SATstuff#8

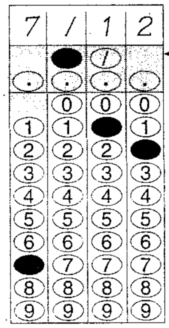
This section concludes the SAT review: grid-ins, coordinate geometry, absolute value, solving radical equations, and a bit of probability and statistics.

GRID-INS

One of the math sections on the SAT contains ten problems without multiple-choice answers; these are called STUDENT-PRODUCED RESPONSES, or GRID-INS.

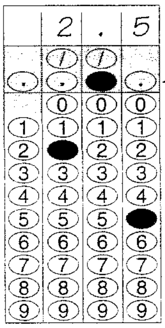
Here are some typical (filled-in) grids:

Answer: $\frac{7}{12}$



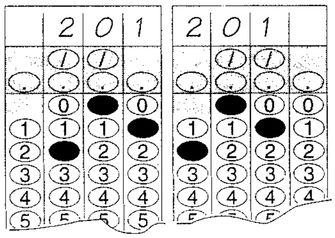
← Fraction line

Answer: 2.5



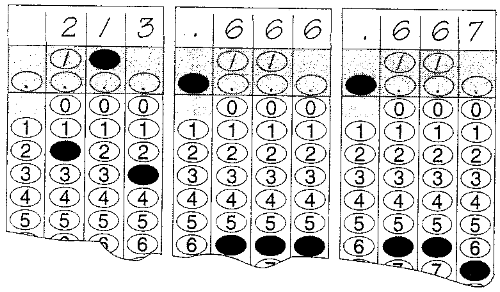
← Decimal point

Answer: 201
Either position is correct.



Note: You may start your answers in any column, space permitting. Columns not needed should be left blank.

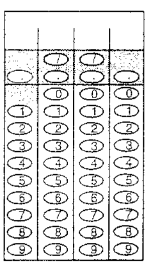
Acceptable ways to grid $\frac{2}{3}$ are:



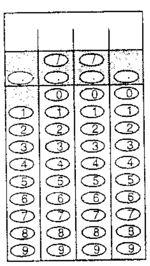
- Write your answer in the boxes at the top; it's a good idea to always start in the first box.
- There are no negative answers; there are no variables in answers.
  (No minus sign or letters appear in the oval choices, so you won't even be tempted.)
- Darken the ovals beneath each number, decimal point, or division symbol—this is what the computer grades.
- Some problems may have more than one correct answer.
  Pick a correct answer, and grid it in!
- Mixed numbers, like $3\frac{1}{2}$, can be gridded as 3.5 or 7/2.
- If your answer comes out longer than what fits in the box, you can round or truncate, but be sure to use all the boxes.
  For example, an answer of 0.87352 would be correctly entered as .873 or .874.
  Don't ever put a zero in front—there isn't even a zero for you to use in the first grid-in column.
- NOTHING is deducted for an incorrect answer on a grid-in.
  Be sure to try every single grid-in question!

GRID-IN THE FOLLOWING ANSWERS:

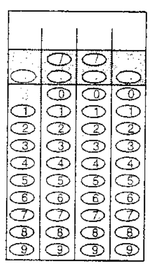
ANS: 7



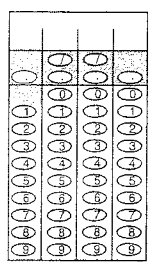
ANS: $2\frac{1}{3}$



ANS: 0.12345

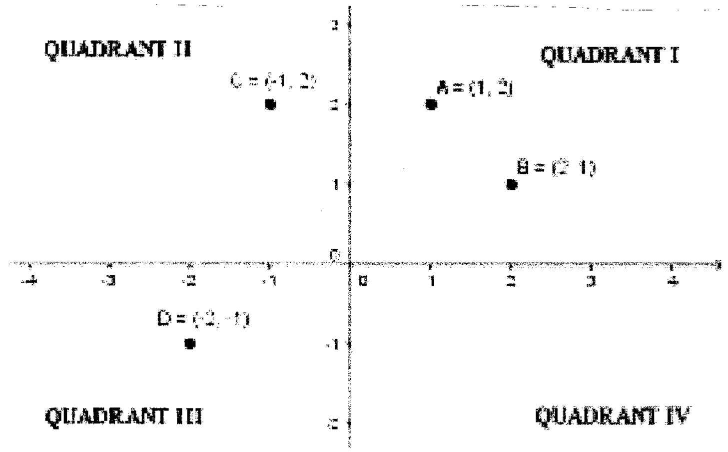


ANS: 1,397



COORDINATE GEOMETRY

Study the *coordinate plane* below:

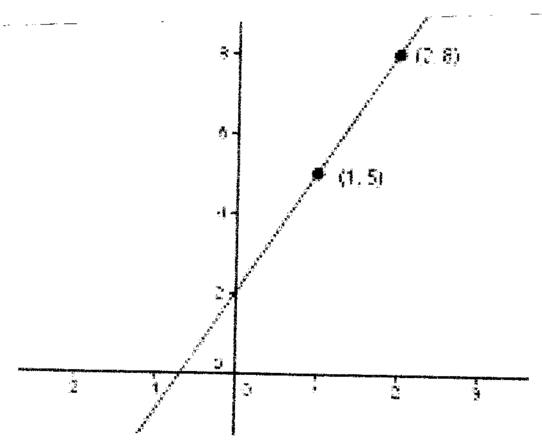


POINTS are of the form (x, y).
The first number (x) is the x-value of the point; it gives left/right information.
The second number (y) is the y-value of the point; it gives up/down information.

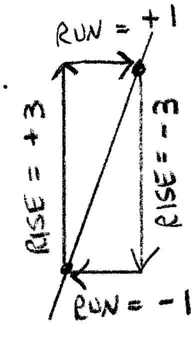
You must know the four QUADRANTS:
Quadrant I: (positive,positive)
Quadrant II: (negative, positive)
Quadrant III: (negative,negative)
Quadrant IV: (positive,negative)

EQUATIONS IN TWO VARIABLES

When you have an equation in two variables (like $y = 3x + 2$) then the solutions are PAIRS of numbers—a choice for x, and a choice for y—that make the equation true.
For example, when $x = 1$, then $y = 3(1) + 2 = 5$, so $(1, 5)$ is a solution.
For example, when $x = 2$, then $y = 3(2) + 2 = 8$, so $(2, 8)$ is a solution.
If you were to plot ALL the solutions to $y = 3x + 2$, then you'd see the line shown below:



The SLOPE of the line is a number that measures its *slant*: slope is $\frac{\text{rise}}{\text{run}}$.



RISE is up/down movement in going from one point to another:
up is positive, *down* is negative.
RUN is left/right movement in going from one point to another:
right is positive, *left* is negative.
Every equation of the form $y = mx + b$ graphs as a line:
the slope is m, and the line crosses the y-axis at b.

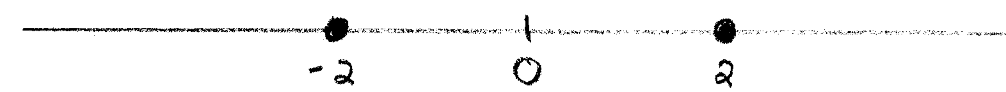
ABSOLUTE VALUE as distance from zero

The expression $|x|$ (read as *the absolute value of x*) gives a number's distance from zero. You can walk from zero in *two directions*—to the right, and to the left!

EXAMPLES:

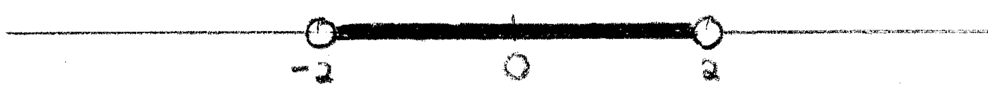
Solve: $|x| = 2$

Shade all numbers whose distance from zero is 2:



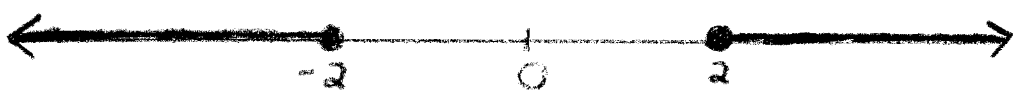
Solve: $|x| < 2$

Shade all numbers whose distance from zero is less than 2:



Solve: $|x| \geq 2$

Shade all numbers whose distance from zero is greater than or equal to 2:



ABSOLUTE VALUE as distance between two numbers

The expression $|x - y|$ gives the distance between the numbers x and y.

For example, $|x - 3|$ gives the distance from x to 3.

Also, $|x + 3| = |x - (-3)|$ gives the distance from x to -3.

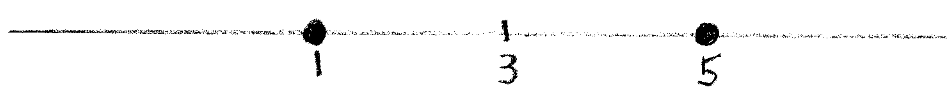
DISTANCE BETWEEN

$|x - 3|$

EXAMPLES:

Solve: $|x - 3| = 2$

Shade all numbers whose distance from 3 is 2:

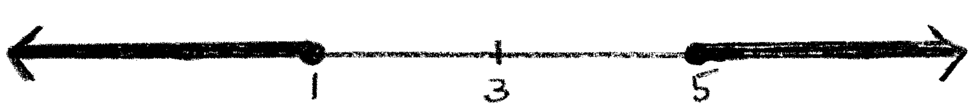


Solve: $|x - 3| < 2$

Shade all numbers whose distance from 3 is less than 2:

Solve: $|x - 3| \geq 2$

Shade all numbers whose distance from 3 is greater than or equal to 2:



ALGEBRAIC DEFINITION OF ABSOLUTE VALUE

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

CAREFUL!!
If $x < 0$, then $-x$ is a POSITIVE NUMBER!
For example, if $x = -2$, then $-x = -(-2) = 2$.
Think of $-x$ as the OPPOSITE of x (not 'negative x').

YOU TRY THESE:

(1) Solve: $|x| - 4 = 1$

(2) Solve: $5 - |x| < 1$

(3) Solve: $3|x| \leq 6$

(4) Solve: $|3 - x| \geq 2$

(5) Solve: $|x - 1| = -5$

(6) Suppose $x < 0$ and $y > 0$. What quadrant is the point $(-x, y)$ in?

(7) In the coordinate plane, show ALL points with x-value equal to 2.

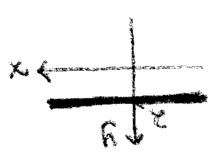
(8) In the coordinate plane, show ALL points with y-value equal to 2.

(9) What does the graph of $3x + y = 2$ look like?

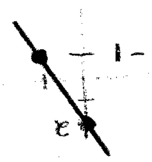
(1) 5, -5 (2) $x > 4$ or $x < -4$ (3) $-2 \leq x \leq 2$

(4) $x \geq 5$ or $x \leq 1$ (5) never true (6) I (7)

(8)



(9)



SOLVING RADICAL EQUATIONS

A *radical equation* is an equation involving a radical (like a square root or a cube root), with a *variable* inside the root.

For example, $x - 2 = \sqrt{3}$ isn't a radical equation.

(To solve this equation, just add 2 to both sides.)

But, $\sqrt{3-x} = 4$ *is* a radical equation.

You 'undo' square roots by squaring, so to solve this equation, square both sides:

$\sqrt{3-x} = 4$

$3 - x = 16$

$-x = 13$

$x = -13$

Check: $\sqrt{3-(-13)} = \sqrt{16} = 4$

To solve a radical equation:

- *Isolate* a radical; that is, get it all by itself on one side of the equation.
- *Undo* the radical to get at the stuff inside: undo a cube root by cubing, undo a fourth root by raising to the fourth power, and so on.
  Be sure to do the *same thing* to both sides.
- *Check your answer(s)* at the end. If something doesn't work, throw it away.
  Here's why you must check your answer(s):
  Note that the equation '$-2 = 2$' is false, but squaring both sides gives the true equation '$(-2)^2 = 2^2$'.
  So, squaring can sometimes add a solution, which you then have to throw away.
- Even roots (square roots, fourth roots, and so on) *can't be negative.*
  So, if you ever arrive at something like $\sqrt{x-3} = -5$, STOP.
  There are no solutions.
- Odd roots (cube roots, fifth roots, etc.) *can be negative*!

EXAMPLE:

Solve: $\sqrt{5-x} - 1 = x$

Isolate the radical: $\sqrt{5-x} = x + 1$

Square both sides: $5 - x = (x+1)^2$

Be careful! The WHOLE right side needs to be squared!

Simplify:

$5 - x = x^2 + 2x + 1$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4$ or $x = 1$

Throw $x = -4$ away, since it doesn't work in the original equation:

$\sqrt{5-(-4)} - 1 = \sqrt{9} - 1 = 3 - 1 = 2$, which is not equal to -4.

However, the number 1 works, and is your only solution.

YOU TRY THESE:

(10) Is $x + \sqrt{5} = 2$ a radical equation? Solve it.

(11) Is $\sqrt{x-5} = 3$ a radical equation? Solve it.

(10) NO!

(11) YES

$\sqrt{x-5} = 3$

$x - 5 = 9$

$x = 14$

check!

a bit of STATISTICS: MEDIAN and MODE

You already know how to find the mean (average) of a collection of numbers.

To find the *median* of a collection of numbers, do this:

- arrange the numbers in order
- the *median* is the number exactly in the middle of the list (if there is a middle)
- if there isn't a middle number, then average the two middle numbers

EXAMPLE:

Find the median of these numbers: 2, 1, 3, 2, 3, 5, 11

Order them: 1, 2, 2, 3, 3, 5, 11

The median is 3 .

Find the median of these numbers: 2, 0, 1, 5, 2, 1000, 4, 3

Order them: 0, 1, 2, 2, 3, 4, 5, 1000

The median is $\frac{2+3}{2} = 2.5$.

Medians are good, because they aren't sensitive to **outliers** (numbers which don't seem to be like the other numbers in the group).

The MODE of a collection of numbers is the most commonly-occurring number (if there is one).

There can be more than one mode!

For example, consider the numbers 1, 1, 2, 3, 4, 4, 5, 6, 7

Both 1 and 4 are modes, because they occur most often.

YOU TRY THESE:

(12) Find both the median and mode of this collection of quiz scores:
100, 0, 100, 90, 85, 100, 70, 50, 100, 80, 90

(13) On a quiz, there were five scores of 100; six scores of 90; and four scores of 80.
Find the mean, median, and mode of this collection.

a bit of PROBABILITY

A PROBABILITY is a number between 0 and 1 that represents how likely it is that something will occur.

When only finitely many things can happen, then a probability of 0 means something definitely WON'T happen; a probability of 1 means something definitely WILL happen.

EXAMPLE:

Suppose a basket contains 5 red marbles and 3 green marbles—that's it.

Nothing else in there. You reach in and choose a marble.

Each marble is equally likely to be chosen—they're all the same size and shape and weight, no sticky marbles, and so on.

There are a total of 8 marbles, and five of them are red, so the probability of choosing a red marble is $\frac{5}{8}$.

The probability of choosing a green marble is $\frac{3}{8}$.

The probability of choosing a red marble or a green marble is 1 . (This will definitely happen).

The probability of choosing a purple marble is 0 . (This definitely won't happen.)

(12) MEDIAN = 90, MODE = 100 (13) MEAN = 90.7, MEDIAN = 90, MODE = 90

the CHOICE PRINCIPLE

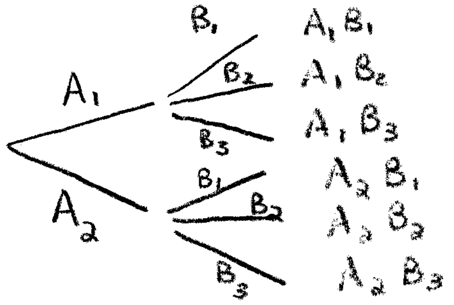
The Choice Principle is used to figure out how many ways to do things in succession, when there are various choices along the way.

The principle is illustrated next with specific numbers:

Suppose there are 2 ways to do something.

After this first thing is done, then there are 3 ways to do something else.

Together, there are $2 \cdot 3$ ways to do the two things in succession, as shown below:



EXAMPLE:

You've got a mix-and-match wardrobe: 7 shirts, 4 pairs of pants, and 2 pairs of shoes.

Everything goes with everything.

How many different outfits can you wear?

Answer: $7 \cdot 4 \cdot 2 = 56$

PERMUTATIONS and FACTORIALS

FACTORIALS are a shorthand for a "multiplication count-down' problem:

$6!$ (read as '6 factorial) means $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$.

A PERMUTATION is an arrangement of objects in a definite order.

Just put the Choice Principle to work:

If you have 5 different letters, how many ways can you arrange them, using each only once?

(In other words, how many *permutations* of five letters are there?)

There are 5 choices for the first, 4 choices for the second, and so on...

so there are $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ different arrangements.

YOU TRY THESE:

(14) In an urn, there are five black marbles, two green marbles, and three purple marbles. They're all equally likely to be chosen. You reach in and choose one marble.

What is the probability that it is black?

What is the probability that it is either black or green?

What is the probability that it is white?

(15) What is the number $3!$?

What is the number $4! - 3!$?

(16) What is the number $\frac{120!}{119!}$?

Show work leading to your answer.

(14) P(black) = $\frac{1}{2}$

P(black or green) = $\frac{7}{10}$

P(white) = 0

(15) $3! = 3 \cdot 2 \cdot 1 = 6$

$4! - 3! = 18$

(16) $\frac{120!}{119!} = \frac{120 \cdot \cancel{119!}}{\cancel{119!}} = 120$