SECTION 3.1 Limits—The Idea

IN-SECTION EXERCISES:

EXERCISE 1.

1. $\lim_{x\to 3} 2x = 2(3) = 6$

   $\lim_{x\to 0} 2x = 2(0) = 0$

   $\lim_{x\to \pi} 2x = 2\pi$

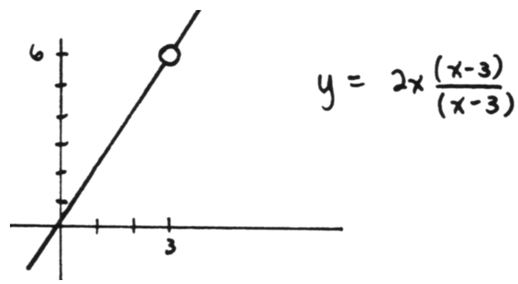
   $\lim_{x\to \frac{2}{3}} 2x = 2(\frac{2}{3}) = \frac{4}{3}$

   $\lim_{x\to -10.1} 2x = 2(-10.1) = -20.2$

2. $\lim_{x\to c} 2x = 2c$

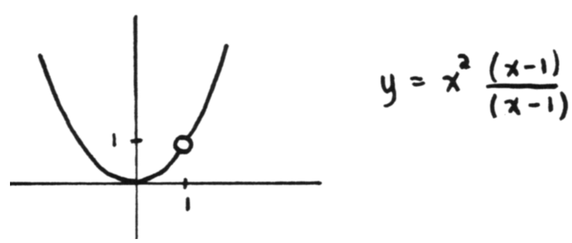
EXERCISE 2.

1. $\lim_{x\to 3} 2x\frac{(x-3)}{(x-3)} = 2(3) = 6$



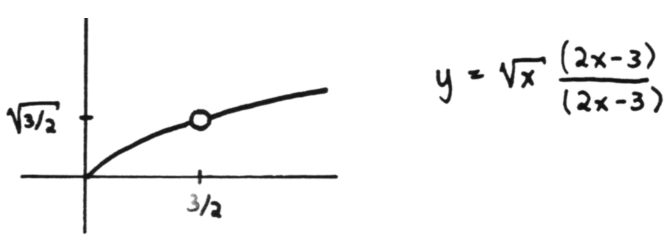
2. $\lim_{x\to 2} 2x\frac{(x-3)}{(x-3)} = 2(2) = 4;$ see the sketch above.

3. $\lim_{x\to 1} x^2\frac{(x-1)}{(x-1)} = 1^2 = 1$



4. $\lim_{x\to 0} x^2\frac{(x-1)}{(x-1)} = 0^2 = 0$

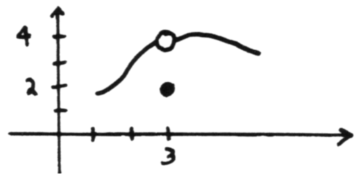
5. $\lim_{x\to \frac{3}{2}} \sqrt{x}\frac{(2x-3)}{(2x-3)} = \sqrt{3/2}$



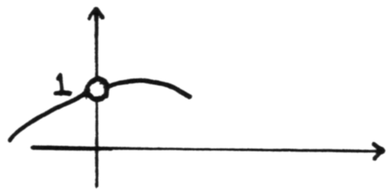
EXERCISE 3.

There are many possible correct graphs.

1. When x is close to 3, $f(x)$ must be close to 4; and the point $(3, 2)$ must be on the graph of f.

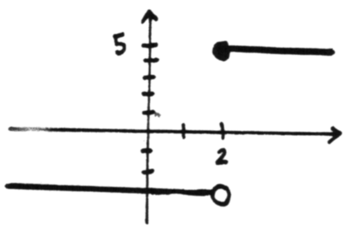


2. When x is close to 0, $f(x)$ must be close to 1; and the function is not defined when $x = 0$.



EXERCISE 4.

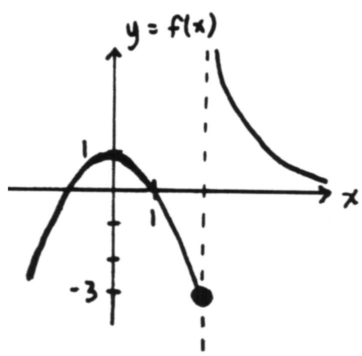
1.



2. a) $g(2) = 5$
   b) $g(1.9782) = -3$
   c) $g(\pi) = 5$
   d) $\lim_{x\to 2} g(x)$ does not exist
   e) $\lim_{x\to 3} g(x) = 5$
   f) $\lim_{x\to 1.99999} g(x) = -3$
   g) $\lim_{z\to 0} g(z) = -3$; when z is close to 0, $f(z)$ is close to -3
   h) $\lim_{y\to \pi} g(y) = 5$
3. if $c > 2$, then $\lim_{x\to c} g(x) = 5$

EXERCISE 5.

1.

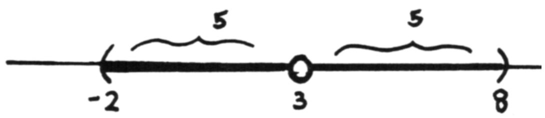


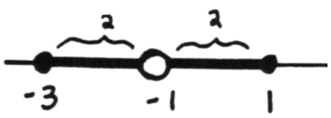
2. $\mathcal{D}(f) = \mathbb{R}$
3. a) $f(2) = 1 - 2^2 = 1 - 4 = -3$
   b) $\lim_{x\to 2} f(x)$ does not exist
   c) for $c > 100$, $f(c) = \frac{1}{c-2}$
   d) for $t < 0$, $f(t) = 1 - t^2$
   e) $\lim_{t\to \pi+1} f(t) = \frac{1}{(\pi+1)-2} = \frac{1}{\pi-1}$
   f) $\lim_{\omega\to 0} f(\omega) = 1 - 0^2 = 1$

EXERCISE 6.

1. Using the variable x: $|x - 4| < 2$
2. Using the variable t: $|t - (-1)| > 5$, that is, $|t + 1| > 5$
3. Using the variable y: $|y - \pi| \geq \delta$

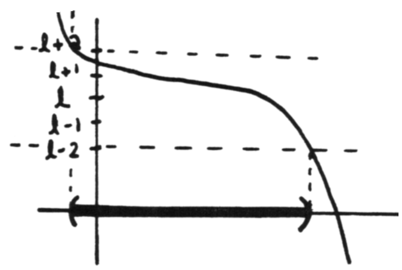
EXERCISE 7.

1. The set shown consists of all numbers whose distance from 2 is less than or equal to 3; using the variable t, $|t - 2| \leq 3$ has the solution set shown.
2. The set shown consists of all numbers that are not equal to 2, and whose distance from 2 is less than 2. Using the variable x, $0 < |x - 2| < 2$ has the solution set shown.
3. 

4. 

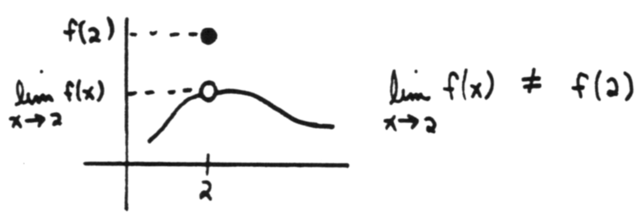
EXERCISE 8.

The set $\{x : |f(x) - l| < 2\}$ is the set of all x-values whose corresponding function values are within 2 of l. These x-values are shaded on the graph below.



END-OF-SECTION EXERCISES:

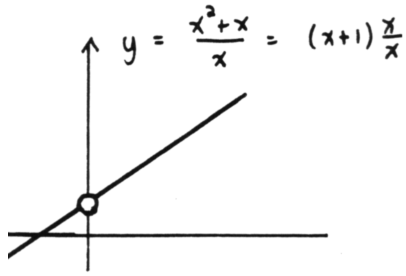
1. EXP; this is another name for the number 3
2. SEN; T. When x is close to 1, $3x$ is close to 3.
3. SEN; T. When t is close to 0, t^2 is also close to 0.
4. EXP; this is another name for the number 0.
5. SEN; C. The truth depends upon the function f and the constants c and l. It is TRUE only if the function f has the property that when x is close to c, $f(x)$ is close to l.
6. SEN; T. When x is close to 1, $2x$ is close to 2.
7. SEN; T. When t is close to 0, $2t + 1$ is close to 1.
8. SEN; C. The truth of this sentence depends upon the function f. A given function f might not even be defined at 2, in which case $f(2)$ doesn't make sense. Or, f may be defined at 2, but when x is close to 2, $f(x)$ need not be close to $f(2)$.



9. SEN; C. See the answer to (8).

10. SEN; T

$$\begin{aligned}\lim_{x\to 0}\frac{x^2+x}{x} &= \lim_{x\to 0}\frac{x(x+1)}{x}\\ &= 0+1=1\end{aligned}$$



The graph of $f(x)=\frac{x^2+x}{x}$ is the same as the graph of $y=x+1$, except it is punctured at $x=0$.

11. EXP; for given numbers x and y, this expression yields the distance between x and y.

12. SEN; C. It is true only for numbers x whose distance from 1 is less than or equal to 2.

13. SEN; (always) T. No matter what real numbers are chosen for x and y, the distance from x to y is the same as the distance from y to x.

14. SEN; (always) T. Putting in an extra step:

$$\begin{aligned}|-2x-2y| &= |-2(x+y)|\\ &= |-2|\cdot|x+y|\\ &= 2|x+y|\end{aligned}$$

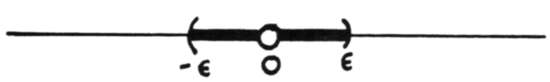
15. SEN; (always) T

16. SEN; C. For example, if a and b are both positive, the sentence is true. However, taking $a=1$ and $b=-1$, the sentence is false: $|a+b|=|1+(-1)|=0$, but $|a|+|b|=|1|+|-1|=1+1=2$.

17. SEN; C. The truth depends on the choices made for a and b. For example, if a and b are both positive numbers with $a>b$, then the sentence is true. However, taking $a=5$ and $b=-7$, the sentence is false: $|a-b|=|5-(-7)|=|12|=12$, but $|a|-|b|=|5|-|-7|=5-7=-2$.

18. SEN; T. The two sentences being compared always have the same truth values. The sentence '$|x|>0$' is true for all nonzero real numbers x; the sentence '$x\in(-\infty,0)\cup(0,\infty)$' is also true for all nonzero real numbers x. These are just two different ways of giving the same information!

19. SEN; T. If ϵ is a positive number, then the sentences '$0<|x|<\epsilon$' and '$x\in(-\epsilon,0)\cup(0,\epsilon)$' always have the same truth values. They are both true for the real numbers shown below, and false elsewhere:



20. SEN; T. As long as a and b are both positive with $a<b$, the sentences being compared always have the same truth values; they are interchangeable. They both represent the real numbers whose distance from 0 is between a and b. They are both true for the numbers x shown below, and false elsewhere: