SATstuff#1

Note: There's lots of information about the SAT (Scholastic Aptitude Test) at the COLLEGEBOARD HOMEPAGE:

`http://www.collegeboard.com`

(You can just google `Collegeboard` .)
Click on `Prepare for the SAT` from the homepage.
Then, click on `Mathematics` in the left column.
If you haven't done so yet, go through all their concept review and practice problems.
(HOMEWORK: Try to do this before next week's session.)

INFORMATION ABOUT THE SAT TEST

What is the SAT? The SAT is a 3 hour 45 minute test that measures verbal and mathematical reasoning skills.
It's divided into ten sections:

- an essay section always comes first
- the stuff in the middle can be in any order
- a 10-minute writing section always comes last

One 25-minute section is always *experimental*: there's no way to decide which section this is, so treat every section as if it is scored.

Who uses the SAT? Many colleges and universities use the SAT to assess the readiness of a student to do college-level work. Other indicators used by colleges may include class rank, high school grade point average, extracurricular activities, personal essay, and teacher recommendations.

Who takes the SAT? The test is typically taken by high school juniors and seniors.

When is the SAT given? The schedule is always posted online.
Go to the Collegeboard homepage.
Click on `Register for the SAT` on the bottom left.
Click on `Calendar Dates & Fees` in the left column.
This online SAT prep material:

- reviews every math concept on the SAT
- talks about test-taking tips and strategies
- has lots of sample problems
  (including randomly-generated online practice and printable worksheets with answers)

How is the test scored? Each subject area (Math, Writing, Critical Reading) is scored on a scale of 200 to 800. The three scores are then totaled, for a combined score between 600 and 2400.
The average SAT score is about 500/section (1500 total).

What does the SAT Math subject area look like?

- There are two types of questions:
  – 44 multiple-choice questions
  – 10 student-produced response (grid-in) questions
- There are three scored math sections:
  – two 25-minute sections with multiple-choice and grid-ins
  – one 20-minute section containing only multiple-choice

May I use a calculator?

- You are allowed (but not required) to use a calculator.
- You need to bring your *own* calculator; definitely do this.
  Make sure you bring one that you've used a lot.
- If you don't know how to solve a problem using pencil-and-paper, then you won't be able to solve it with a calculator either. *Don't waste time just punching numbers.*
- When you *do* know how to solve a problem, the calculator can save you computing time.
- Don't waste time using your calculator for things like $3+4$.

What happens if I leave a question blank? An unanswered question doesn't add to or subtract points from your score. (That is, you get 0 points for every question you leave blank.) You earn 1 point for each answer you get correct. If you have *absolutely no idea at all* what to do on a question, then skip it. Usually, however, you can eliminate at least one answer, which brings us to ...

Should I guess on the SAT? You usually won't even attempt all the math problems on the SAT, unless you're shooting for a near-perfect score. Some test prep books include a chart that shows how many problems you should be attempting if you're trying to get a certain score.

However, if you're doing a problem that you *should* be doing, and if you can eliminate one or more answers as definitely wrong, then you should guess. Here are the penalties for wrong answers:

- You lose $\frac{1}{4}$ point for each wrong multiple-choice question.

  Why is this? Well, if you're purely guessing, then on average you'll get one in five questions correct, since there are five multiple-choice answers. You get one point for the correct answer and lose $\frac{1}{4}$ point for each of the four incorrect answers.

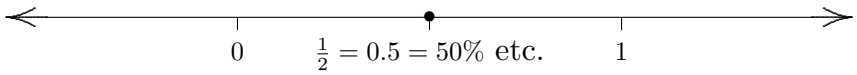
  Thus, your total score on the five questions is: $1 - \frac{1}{4} - \frac{1}{4} - \frac{1}{4} - \frac{1}{4} = 0$.

  The "guessing penalty" is designed to offset any points you may earn purely by guessing.
- You don't lose any points on a wrong Grid-in question.

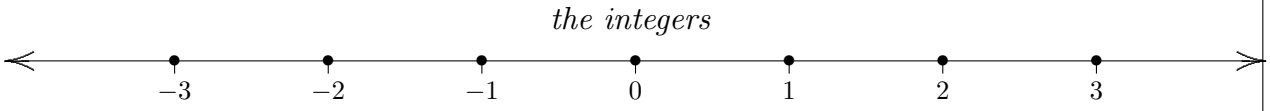
MATH CONCEPTS: SAT Math covers concepts in arithmetic, basic algebra, geometry, and basic algebra II. Let's get started!

REAL NUMBERS: All numbers used on the SAT are *real numbers*. The number line below is a perfect picture of the real numbers. Every point on this line is a real number, and every real number lives somewhere on this line.

NUMBERS HAVE LOTS OF DIFFERENT NAMES! There's only one real number at the position halfway between 0 and 1, but it has lots of different names: for example, $\frac{1}{2}$, 0.5, $\frac{7}{14}$, 50%, $\sqrt{1/4}$, and $\frac{2}{3} - \frac{1}{6}$. Different names are better for different purposes.



INTEGERS: The INTEGERS are the numbers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. Thus, 107 is an integer, but $\frac{1}{2}$ isn't. Between any two integers, there are infinitely many real numbers!

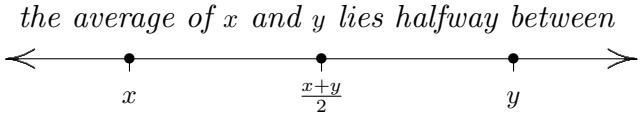


EVEN and ODD integers:
EVEN numbers are divisible by 2: ..., −4, −2, 0, 2, 4, ...
That is, if you divide an even number by 2, the remainder is zero.
Even numbers always end in one of these digits: 0, 2, 4, 6, 8.
ODD numbers leave a remainder of 1 when divided by 2: ..., −5, −3, −1, 1, 3, 5, ...
Properties of ODD and EVEN numbers:

even + even = even	Example: $2 + 4 = 6$
odd + odd = even	Example: $3 + 5 = 8$
even + odd = odd	Example: $2 + 3 = 5$
(even)(even) = even	Example: $2 \cdot 4 = 8$
(odd)(odd) = odd	Example: $3 \cdot 5 = 15$
(even)(odd) = even	Example: $2 \cdot 3 = 6$

CONSECUTIVE INTEGERS... follow one after the other, without any gaps.
For example, −1, 0, 1, and 2 are consecutive integers.
The numbers 1, 3, and 4 are *not* consecutive integers.
If n is an integer, then the next couple integers are $n + 1$ and $n + 2$.

AVERAGE: If x and y are any two different real numbers, then the average, $\frac{x+y}{2}$, lies exactly halfway between x and y.



POSITIVE and NEGATIVE: Positive numbers lie to the right of zero ($x > 0$), and negative numbers lie to the left of zero ($x < 0$). Zero is the *only* number that isn't either positive or negative; it's neutral.

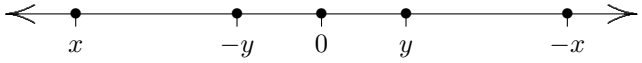
(positive)(positive) = positive
(negative)(negative) = positive
(positive)(negative) = negative

OPPOSITE: The opposite of a number x is denoted by $-x$. Opposites are the same distance from zero, but on opposite sides of zero.

The opposite of a positive number is negative: so if $y > 0$, then $-y < 0$.

The opposite of a negative number is positive: so if $x < 0$, then $-x > 0$.

BE CAREFUL! If x is negative, then its opposite, $-x$, is positive!



GREATER THAN, LESS THAN, GREATEST, LEAST: "Greater than" and "less than" have to do with position on the number line.

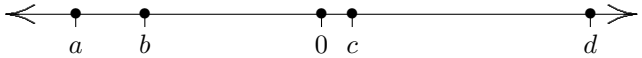
If x is greater than y (written $x > y$), then x lies to the right of y.

If x is less than y (written $x < y$), then x lies to the left of y.

In any collection of numbers, the *greatest* lies farthest to the right; the *least* lies farthest to the left.

EXAMPLE: On the number line below:

- d is the greatest; a is the least
- $-a$ and $-b$ are positive numbers; $-c$ and $-d$ are negative numbers
- These are all true: $a < b$, $-a > -b$, $c < d$, $-d < -c$, $-d < 0$



STANDARD SYMBOLS:

$=$ is equal to
$\neq$ is not equal to
$>$ is greater than
$<$ is less than
$\geq$ is greater than or equal to
$\leq$ is less than or equal to

DISTINCT NUMBERS: Math people use the word *distinct* to mean *different*.
So, 1, 3, and 4 are distinct numbers.
But, 1, 1, and 3 are *not* distinct numbers.

DIGITS and PLACE VALUE: There are ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
In our base ten number system, the *position* of a digit determines its contribution to the total value of the number.
For example:
$732 = 7 \cdot 100 + 3 \cdot 10 + 2 \cdot 1$

FACTORS: (For the discussion of factors, we only consider the numbers 1, 2, 3, ...)
The *factors* of a number are the numbers that go into it evenly.
For example, the factors of 10 are 1, 2, 5, and 10.
Every number has 1 and itself as factors.
A number greater than 1 whose *only* factors are itself and 1 is said to be PRIME.
The first few primes are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

MULTIPLES: The multiples of 2 are 2, 4, 6, 8, 10, 12, and so on.
The multiples of 2 are found by taking the number 2, and multiplying successively by 1, 2, 3, ...
Notice that 2 goes into each of these numbers evenly.

The multiples of 3 are 3, 6, 9, 12, 15, 18, and so on.
The multiples of 3 are found by taking the number 3, and multiplying successively by 1, 2, 3, ...
Notice that 3 goes into each of these numbers evenly.

In general, the multiples of a number x are x, $2x$, $3x$, $4x$, and so on.
To test if something is a multiple of x , just see if x goes into it evenly.

TIPS & STRATEGIES

EASY TO HARD: All the math questions on the SAT test start off basic and gradually increase in difficulty. If you're doing a problem near the beginning of a section and it seems easy, then it probably is! If you're doing a problem near the end of a section and it seems easy, it's probably really NOT!

A POINT IS A POINT IS A POINT: A point earned on a basic question is the same as a point earned on a difficult question. Answer the easy questions first; save harder questions for last. Unless you're going for a near-perfect score, you shouldn't even bother to answer the questions at the very end. Focus your attention on questions you have a better chance of getting correct. SLOW DOWN, and SCORE MORE.

DON'T PUNCH LOTS OF NUMBERS! A calculator is *allowed* on the SAT, but is not *required*. In other words, you never *need* a calculator to solve any SAT problem. If you find yourself doing lots of computations on your calculator, then STOP: you're not doing the problem the easiest way, you're not likely to get the correct answer with what you're doing, and you're wasting precious time.

SAMPLE GRID-IN PROBLEM: (On a grid-in problem, you write the answer in yourself. More on this type of problem later on.) The sum of all the numbers from 1 to 50 is 1275. What is the sum $2+4+\cdots+100$?

WRONG APPROACH: Don't pull out your calculator and start adding! Instead, STOP AND THINK. Note that $2(1+2+\cdots+50)=2+4+\cdots+100$. Thus, the desired sum is (use your calculator here, if you want) $2\cdot 1275=2550$.

KNOW WHAT YOU'RE LOOKING FOR: Read the problem carefully, and CIRCLE what you're being asked to find.

SAMPLE PROBLEM: Four apples plus a pear cost \$1.05. The pear costs 25¢. What is the cost of two apples?

(A) 30¢ (B) 20¢ (C) 15¢ (D) 40¢ (E) 25¢

WRONG: It's too tempting to go like this: $1.05-.25=0.80$ and $\frac{.80}{4}=.20$, so choose (B).

RIGHT: The problem asks for the cost of *two* apples, so the correct answer is (D). If you CIRCLE the words "two apples" as you're reading the problem, then you'll help to avoid this type of mistake.

PICKING NUMBERS: There's more than one way to solve a problem. If a problem seems too abstract because of too many x's and y's, make it more concrete by picking numbers.

SAMPLE PROBLEM: If x is positive and y is negative, which of the following must be negative?

(A) x^2 (B) y^2 (C) $x-y$ (D) $y-x$ (E) $(x-y)^2$

ONE CORRECT APPROACH: Any number, squared, is positive. So eliminate (A), (B), and (E).

Choose $x=2$ and $y=-3$. Then, $x-y=2-(-3)=5$ and $y-x=-3-2=-5$. The correct answer is (D).

PRACTICE PROBLEMS

1. If $x = 1$ and $y = -1$, then which of the following is the greatest?

   (A) $y - x$ (B) $x - y$ (C) $x^2 - y^2$ (D) $-(x + y)$ (E) $(y - x)^2$

2. $\left(\frac{1}{5} + \frac{1}{3}\right) \div \frac{1}{2} =$

   (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{4}{15}$ (D) $\frac{1}{2}$ (E) $\frac{16}{15}$

3. Four plums plus two bananas cost $98¢$. A plum costs $17¢$. How much would three bananas cost?

   (A) $34¢$ (B) $30¢$ (C) $45¢$ (D) $51¢$ (E) $15¢$

4. Which of the following is equal to an even number?

   (A) 17×9 (B) $6 \div 2$ (C) 3^2 (D) $20 - \frac{6}{2}$ (E) $5 + 3$

5. $(3 + 4)^2 =$

   (A) $(2 \times 3) + (2 \times 4)$ (B) $3^2 + 4^2$ (C) 5^2 (D) 7^2 (E) $3^2 \times 4^2$

6. Which of the following is NOT equal to the square of an integer?

   (A) 1 (B) 4 (C) 9 (D) 16 (E) 20

7. If $x + 7$ is an even integer, then x could be which of the following?

   (A) -2 (B) -1 (C) 0 (D) 2 (E) 4

8. Andrea subscribed to four publications that cost \$12.90, \$16.00, \$18.00, and \$21.90 per year, respectively. If she made an initial payment of one-half of the total yearly subscription cost, and paid the rest in four equal monthly payments, how much was each of the four monthly payments?

   (A) \$8.60 (B) \$9.20 (C) \$9.45 (D) \$17.20 (E) \$34.40

ANSWERS: E, E, C, E, D, E, B, A