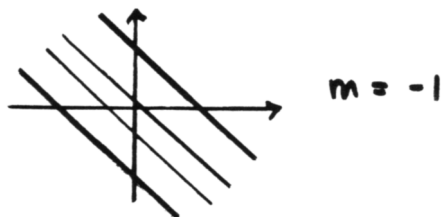


SECTION 6.1 Antiderivatives

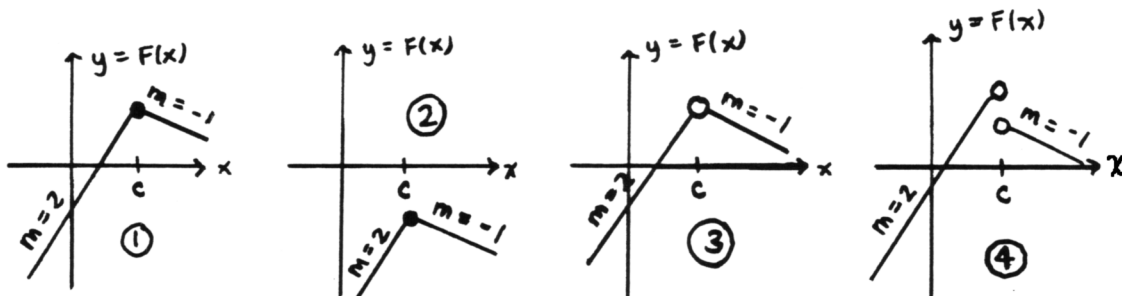
IN-SECTION EXERCISES:

EXERCISE 1.

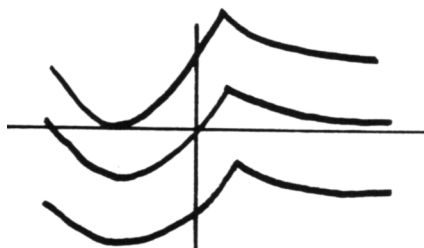
F must be a line with slope -1 ; thus, $F(x) = -x + C$, for any real number C .



EXERCISE 2.



EXERCISE 3.



EXERCISE 4.

1. $\int 3x^2 dx = x^3 + C$
2. $\int 2y dy = y^2 + C$
3. $\int e^t dt = e^t + C$
4. $\int 2e^{2x} dx = e^{2x} + C$
5. $\int \frac{1}{x} dx = \ln x + C$

EXERCISE 5.

Write $\ln 2x + K = \ln x + (\ln 2 + K) = \ln x + C$. Thus, every function of the form $\ln 2x + K$ is also of the form $\ln x + C$.

EXERCISE 6.

1. The phrase ‘the linearity of differentiation’ refers to the fact that the derivative of a sum is the sum of the derivatives, and constants can be ‘slid out’ of the differentiation process.

If you go on to take additional mathematics courses, you will learn that a function f that satisfies both

$$f(x + y) = f(x) + f(y) \text{ for all } x, y \in \mathcal{D}(f)$$

and

$$f(kx) = k \cdot f(x) \text{ for all } k \in \mathbb{R} \text{ and } x \in \mathcal{D}(f)$$

is called a *linear function*. Do these properties look familiar?

2. Linearity was used in going from

$$\frac{d}{dx}(x^2 + 3x^{1/2}) \text{ to } \frac{d}{dx}x^2 + \frac{d}{dx}3x^{1/2};$$

the derivative of a sum is the sum of the derivatives.

Also, linearity was used in going from

$$\frac{d}{dx}3x^{1/2} \text{ to } 3\frac{d}{dx}x^{1/2};$$

the constant 3 was 'slid out' of the differentiation process.

EXERCISE 7.

1. Let $F(x)$ be any antiderivative of $f(x)$. Then, $kF(x)$ is an antiderivative of $kf(x)$, since:

$$\frac{d}{dx}(kF(x)) = k\frac{d}{dx}F(x) = kF'(x) = kf(x)$$

Thus:

$$\int kf(x) dx = kF(x) + C$$

Also:

$$k \int f(x) dx = k(F(x) + K) = kF(x) + (kK) = kF(x) + C$$

Compare!

2. Since $\frac{d}{dx}(\frac{1}{3}x^3) = x^2$:

$$\int x^2 dx = \frac{1}{3}x^3 + C$$

Since $\frac{d}{dx}(\frac{1}{2}x^2) = x$, $\int x dx = \frac{1}{2}x^2 + C$. Thus:

$$x \int x dx = x(\frac{1}{2}x^2 + C) = \frac{1}{2}x^3 + Cx$$

The two integrals do NOT agree! Thus, *variables CANNOT be 'slid out' of an integral.*

EXERCISE 8.

$$\begin{aligned} \int 2x - 3 dx &= \int 2x dx + \int (-3) dx && \text{(linearity of the integral)} \\ &= \int 2x dx - \int 3 dx && \text{('slide out' the constant } -1) \\ &= (x^2 + C_1) - (3x + C_2) && \text{(find antiderivatives)} \\ &= x^2 - 3x + (C_1 - C_2) && \text{(regroup)} \\ &= x^2 - 3x + C && \text{(rename arbitrary constant)} \end{aligned}$$

EXERCISE 9.

1. Assume that $x > 0$, so that $\ln x$ is defined. Then:

$$\int \left(\frac{1}{x} + e^x - 1\right) dx = \ln x + e^x - x + C$$

2.

$$\begin{aligned}\int \frac{3-t}{t} dt &= \int \left(\frac{3}{t} - 1\right) dt \\ &= 3 \ln t - t + C\end{aligned}$$

3. Assume that $x > 2$, so that $\ln(x-2)$ is defined. Then:

$$\int \frac{1}{x-2} dx = \ln(x-2) + C$$

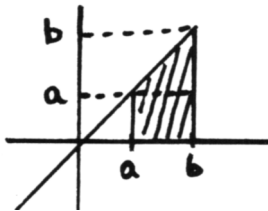
4.

$$\begin{aligned}\int \frac{1}{3x-5} dx &= \int \frac{1}{3\left(x-\frac{5}{3}\right)} dx \\ &= \frac{1}{3} \int \frac{1}{x-\frac{5}{3}} dx \\ &= \frac{1}{3} \ln\left(x-\frac{5}{3}\right) + C\end{aligned}$$

5.

$$\begin{aligned}\int (x+1)^2 dx &= \int (x^2 + 2x + 1) dx \\ &= \frac{x^3}{3} + x^2 + x + C\end{aligned}$$

EXERCISE 10.



3. $\text{AREA} = \frac{1}{2}(b-a)(a+b)$

4. $F(b) = \frac{b^2}{2}$ and $F(a) = \frac{a^2}{2}$. Thus:

$$\begin{aligned}F(b) - F(a) &= \frac{b^2}{2} - \frac{a^2}{2} \\ &= \frac{1}{2}(b^2 - a^2) \\ &= \frac{1}{2}(b-a)(b+a)\end{aligned}$$

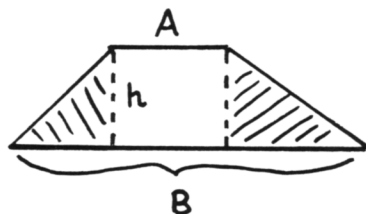
Compare!

END-OF-SECTION EXERCISES:

1. EXP
2. SEN; CONDITIONAL. The truth depends on the choice of function F .
3. EXP
4. EXP
5. SEN; CONDITIONAL. The truth depends on the choice of functions f and F .
6. SEN; TRUE

7. SEN; TRUE
8. SEN; TRUE. The integral of a sum is the sum of the integrals.
9. SEN; TRUE. It is assumed (by normal mathematical conventions) that k is a real number. The sentence states that constants can be 'slid out' of the integration process.
10. SEN; TRUE. To evaluate the indefinite integral, an antiderivative of $f'(x)$ is needed; but $f(x)$ is an antiderivative of $f'(x)$! Then, all other antiderivatives must differ by at most a constant.
11. Let the altitude be denoted by h ; the bases by A and B , as shown. Then:

$$\begin{aligned} \text{AREA} &= Ah + \frac{1}{2}h(B - A) = h\left(A + \frac{1}{2}(B - A)\right) \\ &= h\left(A + \frac{B}{2} - \frac{A}{2}\right) = h\left(\frac{A}{2} + \frac{B}{2}\right) \\ &= \frac{1}{2}h(A + B) \end{aligned}$$



$$= \begin{array}{c} \text{A} \\ \square \\ h \\ \text{AREA} \\ = Ah \end{array} + \begin{array}{c} \text{A} \\ \triangle \\ h \\ \text{B-A} \\ \text{AREA} = \frac{1}{2}(B-A)h \end{array}$$