

SECTION 7.6 Finding the Volume of a Solid of Revolution—Disks

IN-SECTION EXERCISES:

EXERCISE 2.

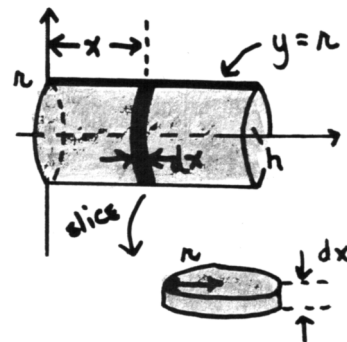
2. A cylinder of height h and radius r is easily generated by taking the graph of $y = r$, and rotating it about the x -axis on the interval $[0, h]$. A typical 'slice' is a disk with volume

$$\pi r^2 dx ;$$

'summing' these disks as x travels from 0 to h yields

$$\int_0^h \pi r^2 dx = \pi r^2 x \Big|_0^h = \pi r^2 h ,$$

which is the desired volume.



END-OF-SECTION EXERCISES:

1.



$$\int_0^1 \pi(2x)^2 dx = \int_0^1 4\pi x^2 dx = 4\pi \frac{x^3}{3} \Big|_0^1 = \frac{4\pi}{3}(1 - 0) = \frac{4\pi}{3}$$

2.

$$\int_1^2 \pi(x^3)^2 dx = \int_1^2 \pi x^6 dx = \pi \frac{x^7}{7} \Big|_1^2 = \frac{\pi}{7}(2^7 - 1) = \frac{127\pi}{7}$$



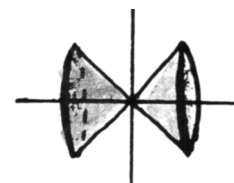
3.



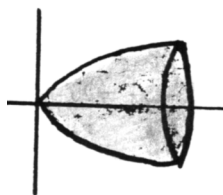
$$\int_1^2 \pi\left(\frac{1}{x}\right)^2 dx = \int_1^2 \pi x^{-2} dx = \pi \cdot \frac{x^{-1}}{-1} \Big|_1^2 = -\pi \cdot \frac{1}{x} \Big|_1^2 = -\pi\left(\frac{1}{2} - 1\right) = \frac{1}{2}\pi$$

4. Take advantage of symmetry; find half the desired volume, then double. For $x \geq 0$, $|x| = x$. The desired volume is:

$$2 \int_0^1 \pi x^2 dx = 2\pi \frac{x^3}{3} \Big|_0^1 = \frac{2\pi}{3}(1 - 0) = \frac{2\pi}{3}$$



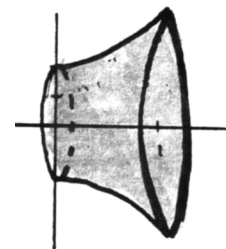
5.



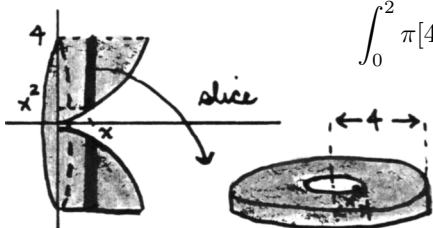
$$\int_0^4 \pi(\sqrt{x})^2 dx = \pi \frac{x^2}{2} \Big|_0^4 = \frac{\pi}{2}(16 - 0) = 8\pi$$

6.

$$\begin{aligned} \int_0^1 \pi(e^x + 1)^2 dx &= \pi \int_0^1 (e^{2x} + 2e^x + 1) dx \\ &= \pi \left(\frac{1}{2}e^{2x} + 2e^x + x \right) \Big|_0^1 \\ &= \pi \left[\left(\frac{1}{2}e^2 + 2e + 1 \right) - \left(\frac{1}{2}e^0 + 2e^0 + 0 \right) \right] \\ &= \pi \left(\frac{1}{2}e^2 + 2e + 1 - \frac{1}{2} - 2 \right) = \pi \left(\frac{1}{2}e^2 + 2e - \frac{3}{2} \right) \end{aligned}$$



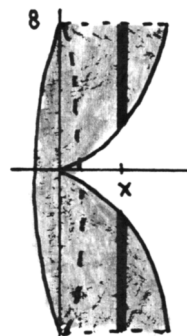
7. intersection point: $4 = x^2 \iff x = \pm 2$. A typical 'slice' has outer radius 4 and inner radius x^2 . The desired volume is:



$$\begin{aligned} \int_0^2 \pi[4^2 - (x^2)^2] dx &= \pi \int_0^2 (16 - x^4) dx \\ &= \pi(16x - \frac{x^5}{5}) \Big|_0^2 \\ &= \pi(32 - \frac{32}{5}) = \frac{128\pi}{5} \end{aligned}$$

8. intersection point: $x^3 = 8 \iff x = 2$. A typical 'slice' has outer radius 8 and inner radius x^3 . The desired volume is:

$$\begin{aligned} \int_0^2 \pi(8^2 - (x^3)^2) dx &= \pi \int_0^2 (64 - x^6) dx \\ &= \pi(64x - \frac{x^7}{7}) \Big|_0^2 \\ &= \frac{768\pi}{7} \end{aligned}$$



9.



$$\int_0^8 \pi y^2 dy = \pi \frac{y^3}{3} \Big|_0^8 = \frac{\pi}{3}(8^3 - 0) = \frac{512\pi}{3}$$

10. Note that $y = 2x \iff x = \frac{y}{2}$. Thus, a typical 'slice at a distance y has radius $\frac{y}{2}$. The desired volume is:

$$\int_1^2 \pi(\frac{y}{2})^2 dy = \frac{\pi}{4} \int_1^2 y^2 dy = \frac{\pi}{4} \cdot \frac{y^3}{3} \Big|_1^2 = \frac{\pi}{12}(8 - 1) = \frac{7\pi}{12}$$



11. Note that $y = \frac{1}{x} \iff x = \frac{1}{y}$. A typical 'slice' at a distance y has outer radius $\frac{1}{y}$ and inner radius $\frac{1}{2}$. The desired volume is:

$$\begin{aligned} \int_1^2 \pi((\frac{1}{y})^2 - (\frac{1}{2})^2) dy &= \pi \int_1^2 (y^{-2} - \frac{1}{4}) dy \\ &= \pi(\frac{y^{-1}}{-1} - \frac{1}{4}y) \Big|_1^2 \\ &= \pi(-\frac{1}{y} - \frac{1}{4}y) \Big|_1^2 \\ &= \pi[(-\frac{1}{2} - \frac{2}{4}) - (-1 - \frac{1}{4})] \\ &= \pi[(-1) - (-\frac{5}{4})] = \pi(-\frac{4}{4} + \frac{5}{4}) = \frac{\pi}{4} \end{aligned}$$

