SECTION 7.4 The Substitution Technique applied to Definite Integrals

IN-SECTION EXERCISES:

EXERCISE 1.

First solution:

$u = 3x^2 - 1$

$du = 6x\,dx$

$$\begin{aligned}\int x(3x^2-1)^5\,dx &= \frac{1}{6}\int (3x^2-1)^5\,6x\,dx \\ &= \frac{1}{6}\int u^5\,du = \frac{1}{6}\cdot\frac{u^6}{6} + C \\ &= \frac{1}{36}(3x^2-1)^6 + C\end{aligned}$$

Then:

$$\int_0^1 x(3x^2-1)^5\,dx = \frac{1}{36}(3x^2-1)^6\,\Big|_0^1 = \frac{1}{36}(2^6-1) = \frac{63}{36} = \frac{21}{12} = \frac{7}{4}$$

Second solution:

$u = 3x^2 - 1$

$du = 6x\,dx$

$x = 0 \Rightarrow u = -1;\quad x = 1 \Rightarrow u = 2$

$$\begin{aligned}\int_0^1 x(3x^2-1)^5\,dx &= \frac{1}{6}\int_0^1 (3x^2-1)^5\,6x\,dx \\ &= \frac{1}{6}\int_{-1}^2 u^5\,du = \frac{1}{6}\cdot\frac{u^6}{6}\,\Big|_{-1}^2 \\ &= \frac{1}{36}(2^6-1) = \frac{7}{4}\end{aligned}$$

Third Solution:

$u = 3x^2 - 1$

$du = 6x\,dx$

$$\begin{aligned}\int_0^1 x(3x^2-1)^5\,dx &= \frac{1}{6}\int_0^1 (3x^2-1)^5\,6x\,dx \\ &= \frac{1}{6}\int_{u(0)}^{u(1)} u^5\,du = \frac{1}{6}\cdot\frac{u^6}{6}\,\Big|_{u(0)}^{u(1)} \\ &= \frac{1}{36}(3x^2-1)^6\,\Big|_0^1 = \frac{1}{36}(2^6-1) = \frac{7}{4}\end{aligned}$$

EXERCISE 2.

$u = x \qquad dv = e^x\,dx$

$du = dx \qquad v = e^x$

$$\begin{aligned}\int_0^1 xe^x\,dx &= xe^x\,\Big|_0^1 - \int_0^1 e^x\,dx \\ &= (1e^1 - 0) - e^x\,\Big|_0^1 \\ &= e - (e^1 - e^0) = e - e + 1 = 1\end{aligned}$$

END-OF-SECTION EXERCISES:

1.

$u = 1 + x^2$

$du = 2x\,dx$

$x = -1 \Rightarrow u = 2;\quad x = 1 \Rightarrow u = 2$

$$\begin{aligned}\int_{-1}^1 x\sqrt{1+x^2}\,dx &= \frac{1}{2}\int_{-1}^1 (1+x^2)^{1/2}\,2x\,dx \\ &= \frac{1}{2}\int_2^2 u^{1/2}\,du = 0\end{aligned}$$

It was not necessary to find an antiderivative, since $\int_c^c f(x)\,dx$ always equals zero.

2.

$u = 3x + 4$
$du = 3dx$

$x = 0 \Rightarrow u = 3 \cdot 0 + 4 = 4$
$x = 3 \Rightarrow u = 3 \cdot 3 + 4 = 13$

$$\begin{aligned}\int_0^3 \frac{2}{3x+4}\,dx &= \frac{2}{3}\int_0^3 \frac{1}{3x+4}\,3\,dx \\ &= \frac{2}{3}\int_4^{13} \frac{1}{u}\,du \\ &= \frac{2}{3}\ln|u|\ \Big|_4^{13} \\ &= \frac{2}{3}(\ln 13 - \ln 4) \approx 0.786\end{aligned}$$

3.

$u = 5 - t$
$du = -dt$

$t = 1 \Rightarrow u = 5 - 1 = 4$
$t = 2 \Rightarrow u = 5 - 2 = 3$

$$\begin{aligned}\int_1^2 \frac{1}{(5-t)^3}\,dt &= -\int_1^2 (5-t)^{-3}(-1)\,dt \\ &= -\int_4^3 u^{-3}\,du \\ &= -\frac{u^{-2}}{-2}\ \Big|_4^3 \\ &= \frac{1}{2}\cdot\frac{1}{u^2}\ \Big|_4^3 = \frac{1}{2}(\frac{1}{9} - \frac{1}{16}) \\ &\approx 0.024\end{aligned}$$

4.

$u = \ln 3x$ $\quad dv = dx$
$du = \frac{1}{3x} \cdot 3\,dx$ $\quad v = x$
$= \frac{1}{x}\,dx$

$$\begin{aligned}\int_1^3 \ln 3x\,dx &= x\ln 3x\ \Big|_1^3 - \int_1^3 x\cdot\frac{1}{x}\,dx \\ &= (3\ln 9 - 1\ln 3) - x\ \Big|_1^3 \\ &= (3\ln 3^2 - \ln 3) - (3-1) \\ &= (6\ln 3 - \ln 3) - 2 = 5\ln 3 - 2 \approx 3.493\end{aligned}$$

5. First, compute $\int_2^3 \ln(x-1)\,dx$; at the last step, multiply by 5. Do not approximate until the final step.

$u = \ln(x-1)$ $\quad dv = dx$
$du = \frac{1}{x-1}\,dx$ $\quad v = x - 1$

$$\begin{aligned}\int_2^3 \ln(x-1)\,dx &= (x-1)\ln(x-1)\ \Big|_2^3 - \int_2^3 (x-1)\frac{1}{x-1}\,dx \\ &= [2\ln 2 - (1)\ln 1] - \int_2^3 (1)\,dx \\ &= 2\ln 2 - x\ \Big|_2^3 = 2\ln 2 - (3-2) = 2\ln 2 - 1\end{aligned}$$

Then:

$$\int_2^3 5\ln(x-1)\,dx = 5\int_2^3 \ln(x-1)\,dx = 5(2\ln 2 - 1) = 10\ln 2 - 5 \approx 1.931$$