SECTION 1.3 Sets and Set Notation

IN-SECTION EXERCISES:

EXERCISE 1.

The set S has four elements: $2,\ \ \pi,\ \ \{2,\pi\},\ \ \{2\}$. Thus, two of the elements are numbers, and two of the elements are sets.

EXERCISE 2.

1. $\{-3,-2,-1,0,1,2,3\} = \{n \in \mathbb{Z} \,|\, -3 \le n \le 3\} = \{j \in \mathbb{Z} \,|\, j > -4 \text{ and } j < 4\}$; there are many other possibilities
2. $\{\dots,-1,0,1,2\} = \{n \in \mathbb{Z} \,|\, j \le 2\} = \{j \in \mathbb{Z} \,|\, j < 2.5\}$; there are many other possibilities

EXERCISE 3.

In order, the interval notation for the sets is: $[-1,4),\ \ (-0.1,\infty),\ \ (-\infty,3]$

EXERCISE 4.

1. DEFINITION. The *solution set of an inequality* is the set of all its solutions. To *solve an inequality* means to find its solution set.
2. a) $x > 4$ has solution set $(4,\infty)$

   b) $y^2 \ge 0$ has solution set $(-\infty,\infty)$. This is because *all* real numbers, when squared, are greater than or equal to zero.

   c) $t^2 < 0$ has an empty solution set. There are no real numbers which, when squared, are less than zero.

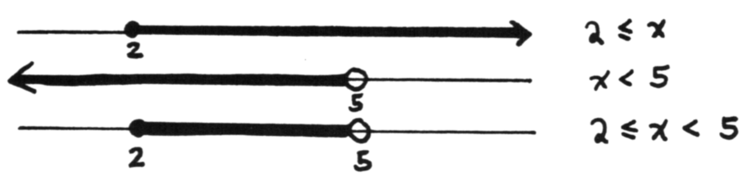
   d) $|x| \le 1$ has solution set $[-1,1]$. Here, we seek all real numbers whose distance from zero is less than or equal to 1.
3. Inequalities usually have an infinite number of solutions (some interval of real numbers).

EXERCISE 5.

1. T
2. T
3. F, since '$-2 < -4$' is false
4. F, since '$1 \in (1,3)$' is false
5. F; here, both component sentences are false
6. The sentence '$x > 0$ and $x > 2$' is true for $x \in (2,\infty)$ and false elsewhere.
7. The sentence '$x > 0$ and $x < 2$' is true for $x \in (0,2)$ and false elsewhere.
8. The sentence '$x > 0$ and $x < -2$' is false for all real numbers.

EXERCISE 6.

1.



2. The inequality $3 < x < 1$ is shorthand for '$3 < x$ and $x < 1$'. There are no real numbers that are both greater than 3, and (at the same time) less than 1. Thus, compound inequalities $a < x < b$ where a is greater than b don't make much sense.

EXERCISE 7.

positive integers $= \{1,2,3,\dots\}$

negative integers $= \{-1,-2,-3,\dots\}$

nonnegative integers $= \{0,1,2,3,\dots\}$

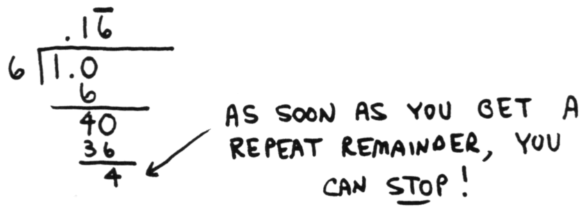
nonpositive integers $= \{0,-1,-2,-3,\dots\}$

EXERCISE 8.

1. Long division shows that: $\frac{1}{6} = 0.1\overline{6}$

   Long division shows that: $\frac{2}{7} = 0.\overline{285714}$

   Long division shows that $\frac{1}{25} = 0.04$. Alternately: $\frac{1}{25} = \frac{1}{25} \cdot \frac{4}{4} = \frac{4}{100} = 0.04$



2. If a fraction is in reduced form, and the denominator has *only* factors of 2's and 5's, then we can always multiply by 1 in an appropriate form to get the *same number* of 2's and 5's downstairs. These are then grouped together to get powers of 10 .
3. $\frac{3}{120} = \frac{1}{40} = \frac{1}{2 \cdot 2 \cdot 2 \cdot 5}$; so, $\frac{3}{120}$ has a finite decimal expansion
4. The number 41 is prime, and is not a factor of 333; therefore $\frac{41}{333}$ is in reduced form. The number 333 has factors other than 2's and 5's (for example, it has a factor of 3). Thus, $\frac{41}{333}$ has an infinite decimal expansion.
5. $\frac{10}{81} = \frac{2 \cdot 5}{3 \cdot 3 \cdot 3 \cdot 3}$; thus, we see that the fraction is in reduced form, and the denominator has factors other than 2 and 5. Therefore, $\frac{10}{81}$ has an infinite decimal expansion.

END-OF-SECTION EXERCISES:

1. EXP; this is a set
2. EXP; this is a set
3. SEN, T
4. SEN, T
5. SEN, F
6. SEN, T (since $\frac{1}{2} < \frac{5}{9}$)
7. SEN, C. The truth of this sentence depends upon the set S and the element x.
8. SEN, F
9. SEN, C. The truth depends on x. If x is 1, 2, or 3, then the sentence is true. Otherwise, it is false.
10. SEN, T
11. EXP; this is a set
12. SEN, F. The set $\{\ \{1\}\ ,\ \{1, \{1\}\}\ \}$ has only two elements, and they are both sets: $\{1\}$ and $\{1, \{1\}\}$
13. SEN, T. See (12).
14. SEN, F. Regardless of what number is chosen for x, this sentence is false, since a number cannot be simultaneously greater than 1 and less than 1 .
15. SEN, C. The only number that makes this true is 1 .
16. SEN, F. No number is simultaneously greater than 5, and less than or equal to 3 .
17. SEN, T. No matter what real number is chosen for x, both component sentences '$|x| \geq 0$' and '$x^2 \geq 0$' are true.
18. SEN, C. This sentence is true for all real number except 0 ; it is false when x is 0 .
19. SEN, T. The two elements are both sets: $\{1\}$ and $\{1, \{2\}\}$
20. SEN, F. The set has only two elements, which are both sets: $\{a\}$ and $\{b, c\}$
21. SEN, F. The number $\frac{3}{7}$ is in reduced form; the denominator has factors other than 2's and 5's.
22. SEN, T; $\frac{7}{35} = \frac{1}{5} = \frac{2}{10} = 0.2$