

SECTION 5.6 Asymptotes—Checking Behavior at Infinity

IN-SECTION EXERCISES:

EXERCISE 1.

The limit statement

$$\lim_{x \rightarrow c^-} f(x) = \infty$$

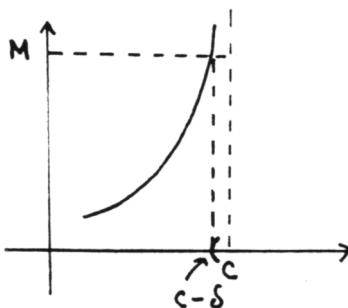
means that $f(x)$ can be made as large and positive as desired, by requiring that x be sufficiently close to c (and less than c).

Precisely:

$$\lim_{x \rightarrow c^-} f(x) = \infty \iff \forall M > 0, \exists \delta > 0 \text{ such that if } x \in (c - \delta, c), \text{ then } f(x) > M$$

The sentence $\lim_{x \rightarrow c^-} f(x) = \infty$ can also be written:

As $x \rightarrow c^-$, $f(x) \rightarrow \infty$



EXERCISE 2.

The limit statement

$$\lim_{x \rightarrow c^+} f(x) = -\infty$$

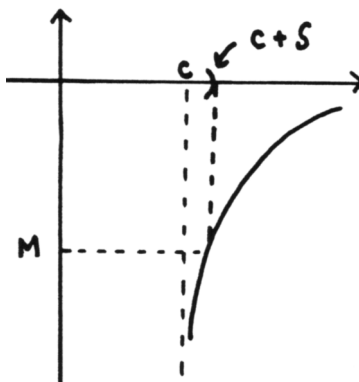
means that $f(x)$ can be made as large and negative as desired, by requiring that x be sufficiently close to c (and greater than c).

Precisely:

$$\lim_{x \rightarrow c^+} f(x) = -\infty \iff \forall M < 0, \exists \delta > 0 \text{ such that if } x \in (c, c + \delta), \text{ then } f(x) < M$$

The sentence $\lim_{x \rightarrow c^+} f(x) = -\infty$ can also be written:

As $x \rightarrow c^+$, $f(x) \rightarrow -\infty$



EXERCISE 3.

PRECISE SOLUTION:

Multiply by 1 in an appropriate form:

$$f(x) = \frac{5x^3}{2x(x^2 - 1)} = \frac{5x^3}{2x^3 - 2x} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \frac{5}{2 - \frac{2}{x^2}}$$

Then:

$$\lim_{x \rightarrow \pm\infty} f(x) = \lim_{x \rightarrow \pm\infty} \frac{5}{2 - \frac{2}{x^2}} = \frac{\lim_{x \rightarrow \pm\infty} 5}{\lim_{x \rightarrow \pm\infty} (2 - \frac{2}{x^2})} = \frac{5}{2}$$

Thus, $y = \frac{5}{2}$ is a horizontal asymptote for f .

ABBREVIATED SOLUTION:

For large values of x :

$$f(x) \approx \frac{5x^3}{2x^3} = \frac{5}{2}$$

Thus, $y = \frac{5}{2}$ is a horizontal asymptote for f .

EXERCISE 4.

The limit statement

$$\lim_{x \rightarrow -\infty} f(x) = L$$

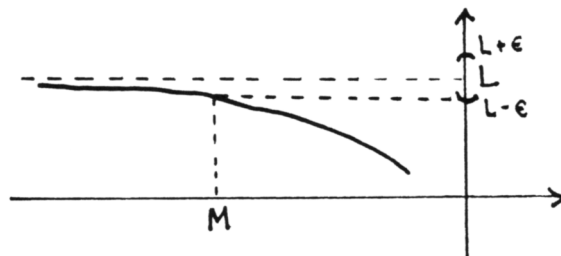
means that the numbers $f(x)$ can be made as close to L as desired, by requiring that x be sufficiently large and negative.

Precisely:

$$\lim_{x \rightarrow -\infty} f(x) = L \iff \forall \epsilon > 0, \exists M < 0 \text{ such that if } x < M, \text{ then } |f(x) - L| < \epsilon$$

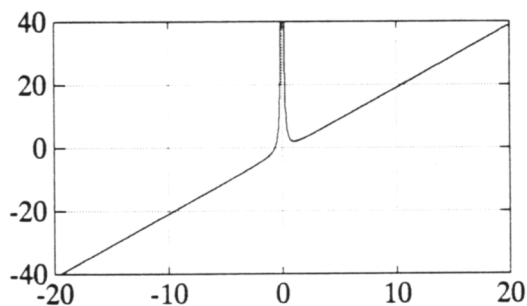
The sentence $\lim_{x \rightarrow -\infty} f(x) = L$ can also be written:

As $x \rightarrow -\infty$, $f(x) \rightarrow L$

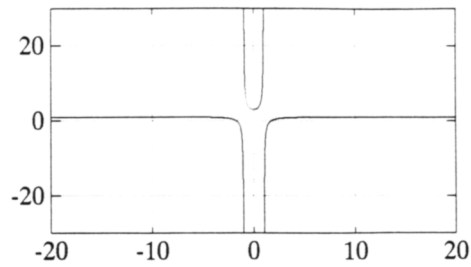


EXERCISE 5.

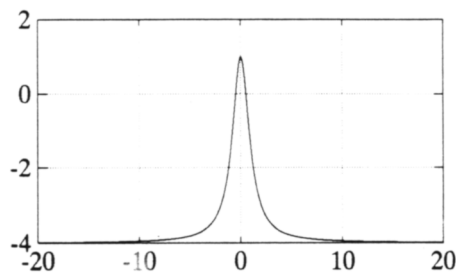
1. vertical asymptote: $x = 0$
oblique asymptote: $y = 2x - 1$



2. vertical asymptotes: $x = 1$ and $x = -1$
horizontal asymptote: $y = 1$



3. horizontal asymptote: $y = -4$



EXERCISE 6.

Note first that $\mathcal{D}(f) = \mathbb{R} - \{1, -1\}$. However, (in the same breath), you must notice that the numerator is also zero when $x = 1$ and when $x = -1$. Indeed:

$$x^3 + 2x^2 - x - 2 = (x - 1)(x + 1)(x + 2)$$

Thus, for $x \neq 1$ and $x \neq -1$:

$$f(x) = \frac{(x - 1)(x + 1)(x + 2)}{(x - 1)(x + 1)} = x + 2$$

The graph of f is the same as the line $y = x + 2$, punctured at $x = 1$ and $x = -1$.

