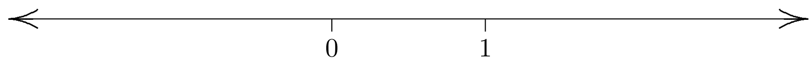
SATstuff#1

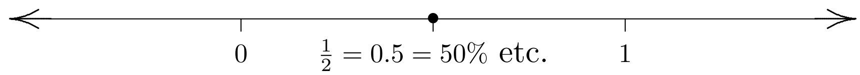
I

MATH CONCEPTS: SAT Math covers concepts in arithmetic, basic algebra, geometry, and basic algebra II. Let's get started!

REAL NUMBERS: All numbers used on the SAT are *real numbers.* The number line below is a perfect picture of the real numbers. Every point on this line is a real number, and every real number lives somewhere on this line.

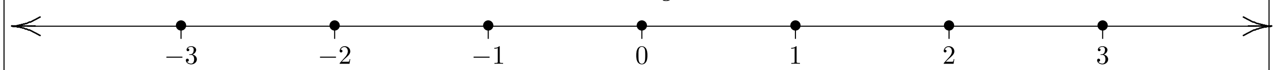


NUMBERS HAVE LOTS OF DIFFERENT NAMES! There's only one real number at the position halfway between 0 and 1, but it has lots of different names: for example, $\frac{1}{2}$, 0.5, $\frac{7}{14}$, 50%, $\sqrt{1/4}$, and $\frac{2}{3} - \frac{1}{6}$. Different names are better for different purposes.



INTEGERS: The INTEGERS are the numbers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. Thus, 107 is an integer, but $\frac{1}{2}$ isn't. Between any two integers, there are infinitely many real numbers!

the integers



EVEN and ODD integers:

EVEN numbers are divisible by 2: $\ldots, -4, -2, 0, 2, 4, \ldots$

That is, if you divide an even number by 2, the remainder is zero.

Even numbers always end in one of these digits: $0, 2, 4, 6, 8$.

ODD numbers leave a remainder of 1 when divided by 2: $\ldots, -5, -3, -1, 1, 3, 5, \ldots$

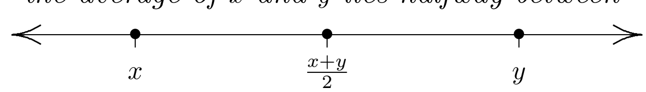
Properties of ODD and EVEN numbers:

even + even = even   Example: $2 + 4 = 6$
odd + odd = even   Example: $3 + 5 = 8$
even + odd = odd   Example: $2 + 3 = 5$
(even)(even) = even   Example: $2 \cdot 4 = 8$
(odd)(odd) = odd   Example: $3 \cdot 5 = 15$
(even)(odd) = even   Example: $2 \cdot 3 = 6$

CONSECUTIVE INTEGERS... follow one after the other, without any gaps.
For example, -1, 0, 1, and 2 are consecutive integers.
The numbers 1, 3, and 4 are *not* consecutive integers.
If n is an integer, then the next couple integers are $n + 1$ and $n + 2$.

AVERAGE: If x and y are any two different real numbers, then the average, $\frac{x+y}{2}$, lies exactly halfway between x and y.

the average of x and y lies halfway between



POSITIVE and NEGATIVE: Positive numbers lie to the right of zero ($x > 0$), and negative numbers lie to the left of zero ($x < 0$). Zero is the *only* number that isn't either positive or negative; it's neutral.

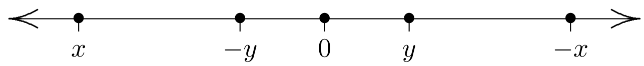
(positive)(positive) = positive
(negative)(negative) = positive
(positive)(negative) = negative

OPPOSITE: The opposite of a number x is denoted by $-x$. Opposites are the same distance from zero, but on opposite sides of zero.

The opposite of a positive number is negative: so if $y > 0$, then $-y < 0$.

The opposite of a negative number is positive: so if $x < 0$, then $-x > 0$.

BE CAREFUL! If x is negative, then its opposite, $-x$, is positive!



GREATER THAN, LESS THAN, GREATEST, LEAST: "Greater than" and "less than" have to do with position on the number line.

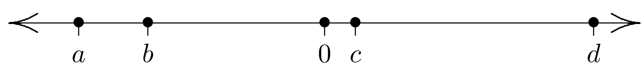
If x is greater than y (written $x > y$), then x lies to the right of y.

If x is less than y (written $x < y$), then x lies to the left of y.

In any collection of numbers, the *greatest* lies farthest to the right; the *least* lies farthest to the left.

EXAMPLE: On the number line below:

- d is the greatest; a is the least
- $-a$ and $-b$ are positive numbers; $-c$ and $-d$ are negative numbers
- These are all true: $a < b$, $-a > -b$, $c < d$, $-d < -c$, $-d < 0$



STANDARD SYMBOLS:
$=$ is equal to
$\neq$ is not equal to
$>$ is greater than
$<$ is less than
$\geq$ is greater than or equal to
$\leq$ is less than or equal to

DISTINCT NUMBERS: Math people use the word *distinct* to mean *different*.
So, 1, 3, and 4 are distinct numbers.
But, 1, 1, and 3 are *not* distinct numbers.

DIGITS and PLACE VALUE: There are ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
In our base ten number system, the *position* of a digit determines its contribution to the total value of the number.
For example:
$732 = 7 \cdot 100 + 3 \cdot 10 + 2 \cdot 1$

FACTORS: (For the discussion of factors, we only consider the numbers 1, 2, 3, ...)
The *factors* of a number are the numbers that go into it evenly.
For example, the factors of 10 are 1, 2, 5, and 10.
Every number has 1 and itself as factors.
A number greater than 1 whose *only* factors are itself and 1 is said to be PRIME.
The first few primes are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

MULTIPLES: The multiples of 2 are 2, 4, 6, 8, 10, 12, and so on.
The multiples of 2 are found by taking the number 2, and multiplying successively by 1, 2, 3, $\ldots$.
Notice that 2 goes into each of these numbers evenly.

The multiples of 3 are 3, 6, 9, 12, 15, 18, and so on.
The multiples of 3 are found by taking the number 3, and multiplying successively by 1, 2, 3, $\ldots$.
Notice that 3 goes into each of these numbers evenly.

In general, the multiples of a number x are x, $2x$, $3x$, $4x$, and so on.
To test if something is a multiple of x , just see if x goes into it evenly.

TIPS & STRATEGIES

EASY TO HARD: All the math questions on the SAT test start off basic and gradually increase in difficulty. If you're doing a problem near the beginning of a section and it seems easy, then it probably is! If you're doing a problem near the end of a section and it seems easy, it's probably really NOT!

A POINT IS A POINT IS A POINT: A point earned on a basic question is the same as a point earned on a difficult question. Answer the easy questions first; save harder questions for last. Unless you're going for a near-perfect score, you shouldn't even bother to answer the questions at the very end. Focus your attention on questions you have a better chance of getting correct. SLOW DOWN, and SCORE MORE.

DON'T PUNCH LOTS OF NUMBERS! A calculator is *allowed* on the SAT, but is not *required*. In other words, you never *need* a calculator to solve any SAT problem. If you find yourself doing lots of computations on your calculator, then STOP: you're not doing the problem the easiest way, you're not likely to get the correct answer with what you're doing, and you're wasting precious time.

SAMPLE GRID-IN PROBLEM: (On a grid-in problem, you write the answer in yourself. More on this type of problem later on.) The sum of all the numbers from 1 to 50 is 1275. What is the sum $2+4+\cdots+100$?

WRONG APPROACH: Don't pull out your calculator and start adding! Instead, STOP AND THINK. Note that $2(1+2+\cdots+50) = 2+4+\cdots+100$. Thus, the desired sum is (use your calculator here, if you want) $2\cdot 1275 = 2550$.

KNOW WHAT YOU'RE LOOKING FOR: Read the problem carefully, and CIRCLE what you're being asked to find.

SAMPLE PROBLEM: Four apples plus a pear cost $1.05. The pear costs 25¢. What is the cost of two apples?

(A) 30¢ (B) 20¢ (C) 15¢ (D) 40¢ (E) 25¢

WRONG: It's too tempting to go like this: $1.05 - .25 = 0.80$ and $\frac{.80}{4} = .20$, so choose (B).

RIGHT: The problem asks for the cost of *two* apples, so the correct answer is (D). If you CIRCLE the words "two apples" as you're reading the problem, then you'll help to avoid this type of mistake.

PICKING NUMBERS: There's more than one way to solve a problem. If a problem seems too abstract because of too many x's and y's, make it more concrete by picking numbers.

SAMPLE PROBLEM: If x is positive and y is negative, which of the following must be negative?

(A) x^2 (B) y^2 (C) $x-y$ (D) $y-x$ (E) $(x-y)^2$

ONE CORRECT APPROACH: Any number, squared, is positive. So eliminate (A), (B), and (E).

Choose $x=2$ and $y=-3$. Then, $x-y = 2-(-3) = 5$ and $y-x = -3-2 = -5$. The correct answer is (D).

PRACTICE PROBLEMS

1. If $x = 1$ and $y = -1$, then which of the following is the greatest?

   (A) $y - x$ (B) $x - y$ (C) $x^2 - y^2$ (D) $-(x + y)$ (E) $(y - x)^2$

2. $(\frac{1}{5} + \frac{1}{3}) \div \frac{1}{2} =$

   (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{4}{15}$ (D) $\frac{1}{2}$ (E) $\frac{16}{15}$

3. Four plums plus two bananas cost 98¢. A plum costs 17¢. How much would three bananas cost?

   (A) 34¢ (B) 30¢ (C) 45¢ (D) 51¢ (E) 15¢

4. Which of the following is equal to an even number?

   (A) 17×9 (B) $6 \div 2$ (C) 3^2 (D) $20 - \frac{6}{2}$ (E) $5 + 3$

5. $(3 + 4)^2 =$

   (A) $(2 \times 3) + (2 \times 4)$ (B) $3^2 + 4^2$ (C) 5^2 (D) 7^2 (E) $3^2 \times 4^2$

6. Which of the following is NOT equal to the square of an integer?

   (A) 1 (B) 4 (C) 9 (D) 16 (E) 20

7. If $x + 7$ is an even integer, then x could be which of the following?

   (A) -2 (B) -1 (C) 0 (D) 2 (E) 4

8. Andrea subscribed to four publications that cost \$12.90, \$16.00, \$18.00, and \$21.90 per year, respectively. If she made an initial payment of one-half of the total yearly subscription cost, and paid the rest in four equal monthly payments, how much was each of the four monthly payments?

   (A) \$8.60 (B) \$9.20 (C) \$9.45 (D) \$17.20 (E) \$34.40

ANSWERS: E, E, C, E, D, E, B, A