SECTION 3.4 Continuity

IN-SECTION EXERCISES:

EXERCISE 1.

1. a) $\lim_{x \to 2} (x^2 - x + 1) = 2^2 - 2 + 1 = 3$

   b) $\lim_{x \to \pi} (x^2 - x + 1) = \pi^2 - \pi + 1$

   c) $\lim_{x \to b} (x^2 - x + 1) = b^2 - b + 1$

   d) $\lim_{x \to n} (x^2 - x + 1) = n^2 - n + 1$

   e) $\lim_{x \to d} (ax^2 + bx + c) = ad^2 + bd + c$

2. There are many correct answers. Choose, say, $f(x) = x^2$ and $c = \sqrt{2}$. Then, $\lim_{x \to \sqrt{2}} x^2 = (\sqrt{2})^2 = 2$.

EXERCISE 2.

1. The function f has removable discontinuities at $x = 1$ and $x = 9$. Both of these discontinuities could be easily 'removed' by appropriately changing f at a single point.

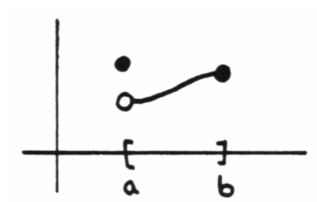
   The function f has nonremovable discontinuities at $x = -1$, $x = 3$, and $x = 7$. 'Patching up' any of these discontinuities would require major reconstruction of the function.

2. The discontinuity at $x = 1$ can be removed by defining $f(1) := 2$. The discontinuity at $x = 9$ can be removed by redefining $f(9) := 2$.

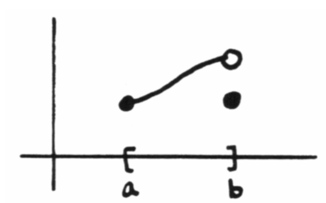
EXERCISE 3.

There are many possible correct graphs for 1–4:

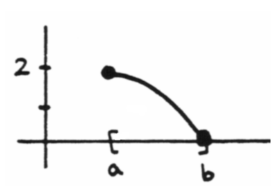
1.



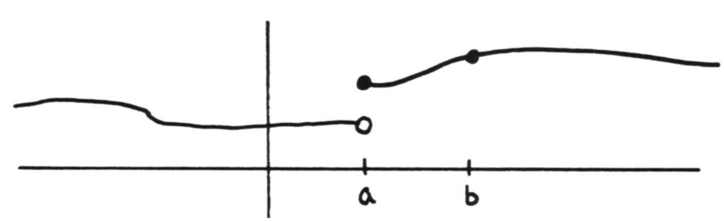
2.



3.



4.



EXERCISE 4.

Suppose that f and g are both continuous at c, and $g(c) \neq 0$. Then:

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}$$

EXERCISE 5.

1. If g is continuous at 3, and f is continuous at $g(3) = 9$, then $f \circ g$ will be continuous at 3.
2. Under the conditions cited above:

$$\lim_{x \to 3} f(g(x)) = f(g(3)) = f(9) = 2$$

END-OF-SECTION EXERCISES:

1. SEN; CONDITIONAL. The truth of this sentence depends upon the choices made for the function f and constant c.
2. SEN; CONDITIONAL. The truth depends upon the choices made for the function f and constant c.
3. EXP. This is the name of the output of a function f, when the input is c.
4. EXP. If the limit exists, it represents the number that $f(x)$ gets close to, as x gets close to c.
5. SEN; CONDITIONAL. This sentence is true if f is continuous at c.
6. SEN; TRUE. Here, c is assumed to be a real number. Every polynomial P is defined on $\mathbb{R}$, and is continuous everywhere.
7. SEN; CONDITIONAL. The truth of this sentence depends upon the choice of function f.
8. SEN; TRUE. The truth of this sentence is a consequence of the definition of a nonremovable discontinuity.
9. SEN; CONDITIONAL. The truth depends on the choice of function f and interval $[a, b]$.
10. SEN; TRUE
11. SEN; TRUE
12. SEN; TRUE
13. SEN; FALSE. The interval $(1, 3]$ is not open, but is also not closed.
14. SEN; FALSE. The interval $(1, 3]$ is not closed, but is also not open.
15. EXP. Out of context, it is not known if this is a POINT (a, b), or an open interval of real numbers. In either case, however, it is an EXPRESSION.
16. SEN; FALSE. As long as $a < b$, the interval $(a, b]$ is not open, and is not closed.