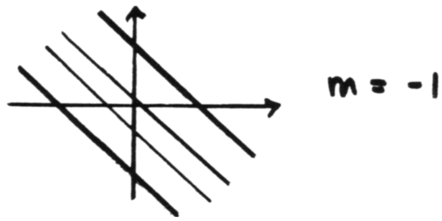
SECTION 6.1 Antiderivatives

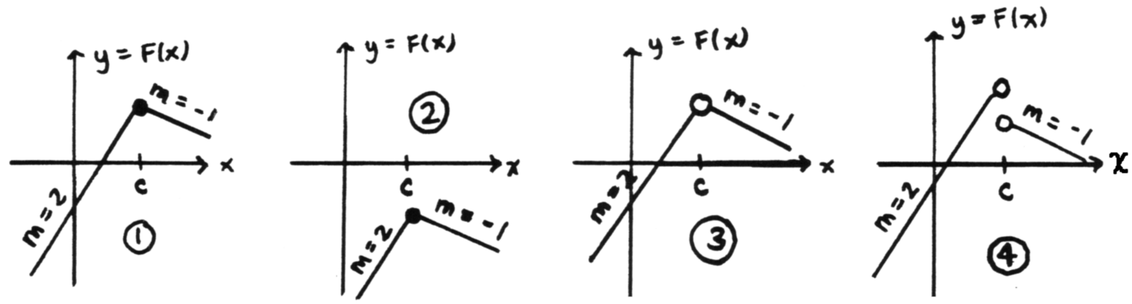
IN-SECTION EXERCISES:

EXERCISE 1.

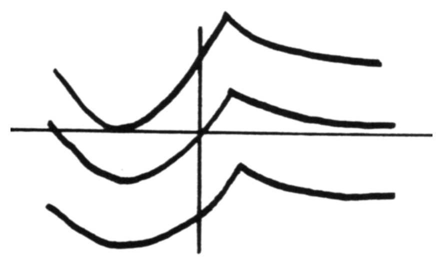
F must be a line with slope -1; thus, $F(x) = -x + C$, for any real number C.



EXERCISE 2.



EXERCISE 3.



EXERCISE 4.

1. $\int 3x^2\,dx = x^3 + C$
2. $\int 2y\,dy = y^2 + C$
3. $\int e^t\,dt = e^t + C$
4. $\int 2e^{2x}\,dx = e^{2x} + C$
5. $\int \frac{1}{x}\,dx = \ln x + C$

EXERCISE 5.

Write $\ln 2x + K = \ln x + (\ln 2 + K) = \ln x + C$. Thus, every function of the form $\ln 2x + K$ is also of the form $\ln x + C$.

EXERCISE 6.

1. The phrase '*the linearity of differentiation*' refers to the fact that the derivative of a sum is the sum of the derivatives, and constants can be 'slid out' of the differentiation process.

   If you go on to take additional mathematics courses, you will learn that a function f that satisfies both

$$f(x+y) = f(x) + f(y) \text{ for all } x, y \in \mathcal{D}(f)$$

and

$$f(kx) = k \cdot f(x) \text{ for all } k \in \mathbb{R} \text{ and } x \in \mathcal{D}(f)$$

   is called a *linear function*. Do these properties look familiar?

2. Linearity was used in going from

$$\frac{d}{dx}(x^2 + 3x^{1/2}) \ \text{ to } \ \frac{d}{dx}x^2 + \frac{d}{dx}3x^{1/2} \ ;$$

the derivative of a sum is the sum of the derivatives.
Also, linearity was used in going from

$$\frac{d}{dx}3x^{1/2} \ \text{ to } \ 3\frac{d}{dx}x^{1/2} \ ;$$

the constant 3 was 'slid out' of the differentiation process.

EXERCISE 7.

1. Let $F(x)$ be any antiderivative of $f(x)$. Then, $kF(x)$ is an antiderivative of $kf(x)$, since

$$\frac{d}{dx}(kF(x)) = k\frac{d}{dx}F(x) = kF'(x) = kf(x) \ .$$

Thus,

$$\int kf(x)\,dx = kF(x) + C \ .$$

Also,

$$k\int f(x)\,dx = k(F(x) + K) = kF(x) + (kK) = kF(x) + C \ .$$

Compare!

2. Since $\frac{d}{dx}(\frac{1}{3}x^3) = x^2$,

$$\int x^2\,dx = \frac{1}{3}x^3 + C \ .$$

Since $\frac{d}{dx}(\frac{1}{2}x^2) = x$, $\int x\,dx = \frac{1}{2}x^2 + C$. Thus,

$$x\int x\,dx = x(\frac{1}{2}x^2 + C) = \frac{1}{2}x^3 + Cx \ .$$

The two integrals do NOT agree! Thus, *variables CANNOT be 'slid out' of an integral.*

EXERCISE 8.

$$\begin{aligned}
\int 2x - 3\,dx &= \int 2x\,dx + \int (-3)\,dx && \text{(linearity of the integral)}\\
&= \int 2x\,dx - \int 3\,dx && \text{('slide out' the constant } -1\text{)}\\
&= (x^2 + C_1) - (3x + C_2) && \text{(find antiderivatives)}\\
&= x^2 - 3x + (C_1 - C_2) && \text{(regroup)}\\
&= x^2 - 3x + C && \text{(rename arbitrary constant)}
\end{aligned}$$

EXERCISE 9.

1. Assume that $x > 0$, so that $\ln x$ is defined. Then,

$$\int (\frac{1}{x} + e^x - 1)\,dx = \ln x + e^x - x + C \ .$$

2.

$$\begin{aligned}\int \frac{3-t}{t}\,dt &= \int (\frac{3}{t} - 1)\,dt \\ &= 3\ln t - t + C\end{aligned}$$

3. Assume that $x > 2$, so that $\ln(x-2)$ is defined. Then,

$$\int \frac{1}{x-2}\,dx = \ln(x-2) + C\ .$$

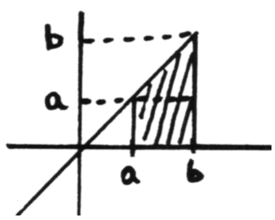
4.

$$\begin{aligned}\int \frac{1}{3x-5}\,dx &= \int \frac{1}{3(x-\frac{5}{3})}\,dx \\ &= \frac{1}{3}\int \frac{1}{x-\frac{5}{3}}\,dx \\ &= \frac{1}{3}\ln(x-\frac{5}{3}) + C\end{aligned}$$

5.

$$\begin{aligned}\int (x+1)^2\,dx &= \int (x^2+2x+1)\,dx \\ &= \frac{x^3}{3} + x^2 + x + C\end{aligned}$$

EXERCISE 10.



3. AREA $= \frac{1}{2}(b-a)(a+b)$
4. $F(b) = \frac{b^2}{2}$ and $F(a) = \frac{a^2}{2}$. Thus:

$$\begin{aligned}F(b) - F(a) &= \frac{b^2}{2} - \frac{a^2}{2} \\ &= \frac{1}{2}(b^2 - a^2) \\ &= \frac{1}{2}(b-a)(b+a)\end{aligned}$$

Compare!

END-OF-SECTION EXERCISES:
1. EXP
2. SEN; CONDITIONAL. The truth depends on the choice of function F.
3. EXP
4. EXP
5. SEN; CONDITIONAL. The truth depends on the choice of functions f and F.
6. SEN; TRUE
7. SEN; TRUE
8. SEN; TRUE. The integral of a sum is the sum of the integrals.

9. SEN; TRUE. It is assumed (by normal mathematical conventions) that k is a real number. The sentence states that constants can be 'slid out' of the integration process.
10. SEN; TRUE. To evaluate the indefinite integral, an antiderivative of $f'(x)$ is needed; but $f(x)$ is an antiderivative of $f'(x)$! Then, all other antiderivatives must differ by at most a constant.
11. Let the altitude be denoted by h; the bases by A and B, as shown. Then:

$$\begin{aligned}\text{AREA} &= Ah + \frac{1}{2}h(B-A) = h(A + \frac{1}{2}(B-A)) \\ &= h(A + \frac{B}{2} - \frac{A}{2}) = h(\frac{A}{2} + \frac{B}{2}) \\ &= \frac{1}{2}h(A+B)\end{aligned}$$