) $1 - 2x \le 3$ or $-3 \le x < -2$

(f) $x^2 = x + 2$

(g) $2x - 3x^2 \le -1$

(h) $3^{2x-1} = 10$

(i) $\log_3(x^2 - 1) = -2$

(j) $\sqrt{3x^2 + 5x - 3} = x$

(k) $y = x^2 + 1$ and $y = 2x + 4$

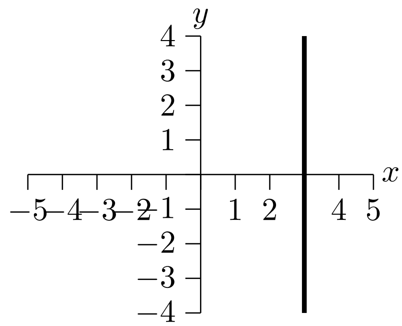
(l) $x + 3 = \dfrac{-2x^2 + 7x - 3}{x - 3}$

(m) Let $f(x) = \begin{cases} x+2\,, & \text{if } x < 0 \\ 2\,, & \text{if } 0 \le x < 1 \\ x - 1 & \text{if } x \ge 1 \end{cases}$. Solve the equation $f(x) = 1$.

3. Graph each of the following equations/inequalities, where each sentence is viewed as a sentence in two variables. (That is, $x = 3$ should be viewed as $x + 0y = 3$.) A sample is done for you.

(sample) $x = 3$

Solution:



(a) $x > 3$

(b) $2y - 3 = 0$

(c) $x = 3$ and $y = 2$

(d) $x = 3$ or $y = 2$

(e) $y - 2x + 1 = 0$

(f) $y = -2\sqrt{x+3} + 1$

(g) $|x| = 2$

(h) $y \le 2$

(i) $\frac{y-2}{3} = 2x - 1$

(j) $\frac{y-2}{3} \ge 2x - 1$

(k) $x^2 + 2x + y^2 - 6y - 15 = 0$

4. Write a list of transformations that takes the graph of $y = f(x)$ to the graph of $y = 5 - 3|f(x+1)|$. There may be more than one correct answer.

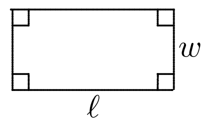
EQUATION	TRANSFORMATION
$y = f(x)$	(starting place)

5. Starting with the equation $y = x^2 - 2x + 1$, apply the specified sequence of transformations.

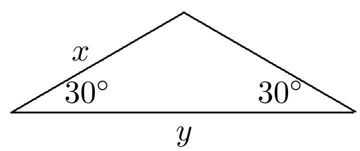
EQUATION	TRANSFORMATION
$y = x^2 - 2x + 1$	(starting place)
	up 1
	left 3
	reflect about the x-axis
	vertical scale by a factor of 2

6. Find the requested measurement(s) of each geometric figure.

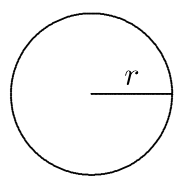
(a) PERIMETER and AREA:



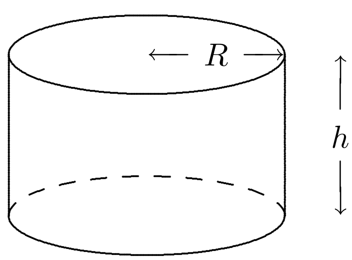
(b) PERIMETER and AREA:



(c) CIRCUMFERENCE and AREA:



(d) VOLUME:



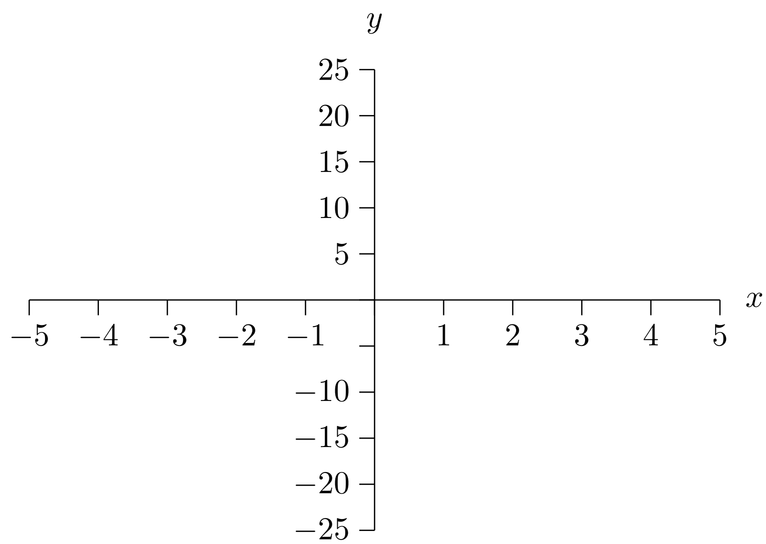
Which of the units below is a unit of length? Of area? Of volume?

cubic feet $\qquad$ cm^2 $\qquad$ meter

7. (a) Let $f(x) = x^2 - 2x + 1$ and $g(x) = 1 - 3x$. Find both $g(f(x))$ and $f(g(x))$.

(b) Find functions f and g such that $f(g(x)) = \sqrt[3]{x^2 - 1}$.

8. Graph the rational function $g(x) = \dfrac{(x^2-1)(x+2)}{(2x-1)(x+3)(x+2)}$ in the space below.



If any of the following do not exist, so state:

x-intercept(s): ______________________

y-intercept(s): ______________________

Equation(s) of any horizontal asymptote(s): ______________________

Equation(s) of any vertical asymptote(s): ______________________

Equation(s) of any slant asymptote(s): ______________________

Puncture point(s): ______________________

Fill in the blank: as $x \to \infty$, $y \to$ ______________________

Fill in the blank: as $x \to -3^+$, $y \to$ ______________________

9. Find the equation of a polynomial P satisfying the following properties: $P(-3) = 0$, 1 is a zero of P , the graph of P crosses the x-axis at $x = 2$, P has degree 5 , and $P(0) = 7$.

10. Write an expression (using the variable x) to represent each sequence of operations.

(a) take a number, multiply by 2, then subtract 3

(b) take a number, subtract 3, then multiply by 2

(c) take a number, multiply it by 2, cube the result, add 1, then divide by the original number

Write the sequence of operations that is being described by each expression.

(d) $3x - 1$

(e) $2(x+1)^3 - 5$

(f) $\dfrac{x-3}{7} - 1$

11. Let $f(x) = x^2 - 2x + 1$. Evaluate each of the following expressions.

(a) $f(0)$

(b) $f(1) - 2$

(c) $f(f(-1))$

12. Find the domain of the function $g(x) = \frac{1}{\sqrt{x-3}}$. Report your answer using interval notation.

13. Write the equation of the line, in $y = mx+b$ form, that satisfies the given conditions.

(a) slope 3, passing through the point $(2, -1)$

(b) the horizontal line that crosses the y-axis at 2

(c) the line that is perpendicular to $x - 3y = 5$ and passes through the point $(0, 3)$

14. (Your calculator is needed for parts of this question.)

(a) What is the domain of the function $f(x) = \dfrac{1-3x}{x-2}$?

(b) Use your graphing calculator to graph the function f in the window $-1 < x < 3$ and $-15 < y < 10$.

(c) Find the x-intercept of the graph.

(d) Use your calculator to estimate a value for x for which $f(x) = 5$. (Zoom, as necessary, to get f(x) within 0.01 of 5.)

15. Estimate (where necessary) each of the following numbers on your calculator. For full credit, each answer must be correct to five decimal places.

(a) $\dfrac{1+\sqrt{2}}{\sqrt[3]{5} - 7}$

(b) $3x^2 - 5x + 1$, where $x = -1.8$

(c) $|1 - 2x|$, where $x = \sqrt{3}$

(d) $(2.03 \times 10^{-9})(-4.1 \times 10^{7})$

SOLUTIONS

Multiplication Tables:
12, 6, 36, 10
64, 27, 70, 8
5, 48, 63, 80
0, 11, 21, 99
24, 77, 21, 20
45, 60, 120, 108

$0\,,\ \ -4\,,\ \ -1$ $\qquad$ $3{,}470\,,\ \ 2.4836\,,\ \ \frac{2}{15}$ $\qquad$ $\frac{1}{15}\,,\ \ \frac{5}{3}\,,\ \ 3{,}024$

1. There are many possible correct answers for some of these problems, but these are the most obvious ones:

$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$23,070,000 = 2.307 \times 10^7$

$x^2 - y^2 = (x-y)(x+y)$

$\frac{x^4 x^{-1}}{(x^2)^3 x} = \frac{x^3}{x^7} = x^{3-7} = x^{-4}$

$300\frac{\text{ft}}{\text{sec}} = 300\frac{\text{ft}}{\text{sec}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \approx 204.5\frac{\text{miles}}{\text{hr}}$

$7,036 = 70 \cdot 10^2 + 360 \cdot 10^{-1}$

$8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{(8^{1/3})^2} = \frac{1}{2^2} = \frac{1}{4}$

Use the technique of completing the square:

$x^2 + 2x + 3 = x^2 + 2x + 1 - 1 + 3 = (x+1)^2 + 2$

When $x < -\frac{3}{2}$, $2x + 3 < 0$. Thus, $|2x+3| = -(2x+3) = -2x - 3$.

$$2\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -2 & 6 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -6 \\ -2 & 2 \end{bmatrix}$$

$2 + 3i - i^2 + (1-i)(3+4i) = 2 + 3i - (-1) + 3 + 4i - 3i - 4(-1) = 10 + 4i$

Use long division of polynomials to get $\frac{x^2-x-1}{x-3} = x + 2 + \frac{5}{x-3}$. Do a "spot-check": when $x = 0$, we have $\frac{x^2-x-1}{x-3} = \frac{0^2-0-1}{0-3} = \frac{1}{3}$; when $x = 0$, we have $x + 2 + \frac{5}{x-3} = 0 + 2 + \frac{5}{0-3} = \frac{6}{3} - \frac{5}{3} = \frac{1}{3}$. They agree when $x = 0$! (A "spot-check" like this catches lots of mistakes.)

Use the change of base formula for logarithms: $\log_b x = \frac{\log_a x}{\log_a b}$

Thus, $\log_7 5 = \frac{\ln 5}{\ln 7}$. Check that $7^{(\log_7 5)} = 5$.

$$\frac{4\log 10^x}{3} = \frac{4}{3}x\log 10 = \frac{4}{3}x(1) = \frac{4}{3}x$$

Use properties of logarithms:

$\ln x^4 - \ln x^2 + \ln(x^2+1) = \ln\frac{x^4}{x^2} + \ln(x^2+1) = \ln x^2 + \ln(x^2+1) = \ln x^2(x^2+1)$

Use the row of Pascal's triangle beginning with "1 4" : Thus, $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$. Since $(x-2y)^4 = (x+(-2y))^4$, we apply this formula with $a = x$ and $b = -2y$ to get:

$$\begin{aligned}(x+(-2y))^4 &= x^4 + 4x(-2y)^3 + 6x^2(-2y)^2 + 4x^3(-2y) + (-2y)^4 \\ &= x^4 - 32xy^3 + 24x^2y^2 - 8x^3y + 16y^4\end{aligned}$$

$(-\infty, -2] \cap (-4, 5] = (-4, -2]$

$\{x \mid x \ge -2\} = [-2, \infty)$

2.

$$\text{(a)} \quad 3x(1-5x)(x^2-16) = 0$$

$$x = 0 \quad \text{or} \quad 1 - 5x = 0 \quad \text{or} \quad x^2 - 16 = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{1}{5} \quad \text{or} \quad x = \pm 4$$

$$\text{Solution set: } \{0, \frac{1}{5}, 4, -4\}$$

$$\text{(b)} \quad \frac{1}{2}x - 7 = 3x + \frac{x}{5}$$

$$5x - 70 = 30x + 2x \quad \text{(clear fractions; multiply by 10)}$$

$$-70 = 27x$$

$$x = \frac{-70}{27}$$

$$\text{Solution set: } \{-\frac{70}{27}\}$$

$$\text{(c)} \quad |2x - 3| > 5$$

$$2x - 3 > 5 \ \text{ or } \ 2x - 3 < -5$$

$$2x > 8 \ \text{ or } \ 2x < -2$$

$$x > 4 \ \text{ or } \ x < -1$$

$$\text{Solution set: } (-\infty, -1) \cup (4, \infty)$$

$$\text{(d)} \quad 2 < |x| < 3$$

solve by inspection; want all #s whose distance from 0 is between 2 and 3

$$-3 < x < -2 \quad \text{or} \quad 2 < x < 3$$

$$\text{Solution set: } (-3, -2) \cup (2, 3)$$

$$(e) \quad 1 - 2x \le 3 \ \text{ or } \ -3 \le x < -2$$
$$x \ge -1 \ \text{ or } \ -3 \le x < -2$$
$$\text{Solution set: } [-3,-2) \cup (-1,\infty)$$

$$(f) \quad x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2 \quad \text{or} \quad x = -1$$
$$\text{Solution set: } \{-1, 2\}$$

$$(g) \quad 2x - 3x^2 \le -1$$
$$-3x^2 + 2x + 1 \le 0$$
$$3x^2 - 2x - 1 \ge 0$$
$$\text{Note: } 3x^2 - 2x - 1 = 0$$
$$x = \frac{2 \pm \sqrt{4 - 4(3)(-1)}}{6} = \frac{2 \pm \sqrt{16}}{6} = \frac{2 \pm 4}{6} = 1\,, -\frac{1}{3}$$

Thus, the graph of $y = 3x^2 - 2x - 1$ crosses the x-axis at $-\frac{1}{3}$ and 1, and holds water.

$$\text{Solution set: } (-\infty, -\frac{1}{3}] \cup [1,\infty)$$

$$(h) \quad 3^{2x-1} = 10$$
$$\ln 3^{2x-1} = \ln 10$$
$$(2x-1)\ln 3 = \ln 10$$
$$2x - 1 = \frac{\ln 10}{\ln 3}$$
$$2x = \frac{\ln 10}{\ln 3} + 1$$
$$x = \frac{1}{2}\Big(\frac{\ln 10}{\ln 3} + 1\Big)$$
$$\text{Solution set: } \Big\{\frac{1}{2}\Big(\frac{\ln 10}{\ln 3} + 1\Big)\Big\}$$

(i) $\log_3(x^2-1) = -2$

$$3^{-2} = x^2 - 1$$
$$\frac{1}{9} = x^2 - 1$$
$$x^2 = \frac{10}{9}$$
$$x = \pm\sqrt{\frac{10}{9}}$$

Solution set: $\{\sqrt{\frac{10}{9}}, -\sqrt{\frac{10}{9}}\}$

(j) $\sqrt{3x^2+5x-3} = x$

square both sides; must check for extraneous solutions at the end

$$3x^2 + 5x - 3 = x^2$$

Solve using the quadratic formula to get:

$$x = \frac{1}{2} \quad \text{or} \quad x = -3$$

Discard $x = -3$; it is an extraneous solution.

Verify that $x = \frac{1}{2}$ is indeed a solution.

Solution set: $\{\frac{1}{2}\}$

(k) $y = x^2 + 1$ and $y = 2x + 4$

A quick sketch verifies that there are two solutions:

$$x^2 + 1 = 2x + 4$$
$$x = 3 \quad \text{or} \quad x = -1$$

When $x = 3$, $y = 10$; when $x = -1$, $y = 2$.

Solution set: $\{(3, 10), (-1, 2)\}$

(l) $x + 3 = \dfrac{-2x^2 + 7x - 3}{x - 3}$

Clear fractions; potential for an extraneous solution when $x = 3$:

$$(x+3)(x-3) = -2x^2 + 7x - 3$$

Solve the quadratic equation, yielding:

$$x = 3 \quad \text{or} \quad x = -\frac{2}{3}$$

Discard $x = 3$; it is an extraneous solution.

Solution set: $\{-\frac{2}{3}\}$

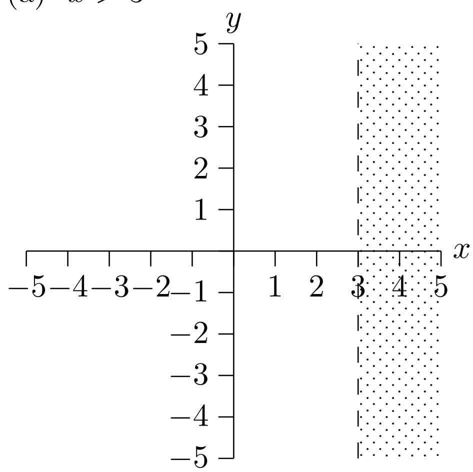
(m) Use a graphical approach to see that there are two solutions:

$$x + 2 = 1 \quad \text{when} \quad x = -1$$

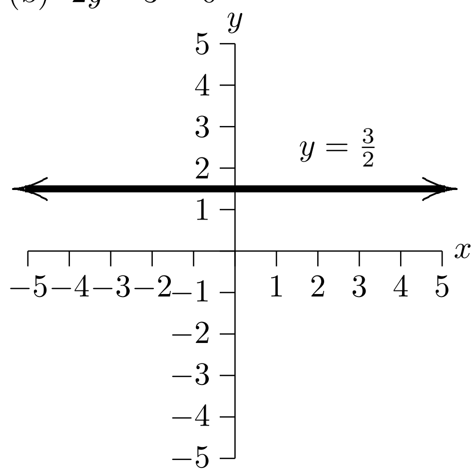
$$x - 1 = 1 \quad \text{when} \quad x = 2$$

Solution set: $\{-1, 2\}$

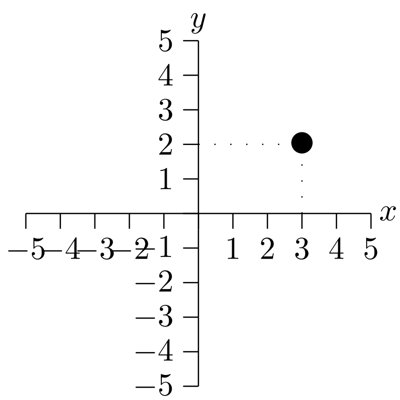
3. (a) $x > 3$



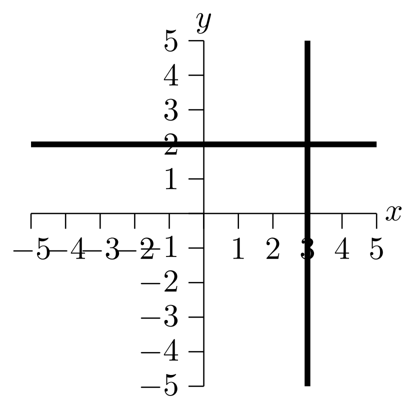
(b) $2y - 3 = 0$



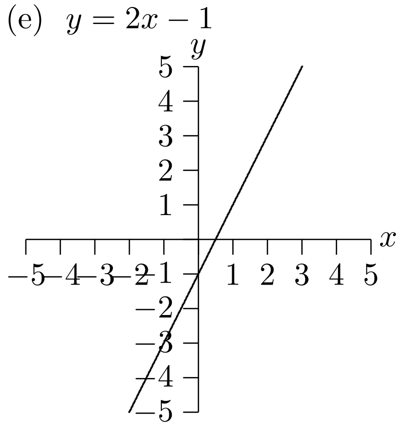
(c) $x = 3 \quad \text{and} \quad y = 2$



(d) $x = 3 \quad \text{or} \quad y = 2$

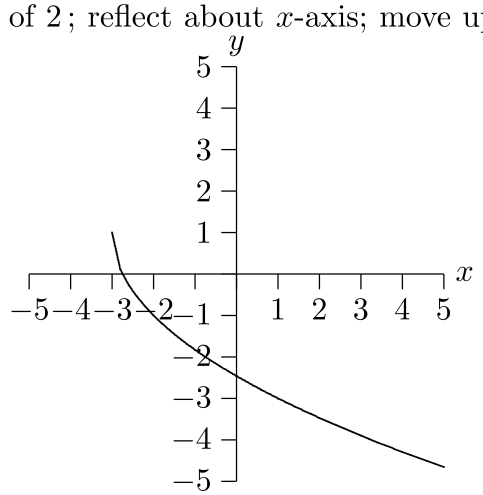


(e) $y = 2x - 1$

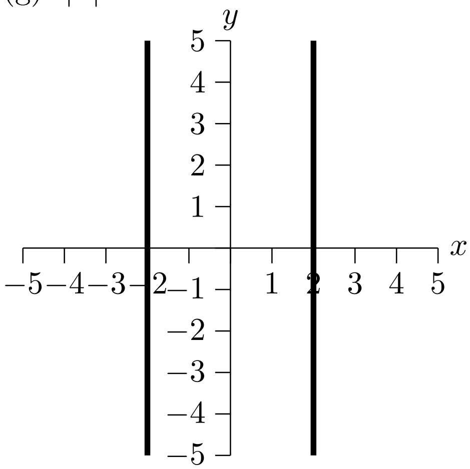


(f) $y = -2\sqrt{x+3} + 1$

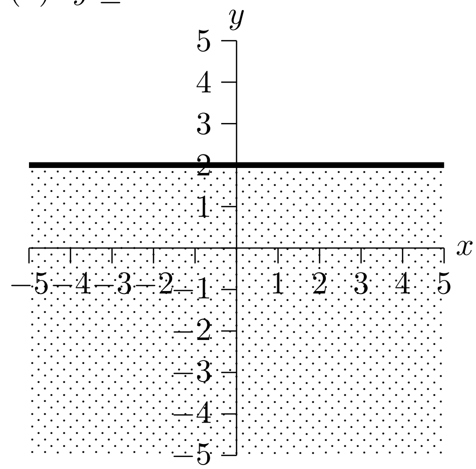
Take $y = \sqrt{x}$, and apply the following transformations: shift left 3; vertical stretch by a factor of 2; reflect about x-axis; move up 1. This gives:



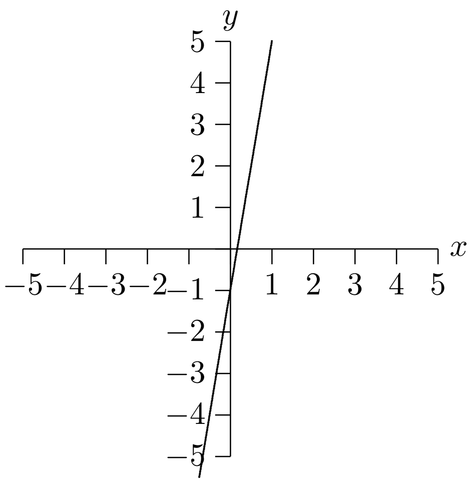
(g) $|x| = 2$



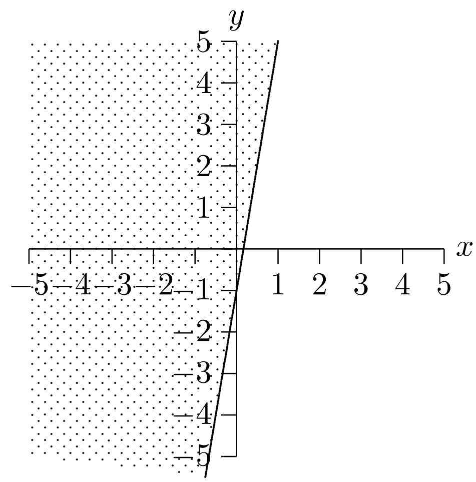
(h) $y \le 2$



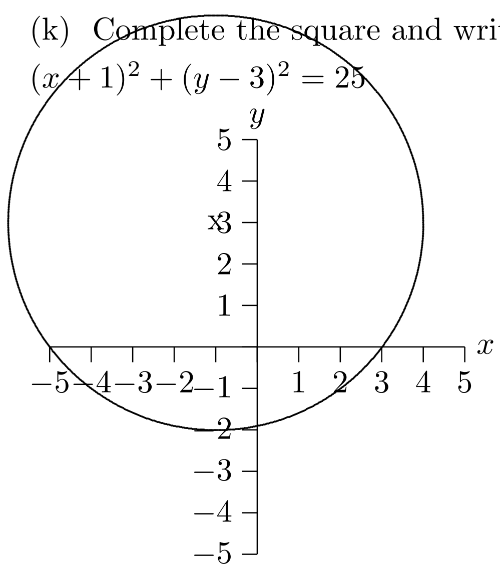
(i) $\frac{y-2}{3} = 2x - 1$ is equivalent to $y = 6x - 1$



(j) $\frac{y-2}{3} \ge 2x - 1$ is equivalent to $y \ge 6x - 1$



(k) Complete the square and write as: $(x + 1)^2 + (y - 3)^2 = 25$



4.

EQUATION	TRANSFORMATION
$y = f(x)$	(starting place)
$y = f(x+1)$	replace x by $x+1$; shift left 1
$y = \lvert f(x+1)\rvert$	take absolute value of y-values; any part below x-axis flips up
$y = 3\lvert f(x+1)\rvert$	multiply previous y-values by 3 ; vertical stretch
$y = -3\lvert f(x+1)\rvert$	multiply previous y-values by -1 ; reflect about x-axis
$y = -3\lvert f(x+1)\rvert + 5$	add 5 to previous y-values; move up 5

5.

EQUATION	TRANSFORMATION
$y = x^2 - 2x + 1$	(starting place)
$y = x^2 - 2x + 2$	up 1
$y = (x+3)^2 - 2(x+3) + 2$	left 3
$y = -(x+3)^2 + 2(x+3) - 2$	reflect about the x-axis
$y = -2(x+3)^2 + 4(x+3) - 4$	vertical scale by a factor of 2

6. PERIMETER $= 2\ell + 2w$, AREA $= \ell w$

PERIMETER $= 2x + y$, AREA $= \frac{1}{2}(y)(\frac{x}{2}) = \frac{1}{4}xy$

CIRCUMFERENCE $= 2\pi r$, AREA $= \pi r^2$

VOLUME $=$ (area of base)(height) $= \pi R^2 h$

Meter is a unit of length; cm^2 is a unit of area; cubic feet is a unit of volume.

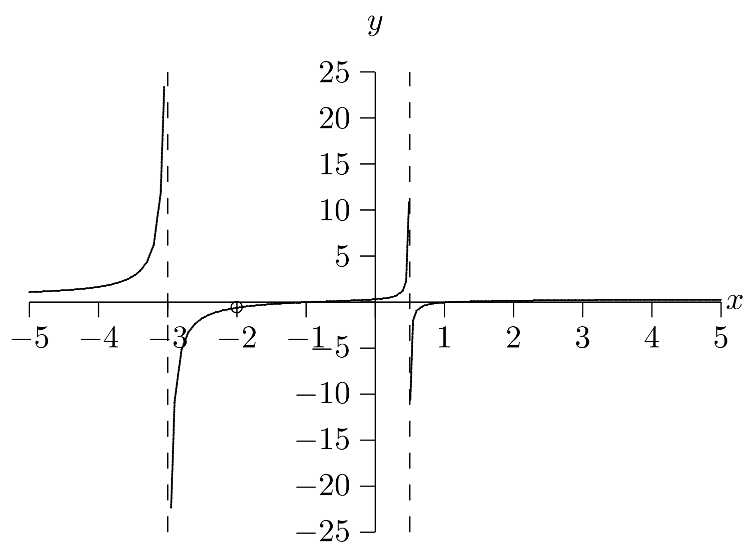
7.

$$
\begin{aligned}
\text{(a)}\quad g(f(x)) &= g(x^2 - 2x + 1) \\
&= 1 - 3(x^2 - 2x + 1) \\
&= 1 - 3x^2 + 6x - 3 \\
&= -3x^2 + 6x - 2
\end{aligned}
$$

$$
\begin{aligned}
f(g(x)) &= f(1 - 3x) \\
&= (1-3x)^2 - 2(1-3x) + 1 \\
&= 1 - 6x + 9x^2 - 2 + 6x + 1 \\
&= 9x^2
\end{aligned}
$$

(b) (There are other possible correct answers.) Let $g(x) = x^2 - 1$ and $f(x) = \sqrt[3]{x}$.

8.



For $x \neq -2$, $g(x) = \dfrac{x^2 - 1}{(2x-1)(x+3)}$.

Note that the point $(-2, -\frac{3}{5})$ is a puncture point.

x-intercepts occur when $x = \pm 1$.

y-intercept: $(0, \frac{1}{3})$

horizontal asymptote: $y = \frac{1}{2}$

vertical asymptotes: $x = \frac{1}{2}$ and $x = -3$

no slant asymptote

As $x \to \infty$, $y \to \frac{1}{2}$.

As $x \to -3^+$, $y \to -\infty$.

9. Since $P(-3) = 0$, P has a factor of $x + 3$.

Since 1 is a zero of P, $x - 1$ is a factor.

Since the graph of P crosses the x-axis at $x = 2$, $x - 2$ is a factor.

Since P must have degree 5, I'll choose to make 1 a zero of multiplicity 3. (There are other possible choices here.) Thus, the polynomial takes on the following form:

$$P(x) = K(x+3)(x-1)^3(x-2)$$

Since $P(0) = 7$, we have:

$$\begin{aligned} K(3)(-1)^3(-2) &= 7 \\ 6K &= 7 \\ K &= \frac{7}{6} \end{aligned}$$

Thus, $P(x) = \frac{7}{6}(x+3)(x-1)^3(x-2)$.

10. (a) $2x-3$

(b) $2(x-3)$

(c) $\frac{(2x)^3+1}{x}$

(d) take a number, multiply by 3, then subtract 1

(e) take a number, add 1, cube the result, multiply by 2, then subtract 5

(f) take a number, subtract 3, divide by 7, then subtract 1

11. (a) $f(0)=0^2-2(0)+1=1$

(b) $f(1)-2=\big(1^2-2\cdot 1+1\big)-2=0-2=-2$

(c) $f(f(-1))=f\big((-1)^2-2(-1)+1\big)=f(4)=4^2-2(4)+1=9$

12. The function g is defined whenever $x-3>0$, that is, whenever $x>3$.
The domain of g is the interval $(3,\infty)$.

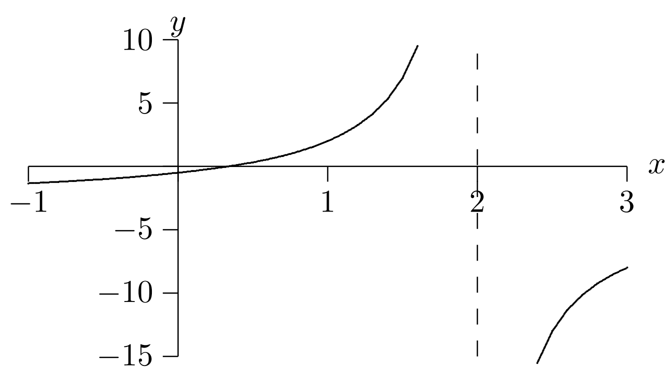
13. (a) $y=3x-7$

(b) $y=2$

(c) The line $x-3y=5$ has slope $\frac{1}{3}$; a perpendicular line will have slope -3.
The line with slope -3 passing through $(0,3)$ has equation $y=-3x+3$.

14. (a) The domain of f is the set of all real numbers except 2.

(b)



(c) The graph crosses the x-axis at $\frac{1}{3}$. (Set $1-3x=0$. Be sure you can get this *exact* answer, not just $x\approx 0.333333$.)

(d) When $x=1.375$ (exactly), then $f(x)=5$. (You could check this, if desired, by solving the equation $5=\frac{1-3x}{x-2}$.)

15. (a) $\frac{1+\sqrt{2}}{\sqrt[3]{5}-7}\approx -0.45637$

(b) 19.72 (this is exact)

(c) $|1-2\sqrt{3}|\approx 2.46410$

(d) $(2.03\times 10^{-9})(-4.1\times 10^{7})=-0.08323$ (this is exact)

SAMPLE PREREQUISITE PROBLEMS: CALCULUS

The following questions arise from actual AP Calculus AB exam questions; I went through lots of questions, and pulled out parts requiring algebra and trigonometry. These problems do an excellent job of illustrating the comfort level that is required with prerequisite material, for both the Honors and AP course.

DO NOT USE A CALCULATOR on these problems, except where a calculator is specifically called for.

I've grouped the problems according to the primary skill needed in the problem.

RATIONAL EXPONENTS:

1. Evaluate $\frac{3}{2}x^{\frac{1}{2}}$ when $x = 4$.

BASIC ALGEBRA SKILLS:

2. Solve for D: $3x^2 + 3(xD + y) + 6y^2 D = 0$

3. Simplify $f(x) = \dfrac{x^2 - 4}{x + 2}$. You may assume that $x \neq -2$.

LINES:

4. Write the equation of the line through the point $(1, 5)$ that has slope -13.

5. Suppose that the tangent line to a curve at a given point has slope 2. Then, what is the slope of the line *normal* to the curve at this point?

FUNCTION COMPOSITION:

6. If h is the function given by $h(x) = f\big(g(x)\big)$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

   (A) $3x^3 - |x|$ (B) $|3x^2 - 1|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$

TRIG FUNCTIONS OF COMMON ANGLES:

7. Evaluate $(x - 1)^2 \cos x + 2(x - 1)\sin x$ when $x = 0$.

FUNCTION NOTATION, and SOLVING EQUATIONS:

8. Suppose that $x(t) = t^3 - t^2 + 4t + K$ and $x(1) = 10$. Find K.

9. Suppose that $\frac{1}{2} \cdot \dfrac{y^{-1}}{-1} = x + C$, and that $y = -1$ when $x = 1$. Solve for C. Then, find y when $x = 2$.

10. Suppose that $F(u) = \cos 2\pi u$. What is $F(x)$?

    Now, suppose that the name of a function is F', and it is known that $F'(u) = -2\pi \sin 2\pi u$. What is $F'(x)$?

11. How many zeros does the function $g(x) = 4(x + 2)^5(x - 3)^3 + 5(x - 3)^4(x + 2)^4$ have?

PROPERTIES OF TRIG FUNCTIONS:

12. The fundamental period of $2\cos(3x)$ is

(A) $\frac{2\pi}{3}$ (B) 2π (C) 6π (D) 2 (E) 3

GRAPHS OF PIECEWISE-DEFINED FUNCTIONS:

13. Let f be the function defined by

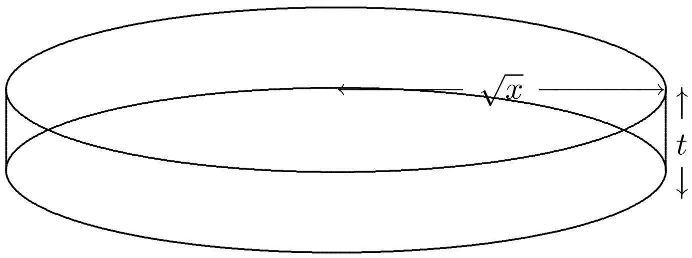
$$f(x) = \begin{cases} x^3 & \text{for } x \geq 0\,, \\ x & \text{for } x < 0\,. \end{cases}$$

Graph f. Is f an odd function?

BASIC GRAPHING SKILLS:

14. Shade the region enclosed by the graph of $y = (x+1)^{\frac{1}{3}}$, the line $x = 7$, the x-axis, and the y-axis.

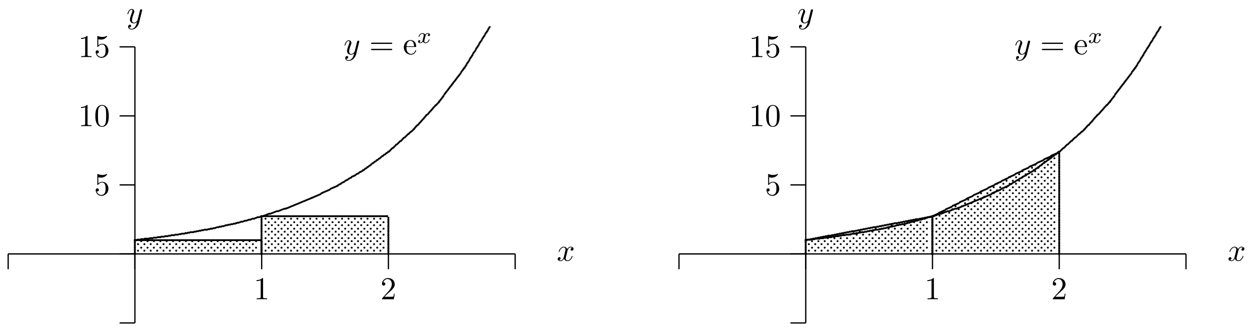
15. Shade the region enclosed by the graph of $y = \sqrt{x}$, the line $x = 3$, and the x-axis. Then, find the volume of the disk shown below. (This disk has radius $\sqrt{x}$ and thickness t.)



GEOMETRY SKILLS:

16. Suppose that a tin can has top and bottom removed. Using metal cutters, it is cut as shown, and unfolded. What is the volume of the resulting sheet of metal? (Hint: Your formula will involve the radius r, the height h, and the metal thickness t.)

17. Find each of the areas shaded below. (Note that each area might be used to approximate the *actual* area under the graph of $y = e^x$ on the interval $[0, 2]$.)



18. The radius of a circle is changing with time. (Think, perhaps, of the circular waves that are generated when a stone is thrown into the water.) Letting $r(t)$ denote the radius at time t, write formulas for the AREA of the circle at time t, and the CIRCUMFERENCE of the circle at time t.

EXPONENT LAWS:

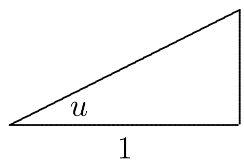
19. (a) Write $\dfrac{1}{x^2} - \dfrac{1}{x^3}$ using negative exponents.

(b) Solve for x: $6x^{-4} - 12x^{-5} = 0$

20. Let $g(x) = \frac{2}{3}(x^2 - 2x - 1)^{-\frac{1}{3}}(2x - 2)$. Find $g(0)$.

INVERSE TRIGONOMETRIC FUNCTIONS:

21. Suppose that $u = \arctan(x - 1)$. Then (fill in the blank) $\tan u =$______________.
Fill in the lengths of the remaining sides of the right triangle in the sketch below.



Find $\dfrac{1}{\sec^2 u}$. (Your formula should involve only x.)

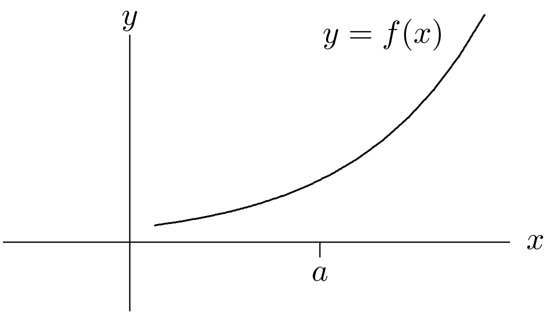
GENERAL FUNCTION KNOWLEDGE:

22. Is 2^x a polynomial function? Is it an exponential function? DEFINE both *polynomial* and *exponential* functions.

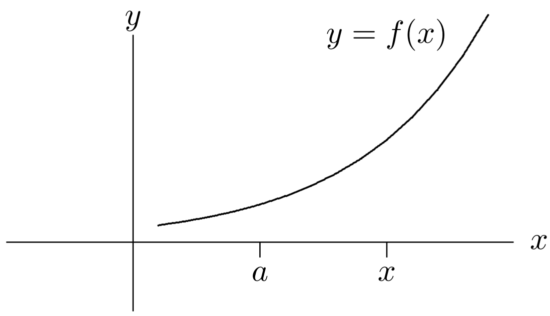
23. Graph the function $f(\theta) = \dfrac{1 - \cos\theta}{2\sin^2\theta}$, using your graphing calculator. As θ approaches 0, what do the numbers $f(\theta)$ approach?

24. On each sketch below, clearly indicate what the given expression represents. Assume that h is a small positive number; a and x are labeled.

$$\frac{f(a+h)-f(a)}{h}$$

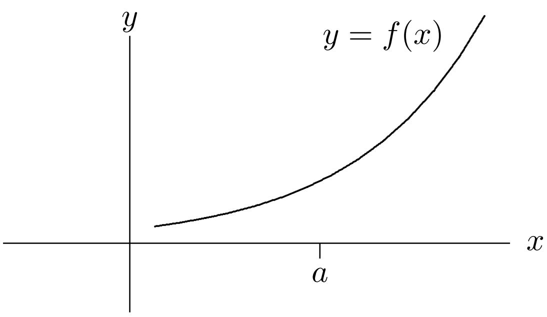


$$\frac{f(x)-f(a)}{x-a}$$

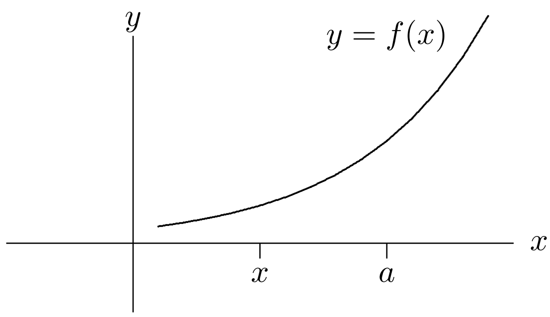


Repeat the instructions above, but this time, assume that h is a small *negative* number, and $x < a$.

$$\frac{f(a+h)-f(a)}{h}$$



$$\frac{f(x)-f(a)}{x-a}$$



PROPERTIES OF LOGARITHMS:

25. Use properties of exponents and logarithms to simplify $e^{3\ln x^2}$.

26. Let $f(x) = x\ln x$. What is the domain of f ? Find $f(\frac{1}{e})$.

RIGHT TRIANGLE TRIGONOMETRY:

27. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, how far is the bottom of the ladder from the wall? (You may use a calculator for this problem.)

RATIONAL FUNCTIONS:

28. If the graph of $y = \frac{ax+b}{x+c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a + c =$

(A) -5 (B) -1 (C) 0 (D) 1 (E) 5

SOLUTIONS

1. 3

2. $D = \dfrac{-x^2 - y}{x + 2y^2}$

3. $x - 2$

4. $y = -13x + 18$

5. $-\frac{1}{2}$

6. E

7. 1

8. $K = 6$

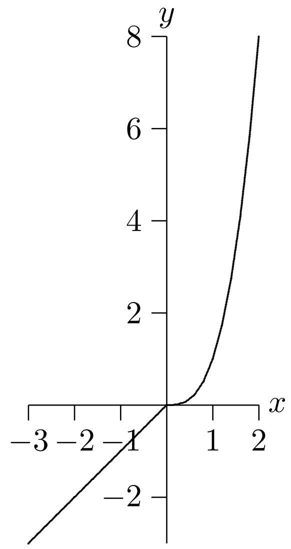
9. $C = -\frac{1}{2}$; $y = -\frac{1}{3}$

10. $F(x) = \cos 2\pi x$; $F'(x) = -2\pi \sin 2\pi x$

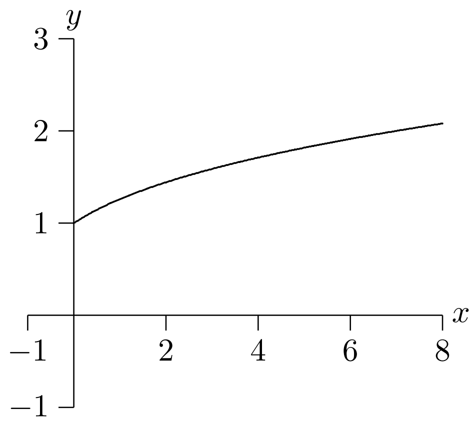
11. The function g has three different zeros: -2 , 3 , and $\frac{7}{9}$.

12. A

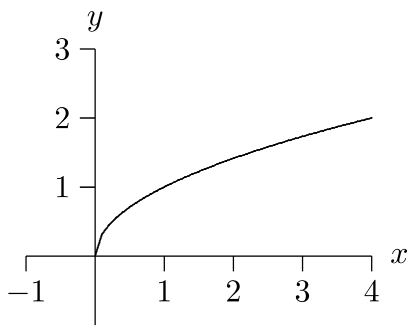
13. The function f is NOT an odd function.



14.



15. The volume of the disk is $\pi x t$.



16. $2\pi r h t$

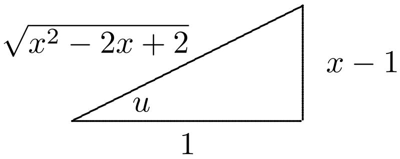
17. $\mathrm{e} + 1$; $\frac{1}{2}(1 + 2\mathrm{e} + \mathrm{e}^2)$

18. $A = \pi(r(t))^2$; $C = 2\pi r(t)$

19. (a) $x^{-2}(1 - x^{-1})$; (b) $x = 2$

20. $g(0) = \frac{4}{3}$

21. $\tan u = x - 1$;

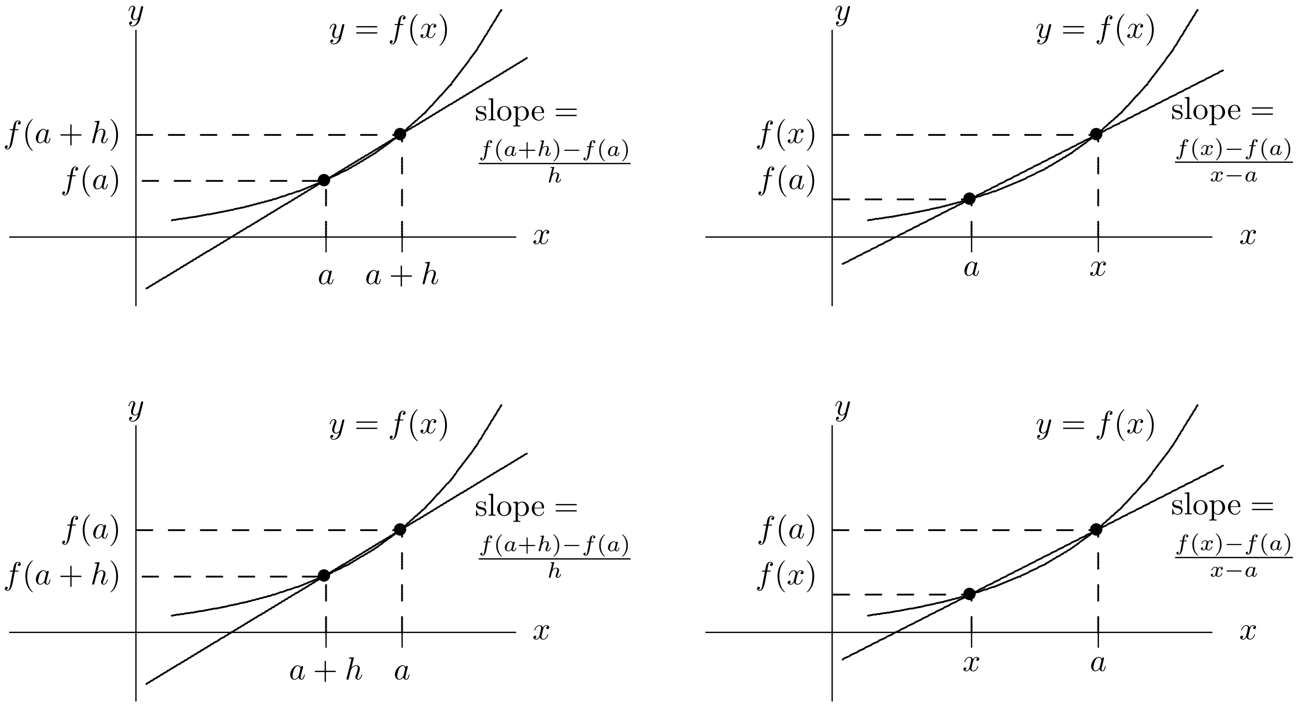


$$\frac{1}{\sec^2 u} = \frac{1}{x^2 - 2x + 2}$$

22. The function 2^x is an exponential function, but not a polynomial. A polynomial is a sum of terms, each of the form ax^n, where $a \in \mathbb{R}$ and n is a nonnegative integer. An exponential function is a function of the form a^x where $a > 0$ and $a \neq 1$.

23. As θ approaches 0, the numbers $f(\theta)$ approach 0.25.

24.



25. x^6

26. The domain of f is $(0, \infty)$. $f(\frac{1}{e}) = -\frac{1}{e}$

27. 24 feet

28. E