SATstuff#3

This week we'll review fractions, decimals, percentages, order of operations, and ratios/proportions.

FRACTIONS:

ADDING/SUBTRACTING FRACTIONS:
– you must have a common denominator
– multiply by 1 in an appropriate form to get a desired denominator

$$\frac{2}{3}+\frac{1}{5}=\frac{2}{3}\cdot\frac{5}{5}+\frac{1}{5}\cdot\frac{3}{3}=\frac{10}{15}+\frac{3}{15}=\frac{13}{15}$$

MULTIPLYING FRACTIONS:
– just multiply across
– cancel BEFORE multiplying; cancelling is getting rid of extra factors of 1

$$\frac{4}{3}\cdot\frac{3}{14}=\frac{2\cdot 2}{3}\cdot\frac{3}{2\cdot 7}=\frac{2}{7}$$

DIVIDING FRACTIONS:
– dividing by 3 is the same as multiplying by $\frac{1}{3}$
– dividing by $\frac{2}{5}$ is the same as multiplying by $\frac{5}{2}$
– dividing by $\frac{a}{b}$ is the same as multiplying by $\frac{b}{a}$

$$\frac{2}{3}\div\frac{1}{5}=\frac{2}{3}\cdot\frac{5}{1}=\frac{10}{3}$$

WRITING FRACTIONS AS DECIMALS:
– the only fractions that can be written as finite decimals are ones with ONLY factors of 2s and 5s in the denominator (when they're in simplest form)
– all other fractions will be infinite repeating decimals; do a long division

$$\frac{3}{20}=\frac{3}{2\cdot 2\cdot 5}=\frac{3}{2\cdot 2\cdot 5}\cdot\frac{5}{5}=\frac{15}{10\cdot 10}=\frac{15}{100}=0.15$$

TRY THESE:

(a) $\frac{1}{3}-\frac{2}{7}$

(b) $\frac{x}{2y}\div\frac{5a}{2b}$

(c) Suppose that n is a power of 2 and m is a power of 5. Is the fraction $\frac{8034}{nm}$ a finite or infinite repeating decimal?

(d) Write as a decimal: $\frac{5}{6}$

PERCENTAGES:

The word *percent* means *per one hundred*.
The symbol % is used for percent.
Whenever you see the symbol $\%$, you can trade it in for a factor of $\frac{1}{100}$.
Whenever you see a factor of $\frac{1}{100}$, it can be traded in for a % symbol.

EXAMPLE: $2\% = 2 \cdot \frac{1}{100} = \frac{2}{100} = 0.02$
(to go from a percent to a decimal, just move the decimal point two places to the left)

EXAMPLE: $5 = 5 \cdot \frac{100}{100} = 500 \cdot \frac{1}{100} = 500\%$
(to go from a decimal to a percent, just move the decimal point two places to the right)

Use this memory device: PuDdLe DiPpeR
(Percent to Decimal, Left; Decimal to Percent, Right)

To do arithmetic with percents, change to decimals first.

PERCENT INCREASE AND DECREASE:
To find a 30% increase, multiply by 1.3: $x + 0.3x = 1.3x$
To find a 7% increase, multiply by 1.07: $x + 0.07x = 1.07x$
If you have a 20% decrease, only 80% remains, so multiply by 0.8: $x - 0.2x = 0.8x$
If you have a 95% decrease, only 5% remains, so multiply by 0.05: $x - 0.95x = 0.05x$

EXAMPLE:
A price x first increases by 30%, then decreases by 25%.
What is the resulting percent increase/decrease?
new price $= x(1.3)(0.75) = 0.975x$
$1 - 0.975 = 0.025$
it is an overall 2.5% decrease

TRY THESE:
(a) find 5% of 200
(b) convert 7% to a decimal
(c) convert 0.0037 to a percent
(d) Suppose an item decreases by 20%, then increases by 20%. What is the overall percent increase/decrease?

ORDER OF OPERATIONS:

The order that operations are to be performed (when not clearly identified) is summarized with the following memory device:

Please **E**xcuse **M**y **D**ear **A**unt **S**ally (PEMDAS)

Do things inside **P**arentheses first (using PEMDAS, if needed, inside the parentheses).

Then do all **E**xponents, in order as they occur, going from left to right.

Then do all **M**ultiplications/**D**ivisions (they have equal weight) in order as they occur, going from left to right.

Finally, do all **A**dditions/**S**ubtractions (they have equal weight) in order as they occur, going from left to right.

In horizontal fractions, there are implied parentheses in both the numerator and denominator.

EXAMPLES:

$-1 + 3 \times 5 - 2 = -1 + (3 \times 5) - 2 = -1 + 15 - 2 = 12$

$2 - 10 \div 5 + 3 = 2 - \frac{10}{5} + 3 = 3$

$\dfrac{2 \times 3}{7 - 4} = \dfrac{6}{3} = 2$ (Key this into your calculator—correctly!)

TRY THESE:

(a) $3 \times 6 \div 2 - 4 \times 7$

(b) $\dfrac{3 + 5}{11 - 9}$ (do both by hand, and on your calculator)

A **RATIO** is a comparison of two things, that gives information about how the quantities of each relate to each other.

For example, suppose you're told:

"the ratio of **girls** to boys in a given group is **3** to 4"

This information is often written as:

$$\textbf{girls} : \text{boys} = \mathbf{3} : 4$$

This means that for every 3 girls in the group, there are 4 boys.

For example, there might be only 3 girls and 4 boys in the group (a group of size 7).

Or, there might be $2 \cdot 3 = 6$ girls and $2 \cdot 4 = 8$ boys (a group of size 14).

Or, there might be $3 \cdot 3 = 9$ girls and $3 \cdot 4 = 12$ boys (a group of size 21).

Fractions are often used to display and work with ratio information:

$$\frac{3 \text{ girls}}{4 \text{ boys}} = \frac{6 \text{ girls}}{8 \text{ boys}} = \frac{9 \text{ girls}}{12 \text{ boys}} = \cdots$$

Notice that the ratio information can be expressed using lots of different names!

Notice that a ratio does not give any absolute information about the size of a group: does our group of girls and boys have size 7, or 14, or 21, or some other multiple of 7?

Quick sketches are often very useful when working with ratio problems:

TRANSLATING RATIO INFORMATION INTO FRACTIONS: Here are some sample questions that might arise from the previous ratio information, and their answers:

Question: What part (fraction) of the group is girls?

Answer: There are 3 girls in a group of size 7, so $\frac{3}{7}$ of the group is girls.

Or, there are 6 girls in a group of size 14, so $\frac{6}{14} = \frac{3}{7}$ of the group is girls.

Notice that it is easiest to use the smallest numbers to answer this question.

Question: What part (fraction) of the group is boys?

Answer: Since there are 4 boys in a group of size 7, $\frac{4}{7}$ of the group is boys.

TRY THESE:

(a) A recipe calls for 3 cups of flour for every 2 cups of sugar.
If these are the only ingredients, what fraction of the mixture is flour?

(b) What is the ratio of sugar to flour in this mixture?

THE CROSS-MULTIPLYING TECHNIQUE: Suppose that $\frac{a}{b} = \frac{c}{d}$. Multiplying both sides by bd and cancelling gives:

$$\begin{aligned} \frac{a}{b} &= \frac{c}{d} \\ \frac{a}{b} \cdot bd &= \frac{c}{d} \cdot bd \\ ad &= bc \end{aligned}$$

This observation is often called the "cross-multiplying technique':

Cross-multiplying is often useful in solving ratio problems.

Here's a sample problem.

Try it on your own; then, several solution approaches will be discussed.

SAMPLE PROBLEM:

When manufacturing widgets, the ratio of defective to nondefective widgets is 2 to 17. How many defective widgets would you expect if N widgets are being produced?

(A) $\frac{2}{19N}$ (B) $\frac{2N}{17}$ (C) $\frac{2N}{19}$ (D) $N + 2$ (E) $\frac{17N}{2}$

SOLUTION #1: Make a quick sketch, noting the total group size:

So, 2 of every 19 widgets is defective.

That is, $\frac{2}{19}$ of the widgets are defective.

So, in a group of size N, we expect $\frac{2}{19}N$ to be defective.

Notice that $\frac{2}{19}N$ can also be written as $\frac{2}{19}N = \frac{2}{19} \cdot \frac{N}{1} = \frac{2N}{19}$. The correct answer is (C).

SOLUTION #2: Let x denote the number of defective widgets in a group of size N.

The question involves comparing defective widgets to the total number in the group. So, we need a ratio that expresses this information:

$$\frac{2 \text{ defective}}{19 \text{ widgets}} = \frac{x \text{ defective}}{N \text{ widgets}}$$

Cross-multiply and solve for x:

$$19x = 2N$$
$$x = \frac{2N}{19}$$

The correct answer is (C).

SOLUTION #3: Choose a number for N, and check the possible answers.

Take the simplest numbers to work with. Suppose there are 2 defective, 17 nondefective, and a total group size of $2 + 17 = 19$. So, let $N = 19$. (The number of defective in a group this size should be 2.)

Substitute $N = 19$ into (A), giving $\frac{2}{19 \cdot 19}$ which is clearly not correct!

Substitute $N = 19$ into (B), giving $\frac{2 \cdot 19}{17}$, which is not correct.

Substitute $N = 19$ into (C), giving $\frac{2 \cdot 19}{19}$, which is correct!

Stop and record answer (C).

Now, you try these:

1. When manufacturing widgets, the ratio of defective to nondefective widgets is 2 to 17. If there are N defective widgets, how many nondefective widgets do you expect to have?

(A) $\frac{19N}{17}$ (B) $\frac{17N}{2}$ (C) $\frac{2N}{17}$ (D) $\frac{17N}{19}$ (E) $\frac{17N}{2}$

2. When manufacturing widgets, the ratio of defective to nondefective widgets is 2 to 17. If there are N nondefective widgets, how many defective widgets do you expect to have?

(A) $\frac{19N}{17}$ (B) $\frac{17N}{2}$ (C) $\frac{2N}{17}$ (D) $\frac{17N}{19}$ (E) $\frac{17N}{2}$

3. When manufacturing widgets, the ratio of defective to nondefective widgets is 2 to 17. If there are N widgets, how many do you expect to be nondefective?

(A) $\frac{19N}{17}$ (B) $\frac{17N}{2}$ (C) $\frac{2N}{17}$ (D) $\frac{17N}{19}$ (E) $\frac{17N}{2}$

4. When manufacturing widgets, the ratio of defective to nondefective widgets is D to N. If 500 widgets are produced, how many do you expect to be defective?

(A) $\frac{500D}{D+N}$ (B) $\frac{D+N}{500}$ (C) $\frac{500D}{N}$ (D) $\frac{N}{500D}$ (E) $\frac{500N}{D+N}$

5. When manufacturing widgets, the ratio of defective to nondefective widgets is D to N. If 500 widgets are produced, how many do you expect to be nondefective?

(A) $\frac{500D}{D+N}$ (B) $\frac{D+N}{500}$ (C) $\frac{500D}{N}$ (D) $\frac{N}{500D}$ (E) $\frac{500N}{D+N}$

6. When manufacturing widgets, the ratio of defective to nondefective widgets is D to N. If a given group has 70 defective widgets, what is the total size of the group?

(A) $\frac{70(D+N)}{D}$ (B) $\frac{D+N}{70D}$ (C) $70ND$ (D) $70+D+N$ (E) $\frac{70D}{D+N}$

7. In a certain game, 12 players form a team. If a team must have at least one male player for every 3 female players, what is the minimum number of male players on a team?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

8. If $x+1=y$, what is the ratio of x to y?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{5}$ (E) It cannot be determined from the information given.

9. A certain recipe for a 6-pound nut mix requires 4 pounds of peanuts, $\frac{1}{2}$ pound of cashews, and $1\frac{1}{2}$ pounds of almonds. How many pounds of almonds would be required to make 16 pounds of this mixture?

(A) $1\frac{1}{3}$ (B) 4 (C) 6 (D) $10\frac{2}{3}$ (E) $11\frac{1}{2}$

10. In a certain class, $\frac{2}{3}$ of the students take Physics and $\frac{2}{3}$ of those students taking Physics also take Chemistry. What fraction of the class takes Physics but <u>not</u> Chemistry?

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{1}{3}$ (D) $\frac{4}{9}$ (E) $\frac{2}{3}$

11. On a certain diagram, 1 inch represents 10 feet. On this diagram, 1.2 inches represents how many feet?

(A) 120 (B) 12 (C) 10.2 (D) 1.2 (E) 0.12

12. If $\frac{a}{b}=\frac{3}{4}$ and $\frac{b}{c}=\frac{2}{5}$, then $\frac{a}{c}=$

(A) $\frac{3}{10}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{4}{5}$ (E) $\frac{5}{3}$

13. Five bags contain the following proportions of marbles:

	Red Marbles	White Marbles
Bag 1	1	2
Bag 2	5	7
Bag 3	3	6
Bag 4	2	3
Bag 5	3	4

In which bag is the ratio of red marbles to white marbles the greatest?

(A) Bag 1 (B) Bag 2 (C) Bag 3 (D) Bag 4 (E) Bag 5

14. A certain insect crawls at the rate of 0.005 inches per second. At this rate, how many seconds will it take for the insect to travel an inch?

(A) 200 (B) 500 (C) 1000 (D) 2000 (E) 5000

15. George is one-half as old as Tom who is one-half as old as Bill. What is the ratio of Bill's age to George's age?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{1}$ (D) $\frac{2}{1}$ (E) $\frac{4}{1}$

16. A line 120 meters long is divided into two portions in a ratio of $1 : 3$. The longer portion is how many meters longer than the shorter portion?

(A) 30 (B) 40 (C) 60 (D) 80 (E) 90

17. In a certain school there are 100 boys and 200 girls. How many students ride the bus if 20 percent of the boys and 30 percent of the girls ride the school bus?

(A) 60 (B) 70 (C) 80 (D) 90 (E) 100

18. A teacher bought some $4 books and some $2 books for a total of $42. If the teacher purchased three $4 books for every $2 book, how many books did the teacher purchase?

(A) 3 (B) 7 (C) 9 (D) 12 (E) 14