

**SECTION 7.4 The Substitution Technique applied to Definite Integrals**

IN-SECTION EXERCISES:

EXERCISE 1.

First solution:

$$\begin{aligned} \int x(3x^2 - 1)^5 dx &= \frac{1}{6} \int (3x^2 - 1)^5 6x dx \\ \mu &= 3x^2 - 1 & &= \frac{1}{6} \int u^5 du = \frac{1}{6} \cdot \frac{u^6}{6} + C \\ d\mu &= 6x dx & &= \frac{1}{36} (3x^2 - 1)^6 + C \end{aligned}$$

Then:

$$\int_0^1 x(3x^2 - 1)^5 dx = \frac{1}{36} (3x^2 - 1)^6 \Big|_0^1 = \frac{1}{36} (2^6 - 1) = \frac{63}{36} = \frac{21}{12} = \frac{7}{4}$$

Second solution:

$$\begin{aligned} \mu &= 3x^2 - 1 & & \int_0^1 x(3x^2 - 1)^5 dx = \frac{1}{6} \int_0^1 (3x^2 - 1)^5 6x dx \\ d\mu &= 6x dx & &= \frac{1}{6} \int_{-1}^2 u^5 du = \frac{1}{6} \cdot \frac{u^6}{6} \Big|_{-1}^2 \\ x=0 \Rightarrow \mu &= -1; \quad x=1 \Rightarrow \mu = 2 & &= \frac{1}{36} (2^6 - 1) = \frac{7}{4} \end{aligned}$$

Third Solution:

$$\begin{aligned} \int_0^1 x(3x^2 - 1)^5 dx &= \frac{1}{6} \int_0^1 (3x^2 - 1)^5 6x dx \\ \mu &= 3x^2 - 1 & &= \frac{1}{6} \int_{u(0)}^{u(1)} u^5 du = \frac{1}{6} \cdot \frac{u^6}{6} \Big|_{u(0)}^{u(1)} \\ d\mu &= 6x dx & &= \frac{1}{36} (3x^2 - 1)^6 \Big|_0^1 = \frac{1}{36} (2^6 - 1) = \frac{7}{4} \end{aligned}$$

EXERCISE 2.

$$\begin{aligned} \mu &= x & & \int_0^1 xe^x dx = xe^x \Big|_0^1 - \int_0^1 e^x dx \\ d\mu &= dx & &= (1e^1 - 0) - e^x \Big|_0^1 \\ & & &= e - (e^1 - e^0) = e - e + 1 = 1 \end{aligned}$$

END-OF-SECTION EXERCISES:

1.

$$\begin{aligned} \mu &= 1 + x^2 & & \int_{-1}^1 x\sqrt{1+x^2} dx = \frac{1}{2} \int_{-1}^1 (1+x^2)^{1/2} 2x dx \\ d\mu &= 2x dx & &= \frac{1}{2} \int_2^2 u^{1/2} du = 0 \\ x=-1 \Rightarrow \mu &= 2; \quad x=1 \Rightarrow \mu = 2 \end{aligned}$$

It was not necessary to find an antiderivative, since  $\int_c^c f(x) dx$  always equals zero.

2.

$$\begin{aligned} \int_0^3 \frac{2}{3x+4} dx &= \frac{2}{3} \int_0^3 \frac{1}{3x+4} 3 dx \\ &= \frac{2}{3} \int_4^{13} \frac{1}{u} du \\ &= \frac{2}{3} \ln|u| \Big|_4^{13} \\ &= \frac{2}{3} (\ln 13 - \ln 4) \approx 0.786 \end{aligned}$$

$u = 3x + 4$   
 $du = 3 dx$   
 $x = 0 \Rightarrow u = 3 \cdot 0 + 4 = 4$   
 $x = 3 \Rightarrow u = 3 \cdot 3 + 4 = 13$

3.

$$\begin{aligned} \int_1^2 \frac{1}{(5-t)^3} dt &= - \int_1^2 (5-t)^{-3} (-1) dt \\ &= - \int_4^3 u^{-3} du \\ &= - \frac{u^{-2}}{-2} \Big|_4^3 \\ &= \frac{1}{2} \cdot \frac{1}{u^2} \Big|_4^3 = \frac{1}{2} \left( \frac{1}{9} - \frac{1}{16} \right) \\ &\approx 0.024 \end{aligned}$$

$u = 5 - t$   
 $du = - dt$   
 $t = 1 \Rightarrow u = 5 - 1 = 4$   
 $t = 2 \Rightarrow u = 5 - 2 = 3$

4.

$$\begin{aligned} \int_1^3 \ln 3x dx &= x \ln 3x \Big|_1^3 - \int_1^3 x \cdot \frac{1}{x} dx \\ &= (3 \ln 9 - 1 \ln 3) - x \Big|_1^3 \\ &= (3 \ln 3^2 - \ln 3) - (3 - 1) \\ &= (6 \ln 3 - \ln 3) - 2 = 5 \ln 3 - 2 \approx 3.493 \end{aligned}$$

$u = \ln 3x$   
 $du = \frac{1}{3x} \cdot 3 dx$   
 $= \frac{1}{x} dx$   
 $dv = dx$   
 $v = x$

5. First, compute  $\int_2^3 \ln(x-1) dx$ ; at the last step, multiply by 5. Do not approximate until the final step.

$$\begin{aligned} \int_2^3 \ln(x-1) dx &= (x-1) \ln(x-1) \Big|_2^3 - \int_2^3 (x-1) \frac{1}{x-1} dx \\ &= [2 \ln 2 - (1) \ln 1] - \int_2^3 (1) dx \\ &= 2 \ln 2 - x \Big|_2^3 = 2 \ln 2 - (3 - 2) = 2 \ln 2 - 1 \end{aligned}$$

$u = \ln(x-1)$   
 $du = \frac{1}{x-1} dx$   
 $dv = dx$   
 $v = x-1$

Then:

$$\int_2^3 5 \ln(x-1) dx = 5 \int_2^3 \ln(x-1) dx = 5(2 \ln 2 - 1) = 10 \ln 2 - 5 \approx 1.931$$