# SAMPLE PREREQUISITE PROBLEMS: CALCULUS

The following questions arise from actual AP Calculus AB exam questions; I went through lots of questions, and pulled out parts requiring algebra and trigonometry. These problems do an excellent job of illustrating the comfort level that is required with prerequisite material, for both the Honors and AP course.

DO NOT USE A CALCULATOR on these problems, except where a calculator is specifically called for.

I've grouped the problems according to the primary skill needed in the problem.

RATIONAL EXPONENTS:

1. Evaluate $\frac{3}{2}x^{\frac{1}{2}}$ when $x = 4$.

BASIC ALGEBRA SKILLS:

2. Solve for $D$: $3x^2 + 3(xD + y) + 6y^2 D = 0$

3. Simplify $f(x) = \dfrac{x^2 - 4}{x + 2}$. You may assume that $x \neq -2$.

LINES:

4. Write the equation of the line through the point $(1, 5)$ that has slope $-13$.

5. Suppose that the tangent line to a curve at a given point has slope $2$. Then, what is the slope of the line *normal* to the curve at this point?

FUNCTION COMPOSITION:

6. If $h$ is the function given by $h(x) = f\big(g(x)\big)$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

   (A) $3x^3 - |x|$ (B) $|3x^2 - 1|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$

TRIG FUNCTIONS OF COMMON ANGLES:

7. Evaluate $(x-1)^2 \cos x + 2(x-1)\sin x$ when $x = 0$.

FUNCTION NOTATION, and SOLVING EQUATIONS:

8. Suppose that $x(t) = t^3 - t^2 + 4t + K$ and $x(1) = 10$. Find $K$.

9. Suppose that $\frac{1}{2} \cdot \frac{y^{-1}}{-1} = x + C$, and that $y = -1$ when $x = 1$. Solve for $C$. Then, find $y$ when $x = 2$.

10. Suppose that $F(u) = \cos 2\pi u$. What is $F(x)$?

    Now, suppose that the name of a function is $F'$, and it is known that $F'(u) = -2\pi \sin 2\pi u$. What is $F'(x)$?

11. How many zeros does the function $g(x) = 4(x+2)^5(x-3)^3 + 5(x-3)^4(x+2)^4$ have?

PROPERTIES OF TRIG FUNCTIONS:

12. The fundamental period of $2\cos(3x)$ is

(A) $\frac{2\pi}{3}$ (B) $2\pi$ (C) $6\pi$ (D) $2$ (E) $3$

GRAPHS OF PIECEWISE-DEFINED FUNCTIONS:

13. Let $f$ be the function defined by

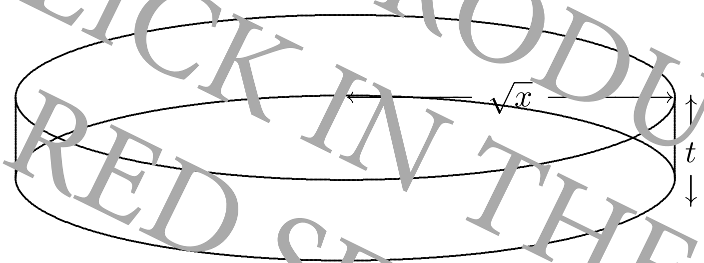
$$f(x) = \begin{cases} x^3 & \text{for } x \ge 0\,, \\ x & \text{for } x < 0\,. \end{cases}$$

Graph $f$. Is $f$ an odd function?

BASIC GRAPHING SKILLS:

14. Shade the region enclosed by the graph of $y = (x+1)^{\frac{1}{3}}$, the line $x = 7$, the $x$-axis, and the $y$-axis.

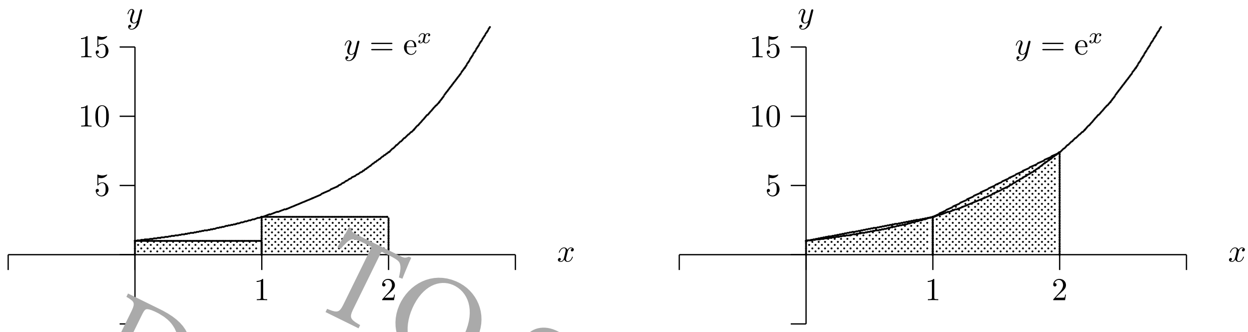
15. Shade the region enclosed by the graph of $y = \sqrt{x}$, the line $x = 3$, and the $x$-axis. Then, find the volume of the disk shown below. (This disk has radius $\sqrt{x}$ and thickness $t$.)




GEOMETRY SKILLS:

16. Suppose that a tin can has top and bottom removed. Using metal cutters, it is cut as shown, and unfolded. What is the volume of the resulting sheet of metal? (Hint: Your formula will involve the radius $r$, the height $h$, and the metal thickness $t$.)

17. Find each of the areas shaded below. (Note that each area might be used to approximate the *actual* area under the graph of $y = e^x$ on the interval $[0, 2]$.)




18. The radius of a circle is changing with time. (Think, perhaps, of the circular waves that are generated when a stone is thrown into the water.) Letting $r(t)$ denote the radius at time $t$, write formulas for the AREA of the circle at time $t$, and the CIRCUMFERENCE of the circle at time $t$.

EXPONENT LAWS:

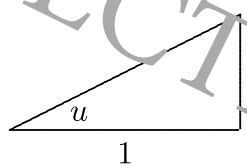
19. (a) Write $\frac{1}{x^2} - \frac{1}{x^3}$ using negative exponents.

(b) Solve for $x$: $6x^{-4} - 12x^{-5} = 0$

20. Let $g(x) = \frac{2}{3}(x^2 - 2x - 1)^{-\frac{1}{3}}(2x - 2)$. Find $g(0)$.

INVERSE TRIGONOMETRIC FUNCTIONS:

21. Suppose that $u = \arctan(x - 1)$. Then (fill in the blank) $\tan u =$_______________.
Fill in the lengths of the remaining sides of the right triangle in the sketch below.




Find $\frac{1}{\sec^2 u}$. (Your formula should involve only $x$.)

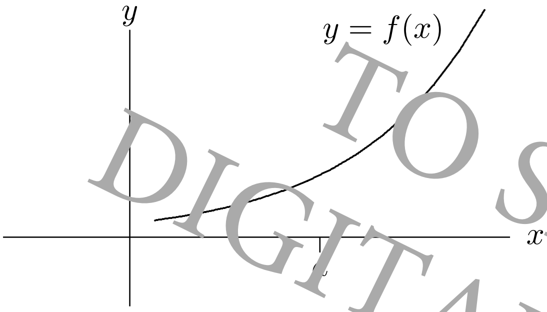
GENERAL FUNCTION KNOWLEDGE:

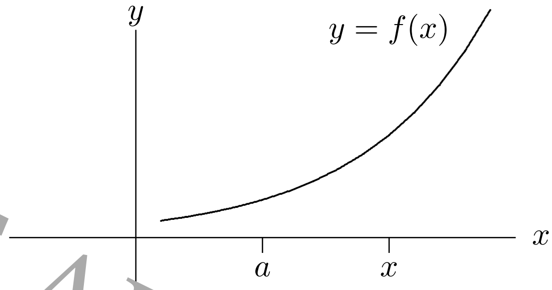
22. Is $2^x$ a polynomial function? Is it an exponential function? DEFINE both *polynomial* and *exponential* functions.

23. Graph the function $f(\theta) = \frac{1 - \cos\theta}{2\sin^2\theta}$, using your graphing calculator. As $\theta$ approaches $0$, what do the numbers $f(\theta)$ approach?

24. On each sketch below, clearly indicate what the given expression represents. Assume that $h$ is a small positive number; $a$ and $x$ are labeled.

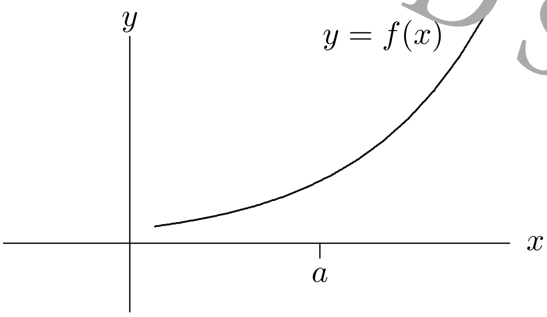
$$\frac{f(a+h)-f(a)}{h} \qquad\qquad \frac{f(x)-f(a)}{x-a}$$

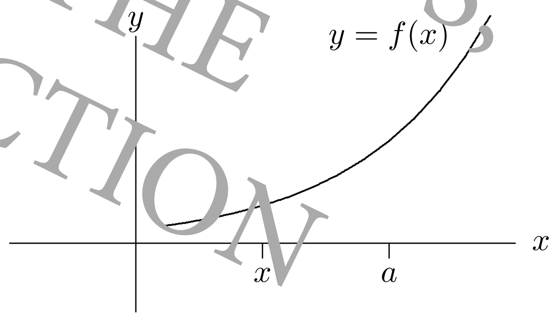



Repeat the instructions above, but this time, assume that $h$ is a small *negative* number, and $x < a$.

$$\frac{f(a+h)-f(a)}{h} \qquad\qquad \frac{f(x)-f(a)}{x-a}$$



PROPERTIES OF LOGARITHMS:

25. Use properties of exponents and logarithms to simplify $e^{3\ln x^2}$.

26. Let $f(x) = x\ln x$. What is the domain of $f$? Find $f(\frac{1}{e})$.

RIGHT TRIANGLE TRIGONOMETRY:

27. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, how far is the bottom of the ladder from the wall? (You may use a calculator for this problem.)

RATIONAL FUNCTIONS:

28. If the graph of $y = \dfrac{ax+b}{x+c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a + c =$

(A) -5 (B) -1 (C) 0 (D) 1 (E) 5

# SOLUTIONS

1. 3

2. $D = \dfrac{-x^2 - y}{x + 2y^2}$

3. $x - 2$

4. $y = -13x + 18$

5. $-\frac{1}{2}$

6. E

7. 1

8. $K = 6$

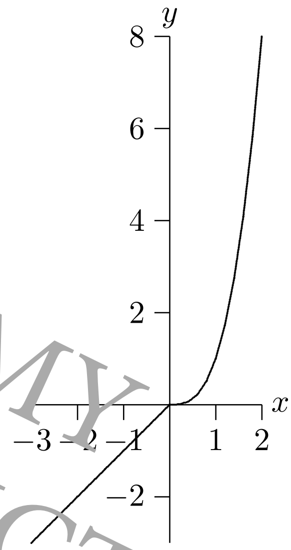
9. $C = -\frac{1}{2}$; $y = -\frac{1}{3}$

10. $F(x) = \cos 2\pi x$; $F'(x) = -2\pi \sin 2\pi x$

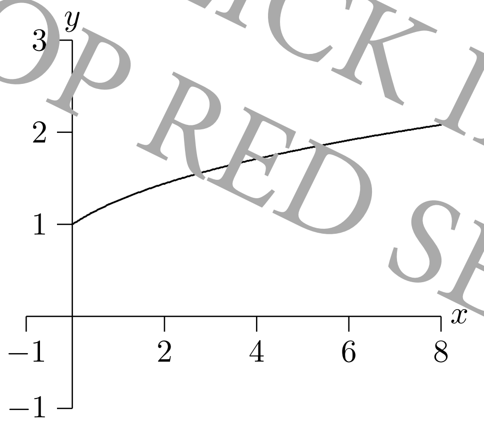
11. The function $g$ has three different zeros: $-2$, $3$, and $\frac{7}{9}$.

12. A

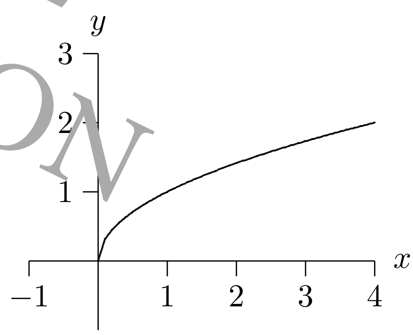
13. The function $f$ is NOT an odd function.




14.




15. The volume of the disk is $\pi x t$.




16. $2\pi r h t$

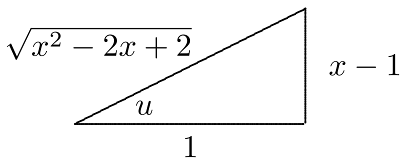
17. $e + 1$; $\frac{1}{2}(1 + 2e + e^2)$

18. $A = \pi(r(t))^2$; $C = 2\pi r(t)$

19. (a) $x^{-2}(1 - x^{-1})$; (b) $x = 2$

20. $g(0) = \frac{4}{3}$

21. $\tan u = x - 1$;

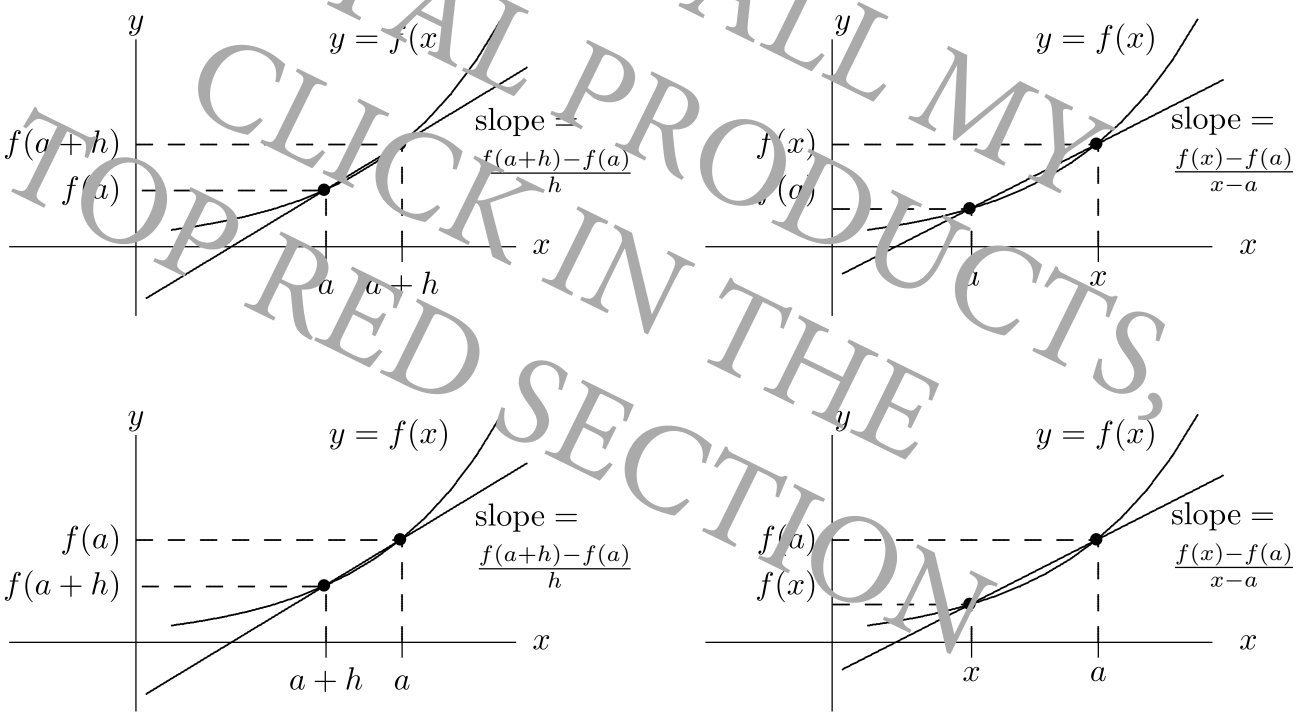



$$\frac{1}{\sec^2 u} = \frac{1}{x^2 - 2x + 2}$$

22. The function $2^x$ is an exponential function, but not a polynomial. A polynomial is a sum of terms, each of the form $ax^n$, where $a \in \mathbb{R}$ and $n$ is a nonnegative integer. An exponential function is a function of the form $a^x$ where $a > 0$ and $a \neq 1$.

23. As $\theta$ approaches $0$, the numbers $f(\theta)$ approach $0.25$.

24.




25. $x^6$

26. The domain of $f$ is $(0, \infty)$. $f(\frac{1}{e}) = -\frac{1}{e}$

27. 24 feet

28. E