CHAPTER 6. ANTIDIFFERENTIATION

Section 6.1 Antiderivatives

Quick Quiz:

1. The graph of f is a line with slope 2. Thus, $f(x) = 2x + C$, for some constant C.
2. Specifying the derivative of a function completely determines its SHAPE.
3. $\int 2\,dt = 2t + C$
4. The antiderivatives of a function can be used to find the area trapped between the graph of the function and the x-axis.
5. The phrase refers to the facts that the derivative of a sum is the sum of the derivatives; and constants can be 'slid out' of the differentiation process.

END-OF-SECTION EXERCISES:

1. EXP
3. EXP
5. SEN; CONDITIONAL
7. SEN; TRUE
9. SEN; TRUE

Section 6.2

Quick Quiz:

1. The 'counterpart' is:

$$\int ke^{kx}\,dx = e^{kx} + C$$

A more useful version of this formula is found as follows:

$$\int ke^{kx}\,dx = e^{kx} + C \quad \iff k\int e^{kx}\,dx = e^{kx} + C \iff \quad \int e^{kx}\,dx = \frac{1}{k}e^{kx} + K$$

2. Rewrite the integrand, and use the Simple Power Rule:

$$\int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} + C = \frac{2}{3}\sqrt{x^3} + C$$

3. $$\int \frac{1}{2t}\,dt = \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln|t| + C$$

4. If $f'(x) = x$, then antidifferentiating yields $f(x) = \frac{x^2}{2} + C$. Then:

$$f(0) = 3 \quad \iff \quad 0 + C = 3 \quad \iff \quad C = 3$$

Take: $f(x) = \frac{x^2}{2} + 3$

Section 6.3

Quick Quiz:

1. 'Speed' has only magnitude (size); 'velocity' has both magnitude and direction.
2. Position at $t = 1$: $d(1) = 1^2 + 2(1) = 3$ feet

   $v(t) = d'(t) = 2t + 2$; Velocity at $t = 1$: $v(1) = 2(1) + 2 = 4$ feet/second

   Speed at time $t = 1$: $|v(1)| = |4| = 4$ feet/second

   $a(t) = v'(t) = 2$; Acceleration at $t = 1$: $a(1) = 2$ feet/second^2

3. A ‘vector’ is a mathematical object that is completely described by two pieces of information: a magnitude (size), and a direction. Vectors are conveniently represented by arrows.
4. A *free-body diagram* is a picture that illustrates the forces acting on an object.
5. ‘$v(2)$’ means the velocity function, *acting on* the input 2; this is function notation. However, ‘$g(2)$’ means the constant g, *times* the number 2. Context is important!

END-OF-SECTION EXERCISES:

1. If ‘down’ is chosen as the positive direction, and ‘ 0 ’ coincides with the ground, then: $d(t) = g\frac{t^2}{2} - 20t - 75$
3. approximately 0.63 seconds
5. approximately 1.25 seconds

Section 6.4

Quick Quiz:

1. With appropriate renaming, transform a difficult integration problem into one that is easier to handle. Solve the ‘new’ integral, then transform the solution back into a solution of the original problem.
2. Substitution:

$$\boxed{\begin{aligned} u &= 2x-1 \\ du &= 2\,dx \end{aligned}}$$

$$\begin{aligned} \int \frac{1}{2x-1}\,dx &= \frac{1}{2}\int \frac{1}{2x-1}2\,dx \\ &= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C \\ &= \frac{1}{2}\ln|2x-1| + C \end{aligned}$$

Without substitution:

$$\int \frac{1}{2x-1}\,dx = \int \frac{1}{2(x-\frac{1}{2})}\,dx = \frac{1}{2}\int \frac{1}{x-\frac{1}{2}}\,dx = \frac{1}{2}\ln|x-\frac{1}{2}| + C$$

To see that the answers differ by only a constant, write:

$$\frac{1}{2}\ln|2x-1| = \frac{1}{2}\ln|2(x-\frac{1}{2})| = \frac{1}{2}[\ln 2 + \ln|x-\frac{1}{2}|] = \frac{1}{2}\ln 2 + \frac{1}{2}\ln|x-\frac{1}{2}|$$

Thus, the two answers differ only by the constant $\frac{1}{2}\ln 2$.

3. After multiplying by ‘1’ in an appropriate form, the linearity of the integral is used to ‘pull’ the unwanted constant part out of the integral.
4. $$\int e^{3x}\,dx = \frac{1}{3}\int e^{3x}3\,dx = \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C = \frac{1}{3}e^{3x} + C$$

$$\boxed{\begin{aligned} u &= 3x \\ du &= 3dx \end{aligned}}$$

5. We need only check if $\frac{(3x+\pi)^6}{18}$ is an antiderivative of $(3x+\pi)^5$:

$$\frac{d}{dx}\Big(\frac{(3x+\pi)^6}{18}\Big) = \frac{1}{18}(6)(3x+\pi)^5 \cdot (3) = (3x+\pi)^5$$

Thus, it IS true that: $\int (3x+\pi)^5\,dx = \frac{(3x+\pi)^6}{18} + C$

END-OF-SECTION EXERCISES:

1. $\frac{1}{36}(2x-1)^{18} + C$
3. $\frac{3}{2}(\ln 4x)^2 + C$
5. $2e^{\sqrt{x}} + C$
7. $\frac{-4}{\sqrt{t^2+t+1}} + C$

9. $f(x) = \dfrac{(e^x + 1)^4}{4}$

Section 6.5

Quick Quiz:

1. In general, integration is harder than differentiation.
2. $\boxed{\begin{array}{l} u = 2 + x; \quad x = u - 2 \\ du = dx \end{array}}$

$$\int \frac{x}{2+x}\,dx = \int \frac{u-2}{u}\,du = \int 1 - \frac{2}{u}\,du = u - 2\ln|u| + C$$
$$= (2+x) - 2\ln|2+x| + C = x - 2\ln|2+x| + K$$

3. There are extensive tables of integrals, and computer programs that can do symbolic integration.

END-OF-SECTION EXERCISES:

1. $\frac{1}{5}(\frac{1}{2}e^{2x} + x) + C$
3. $\frac{3}{16}\sqrt[3]{(4t^2 - 1)^2} + C$
5. $\frac{(x^2 - 1)^4}{8} + C$
7. $\frac{(\ln x)^4}{12} + C$

Section 6.6

Quick Quiz:

1. The Integration By Parts formula is:

$$\int u\,dv = uv - \int v\,du$$

It is a sort of 'integration counterpart' to the product rule for differentiation.

2. $\boxed{\begin{array}{ll} u = \ln 2t & dv = dt \\ du = \frac{1}{2t}\cdot 2\,dt & v = t \\ \quad = \frac{1}{t}\,dt & \end{array}}$

$$\int \ln 2t\,dt = (\ln 2t)(t) - \int t \cdot \frac{1}{t}\,dt$$
$$= t\ln 2t - \int (1)\,dt$$
$$= t\ln 2t - t + C$$

3.

$$\int \ln(x-1)\,dx = (x-1)\ln(x-1) - \int \frac{1}{x-1}(x-1)\,dx$$
$$= (x-1)\ln(x-1) - x + C$$

$\boxed{\begin{array}{ll} u = \ln(x-1) & dv = dx \\ du = \frac{1}{x-1}\,dx & v = x-1 \end{array}}$

4. The choice for 'dv' must be something that you *know how to integrate*!

END-OF-SECTION EXERCISES:

1. $\frac{1}{2}e^{2x} - 2e^x + x + C$
3. $\ln|1 + e^x| + C$
5. $\sqrt{2e^t} + C$