**SATstuff#1**

**MATH CONCEPTS:** SAT Math covers concepts in arithmetic, basic algebra, geometry, and basic algebra II. Let’s get started!

**REAL NUMBERS:** All numbers used on the SAT are *real numbers*. The number line below is a perfect picture of the real numbers. Every point on this line is a real number, and every real number lives somewhere on this line.

- **Numbers Have Lots of Different Names!** There’s only one real number at the position halfway between 0 and 1, but it has lots of different names: for example, $\frac{1}{2}$, 0.5, $\frac{7}{14}$, 50%, $\sqrt{\frac{1}{4}}$, and $\frac{2}{3} - \frac{1}{6}$. Different names are better for different purposes.

**INTEGERS:** The INTEGERS are the numbers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}$. Thus, 107 is an integer, but $\frac{1}{2}$ isn’t. Between any two integers, there are infinitely many real numbers!

**EVEN and ODD integers:**
- **EVEN** numbers are divisible by 2: $\ldots, -4, -2, 0, 2, 4, \ldots$
  - That is, if you divide an even number by 2, the remainder is zero.
  - Even numbers always end in one of these digits: 0, 2, 4, 6, 8.
- **ODD** numbers leave a remainder of 1 when divided by 2: $\ldots, -5, -3, -1, 1, 3, 5, \ldots$

**Properties of ODD and EVEN numbers:**
- even + even = even  
  - Example: $2 + 4 = 6$
- odd + odd = even  
  - Example: $3 + 5 = 8$
- even + odd = odd  
  - Example: $2 + 3 = 5$
- (even)(even) = even  
  - Example: $2 \cdot 4 = 8$
- (odd)(odd) = odd  
  - Example: $3 \cdot 5 = 15$
- (even)(odd) = even  
  - Example: $2 \cdot 3 = 6$

**CONSECUTIVE INTEGERS...** follow one after the other, without any gaps.

- For example, $-1$, 0, 1, and 2 are consecutive integers.
- The numbers 1, 3, and 4 are *not* consecutive integers.
- If $n$ is an integer, then the next couple integers are $n + 1$ and $n + 2$.

**AVERAGE:** If $x$ and $y$ are any two different real numbers, then the average, $\frac{x+y}{2}$, lies exactly halfway between $x$ and $y$.
**POSITIVE and NEGATIVE:** Positive numbers lie to the right of zero \((x > 0)\), and negative numbers lie to the left of zero \((x < 0)\). Zero is the *only* number that isn’t either positive or negative; it’s neutral.

\[
\text{(positive)(positive) = positive} \\
\text{(negative)(negative) = positive} \\
\text{(positive)(negative) = negative}
\]

**OPPOSITE:** The opposite of a number \(x\) is denoted by \(-x\). Opposites are the same distance from zero, but on opposite sides of zero.

The opposite of a positive number is negative: so if \(y > 0\), then \(-y < 0\).

The opposite of a negative number is positive: so if \(x < 0\), then \(-x > 0\).

**BE CAREFUL!** If \(x\) is negative, then its opposite, \(-x\), is positive!

**GREATER THAN, LESS THAN, GREATEST, LEAST:** “Greater than” and “less than” have to do with position on the number line.

If \(x\) is greater than \(y\) (written \(x > y\)), then \(x\) lies to the right of \(y\).

If \(x\) is less than \(y\) (written \(x < y\)), then \(x\) lies to the left of \(y\).

In any collection of numbers, the *greatest* lies farthest to the right; the *least* lies farthest to the left.

**EXAMPLE:** On the number line below:

- \(d\) is the greatest; \(a\) is the least
- \(-a\) and \(-b\) are positive numbers; \(-c\) and \(-d\) are negative numbers
- These are all true: \(a < b\), \(-a > -b\), \(c < d\), \(-d < -c\), \(-d < 0\)

**STANDARD SYMBOLS:**

- \(=\) is equal to
- \(\neq\) is not equal to
- \(>\) is greater than
- \(<\) is less than
- \(\geq\) is greater than or equal to
- \(\leq\) is less than or equal to

**DISTINCT NUMBERS:** Math people use the word *distinct* to mean *different*.

So, 1, 3, and 4 are distinct numbers.

But, 1, 1, and 3 are *not* distinct numbers.

**DIGITS and PLACE VALUE:** There are ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

In our base ten number system, the *position* of a digit determines its contribution to the total value of the number.

For example:

\[732 = 7 \cdot 100 + 3 \cdot 10 + 2 \cdot 1\]

**FACTORS:** (For the discussion of factors, we only consider the numbers 1, 2, 3, \ldots.) The *factors* of a number are the numbers that go into it evenly.

For example, the factors of 10 are 1, 2, 5, and 10.

Every number has 1 and itself as factors.

A number greater than 1 whose *only* factors are itself and 1 is said to be PRIME.

The first few primes are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.
MULTIPLES: The multiples of 2 are 2, 4, 6, 8, 10, 12, and so on. The multiples of 2 are found by taking the number 2, and multiplying successively by 1, 2, 3, \ldots . Notice that 2 goes into each of these numbers evenly. The multiples of 3 are 3, 6, 9, 12, 15, 18, and so on. The multiples of 3 are found by taking the number 3, and multiplying successively by 1, 2, 3, \ldots . Notice that 3 goes into each of these numbers evenly. In general, the multiples of a number $x$ are $x$, $2x$, $3x$, $4x$, and so on. To test if something is a multiple of $x$, just see if $x$ goes into it evenly.
**TIPS & STRATEGIES**

**EASY TO HARD:** All the math questions on the SAT test start off basic and gradually increase in difficulty. If you’re doing a problem near the beginning of a section and it seems easy, then it probably is! If you’re doing a problem near the end of a section and it seems easy, it’s probably really NOT!

**A POINT IS A POINT IS A POINT:** A point earned on a basic question is the same as a point earned on a difficult question. Answer the easy questions first; save harder questions for last. Unless you’re going for a near-perfect score, you shouldn’t even bother to answer the questions at the very end. Focus your attention on questions you have a better chance of getting correct. SLOW DOWN, and SCORE MORE.

**DON’T PUNCH LOTS OF NUMBERS!** A calculator is *allowed* on the SAT, but is not *required*. In other words, you never *need* a calculator to solve any SAT problem. If you find yourself doing lots of computations on your calculator, then STOP: you’re not doing the problem the easiest way, you’re not likely to get the correct answer with what you’re doing, and you’re wasting precious time.

**SAMPLE GRID-IN PROBLEM:** (On a grid-in problem, you write the answer in yourself. More on this type of problem later on.) The sum of all the numbers from 1 to 50 is 1275. What is the sum $2 + 4 + \cdots + 100$?

**WRONG APPROACH:** Don’t pull out your calculator and start adding! Instead, STOP AND THINK. Note that $2(1 + 2 + \cdots + 50) = 2 + 4 + \cdots + 100$. Thus, the desired sum is (use your calculator here, if you want) $2 \cdot 1275 = 2550$.

**KNOW WHAT YOU’RE LOOKING FOR:** Read the problem carefully, and CIRCLE what you’re being asked to find.

**SAMPLE PROBLEM:** Four apples plus a pear cost $1.05. The pear costs 25¢. What is the cost of two apples?

(A) 30¢ (B) 20¢ (C) 15¢ (D) 40¢ (E) 25¢

**WRONG:** It’s too tempting to go like this: $1.05 - 0.25 = 0.80$ and $\frac{80}{4} = 0.20$, so choose (B).

**RIGHT:** The problem asks for the cost of two apples, so the correct answer is (D). If you CIRCLE the words “two apples” as you’re reading the problem, then you’ll help to avoid this type of mistake.

**PICKING NUMBERS:** There’s more than one way to solve a problem. If a problem seems too abstract because of too many $x$’s and $y$’s, make it more concrete by picking numbers.

**SAMPLE PROBLEM:** If $x$ is positive and $y$ is negative, which of the following must be negative?

(A) $x^2$ (B) $y^2$ (C) $x - y$ (D) $y - x$ (E) $(x - y)^2$

**ONE CORRECT APPROACH:** Any number, squared, is positive. So eliminate (A), (B), and (E).

Choose $x = 2$ and $y = -3$. Then, $x - y = 2 - (-3) = 5$ and $y - x = -3 - 2 = -5$. The correct answer is (D).
### Practice Problems

1. If \( x = 1 \) and \( y = -1 \), then which of the following is the greatest?
   - (A) \( y - x \)
   - (B) \( x - y \)
   - (C) \( x^2 - y^2 \)
   - (D) \(-(x + y)\)
   - (E) \((y - x)^2\)

2. \( \left( \frac{1}{5} + \frac{1}{3} \right) \div \frac{1}{2} = \)
   - (A) \( \frac{1}{8} \)
   - (B) \( \frac{1}{4} \)
   - (C) \( \frac{4}{15} \)
   - (D) \( \frac{1}{2} \)
   - (E) \( \frac{16}{15} \)

3. Four plums plus two bananas cost 98¢. A plum costs 17¢. How much would three bananas cost?
   - (A) 34¢
   - (B) 30¢
   - (C) 45¢
   - (D) 51¢
   - (E) 15¢

4. Which of the following is equal to an even number?
   - (A) \( 17 \times 9 \)
   - (B) \( 6 \div 2 \)
   - (C) \( 3^2 \)
   - (D) \( 20 - \frac{6}{2} \)
   - (E) \( 5 + 3 \)

5. \( (3 + 4)^2 = \)
   - (A) \( (2 \times 3) + (2 \times 4) \)
   - (B) \( 3^2 + 4^2 \)
   - (C) \( 5^2 \)
   - (D) \( 7^2 \)
   - (E) \( 3^2 \times 4^2 \)

6. Which of the following is NOT equal to the square of an integer?
   - (A) 1
   - (B) 4
   - (C) 9
   - (D) 16
   - (E) 20

7. If \( x + 7 \) is an even integer, then \( x \) could be which of the following?
   - (A) -2
   - (B) -1
   - (C) 0
   - (D) 2
   - (E) 4

8. Andrea subscribed to four publications that cost $12.90, $16.00, $18.00, and $21.90 per year, respectively. If she made an initial payment of one-half of the total yearly subscription cost, and paid the rest in four equal monthly payments, how much was each of the four monthly payments?
   - (A) $8.60
   - (B) $9.20
   - (C) $9.45
   - (D) $17.20
   - (E) $34.40

**Answers:** E, E, C, E, D, E, B, A